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LIMITING BEHAVIORS OF LINEAR PROCESSES WITH RANDOM COEFFICIENTS BASED ON m–ANA RANDOM VARIABLES

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Abstract. In this paper, the complete convergence and complete moment convergence of linear processes with random coefficients based on m-ANA random variables are investigated. The results improve and generalise some former results in the literature. As corollaries, the Marcinkiewicz-Zygmund type and the Kolmogorov type strong law of large numbers are also established for linear processes of m-ANA random variables with random coefficients.

1. Introduction

Suppose that $\{\varepsilon_n, n \in \mathbb{Z}\}$ is a sequence of identically distributed random variables and $\{a_n, n \in \mathbb{Z}\}$ is a sequence of absolutely summable real numbers. A linear process (or moving average process of infinite order) is defined as

$$Y_t = \sum_{j=-\infty}^{\infty} a_j \varepsilon_{t-j}.$$

Linear process is one of the most essential topics in various applications such as electronic, financial mathematics and time series. Under the assumption that $\{\varepsilon_n, n \in \mathbb{Z}\}$ is a sequence of independent and identically distributed random variables, many limiting results have been established for the linear process $\{Y_t, t \geqslant 1\}$. For example, Ibragimov (1962) established the central limit theorem, Burton and Dehling (1990) obtained the results on large deviation principle, Li et al. (1992) obtained the complete convergence, and so on. It is obvious that even if $\{\varepsilon_n, n \in \mathbb{Z}\}$ is an independent and identically distributed sequence, the linear process $\{Y_t, t \geqslant 1\}$ is still dependent, which is usually called weakly dependent. Under different dependence assumptions on $\{\varepsilon_n, n \in \mathbb{Z}\}$, Zhang (1996) obtained the complete convergence result based on φ-mixing random variables; Baek et al. (2003) as well as Liang et al. (2003) obtained the complete convergence results when the linear process consists of negatively associated random variables; Budsaba et al. (2007a, 2007b) investigated the complete convergence and

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strong law of large numbers based on a sequence of ρ^- mixing random variables; Chen et al. (2007) improved the result of complete convergence in Baek et al. (2003); Chen et al. (2009) obtained the complete convergence for linear processes based on φ-mixing random variables, and so on.

The concept of complete convergence was introduced by Hsu and Robbins (1947) as follows: a sequence of random variables $\{X_n, n \geqslant 1\}$ is said to converge completely to a constant C (write $X_n \to C$ *completely*) if

$$\sum_{n=1}^{\infty} P(|X_n - C| > \varepsilon) < \infty$$

for all $\varepsilon > 0$. Thanks to the Borel-Cantelli lemma, this implies that $X_n \to C$ almost surely (a.s.). The converse is true if the $\{X_n, n \geqslant 1\}$ are independent.

The concept of complete moment convergence was first appeared in Chow (1988) as follows. Let $\{X_n, n \geqslant 1\}$ be a sequence of random variables and $a_n > 0$, $b_n > 0$, $q > 0$. If for any $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} a_n E\{b_n^{-1}|X_n| - \varepsilon\}_+^q < \infty,$$

then $\{X_n, n \geqslant 1\}$ is said to exhibit complete moment convergence. It is well known that the complete moment convergence is stronger than complete convergence. For the linear processes, there are also some results investigating the complete moment convergence, such as Kim and Ko (2008), Ko et al. (2008), Zhou (2010), and so on.

Recently, there are many scholars paying attention to the study of the properties of linear processes with random coefficients. For example, Kulik (2006) obtained the limit theorem for moving averages with random coefficients and heavy tailed noise; Saavedra et al. (2008) established the estimation of population spectrum for linear processes with random coefficients; Hosseini and Nezakati (2019) established the complete moment convergence for extended negatively dependent linear processes with random coefficients; Hosseini and Nezakati (2020) obtained the convergence rate of the Marcinkiewicz-Zygmund strong law of large numbers for extended negatively dependent linear processes with random coefficients; Lu and Wang (2022) extended the result of Hosseini and Nezakati (2019) to widely orthant dependent settings.

Now let us recall the concept of linear process with random coefficients as follows.

DEFINITION 1.1. Let $\{\varepsilon_n, n \in \mathbb{Z}\}$ and $\{A_n, n \in \mathbb{Z}\}$ be two sequences of random variables and

$$X_t = \sum_{j=-\infty}^{\infty} A_j \varepsilon_{t-j},$$

then $\{X_t, t \geqslant 1\}$ is a linear process with random coefficients.

This paper aims to study the limit properties of linear processes with random coefficients based on m-asymptotically negatively associated random variables. Now let us recall some concepts of dependent random variables as follows.

DEFINITION 1.2. A finite family of random variables $X_1,\cdots,X_n$ is said to be negatively associated (NA) if for every pair of disjoint subsets A and B of $\{1,2,\cdots,n\}$ and any real coordinatewise nondecreasing functions f_1 on $\mathbb{R}^A$ and f_2 on $\mathbb{R}^B$,

$$Cov(f_1(X_i,i\in A),f_2(X_j,j\in B))\leqslant 0,$$

whenever the covariance above exists. An infinite family of random variables is NA if every finite subfamily is NA.

Joag-Dev and Proschan (1983) proposed the above concept of NA random variables and pointed out that a number of well known multivariate distributions all possess the NA property.

DEFINITION 1.3. A sequence $\{X_n,n\geqslant 1\}$ of random variables is called ρ^*-mixing if

$$\rho^*(n)=\sup\{\rho(S,T);S,T\subset\mathbb{N},dist(S,T)\geqslant n\}\to 0$$

as $n\to\infty$, where

$$\rho(S,T)=\sup\left\{\frac{|Cov(X,Y)|}{\sqrt{Var(X)Var(Y)}}:X\in L_2(\sigma(X_i,i\in S)),Y\in L_2(\sigma(X_j,j\in T))\right\}.$$

The concept of ρ^*-mixing random variables was introduced by Bradley (1992). Some examples therein show that moving average processes, Markov chains with regular conditions, and so on are all ρ^*-mixing.

Zhang and Wang (1999) introduced the following concept of asymptotic negatively associated (ANA) or ρ^--mixing random variables, which includes NA random variables and ρ^*-mixing random variables as special cases.

DEFINITION 1.4. A sequence $\{X_n,n\geqslant 1\}$ of random variables is called ANA if

$$\rho^-(n)=\sup\{\rho^-(S,T):S,T\subset\mathbb{N},dist(S,T)\geqslant n\}\to 0$$

as $n\to\infty$, where

$$\rho^-(S,T)=0\vee\left\{\frac{Cov(f_1(X_i,i\in S),f_2(X_j,j\in T))}{\sqrt{Var(f_1(X_i,i\in S))Var(f_2(X_j,j\in T))}}:f_1,f_2\in\mathscr{C}\right\}$$

and $\mathscr{C}$ is the set of nondecreasing functions.

Wu et al. (2021) extended the concept of ANA random variables to m-ANA random variables, which is presented as follows.

DEFINITION 1.5. Let $m\geqslant 1$ be a fixed integer. A sequence $\{X_n,n\geqslant 1\}$ of random variables is said to be m-asymptotic negatively associated (m-ANA) if for any $n\geqslant 2$, and any $i_1,i_2,\cdots,i_n$ such that $|i_k-i_j|\geqslant m$ for all $1\leqslant k\neq j\leqslant n$, we have that $X_{i_1},X_{i_2},\cdots,X_{i_n}$ are ANA.

Wu et al. (2021) stated that m-ANA degenerates to ANA if $m=1$. In conclusion, the concept of m-ANA random variables is very general and maybe much more reasonable in realistic applications. Hence, the study of the limiting behavior of m-ANA random variables is of great interest. This paper will be concerned with this topic.

In this study, we mainly investigate the complete convergence and complete moment convergence of linear processes with random coefficients based on m-ANA random variables. The results improve and generalise some former results in the literature. As corollaries, the Marcinkiewicz-Zygmund type and the Kolmogorov type strong law of large numbers are also established for linear processes of m-ANA random variables with random coefficients.

The paper is organized as follows: main results of the paper are presented in Section 2, including the complete convergence, complete moment convergence, Marcinkiewicz-Zygmund type and the Kolmogorov type strong law of large numbers. Some basic properties and important lemmas are provided in Section 3. In Section 4, we give the proofs of the main results.

Throughout the paper, let C be a positive constant not depending on n, which may be different in various places. Let $a_+ = \max\{a,0\}$ and $\mathbb{Z} = \{\cdots,-2,-1,0,1,2,\cdots\}$. Denote $\log x = \ln\max(x,e)$.

2. Main results

Now we give our main results as follows.

THEOREM 2.1. *Suppose* $r \geqslant 1$, $1 \leqslant p < 2$, $rp \neq 1$, *and* $X_t = \sum_{j=-\infty}^{\infty} A_j \varepsilon_{t-j}$ *is a linear process with random coefficients, where* $\{\varepsilon_n, n \in \mathbb{Z}\}$ *is a sequence of zero mean m-ANA random variables stochastically dominated by a random variable* ε *with* $E|\varepsilon|^{rp} < \infty$, *and* $\{A_n, n \in \mathbb{Z}\}$ *is a sequence of random variables independent of* $\{\varepsilon_n, n \in \mathbb{Z}\}$ *with*

$$E\left(\sum_{j=-\infty}^{\infty} |A_j|\right)^q < \infty \tag{2.1}$$

for some $q > \max\{rp, 2p(r-1)/(2-p)\}$. *Then for any* $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| > \varepsilon n^{1/p}\right) < \infty. \tag{2.2}$$

REMARK 2.1. Chen et al. (2008) obtained the corresponding result for linear process with summable non-random coefficients based on NA random variables. Noting that if $\{\varepsilon_n, n \in \mathbb{Z}\}$ is NA and $A_j = a_j$ for each $j \in \mathbb{Z}$, then (2.1) holds trivially and thus Theorem 2.1 reduces to the result of Chen et al. (2008). Hence, Theorem 2.1 improves and extends the corresponding result of Chen et al. (2008) from linear process with non-random coefficients based on NA random variables to random coefficients based on m-ANA random variables. Hosseini and Nezakati (2020) also established the complete convergence for linear process with random coefficients based on extended negatively dependent random variables. However, the conditions $r > 1$, $1 \leqslant p < 2$, $1 < rp < 2$ are required, which result in that the conclusion $n^{-1/p}\sum_{t=1}^{n} X_t \to 0$ *completely* could not be obtained. Hence, Theorem 2.1 also improves and extends the corresponding one of Hosseini and Nezakati (2020).

By Theorem 2.1, we can obtain the following Marcinkiewicz-Zygmund type strong law of large numbers for linear processes of m-ANA random variables with random coefficients.

COROLLARY 2.1. *Suppose* $1 < p < 2$ *and* $X_t = \sum_{j=-\infty}^{\infty} A_j \varepsilon_{t-j}$ *is a linear process with random coefficients, where* $\{\varepsilon_n, n \in \mathbb{Z}\}$ *is a sequence of zero mean* m*-ANA random variables stochastically dominated by a random variable* ε *with* $E|\varepsilon|^p < \infty$, *and* $\{A_n, n \in \mathbb{Z}\}$ *is a sequence of random variables independent of* $\{\varepsilon_n, n \in \mathbb{Z}\}$ *such that (2.1) holds for some* $q > p$. *Then*

$$n^{-1/p} \sum_{t=1}^{n} X_t \to 0 \ a.s.$$

For the meaningful case $r = p = 1$, we also obtain the following result.

THEOREM 2.2. *Let* $X_t = \sum_{j=-\infty}^{\infty} A_j \varepsilon_{t-j}$ *be a linear process with random coefficients, where* $\{\varepsilon_n, n \in \mathbb{Z}\}$ *is a sequence of zero mean* m*-ANA random variables stochastically dominated by a random variable* ε *with* $E|\varepsilon|\log(1+|\varepsilon|) < \infty$, *and* $\{A_n, n \in \mathbb{Z}\}$ *is a sequence of random variables independent of* $\{\varepsilon_n, n \in \mathbb{Z}\}$ *such that (2.1) holds for some* $q > 1$. *Then for any* $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} n^{-1} P\left(\max_{1 \leqslant k \leqslant n} \left| \sum_{t=1}^{k} X_t \right| > \varepsilon n \right) < \infty. \tag{2.3}$$

REMARK 2.2. Chen et al. (2009) investigated the case $r = p = 1$ for linear process with non-random coefficients satisfying $\sum_{j=-\infty}^{\infty} |a_j|^{\vartheta} < \infty$ for some $0 < \vartheta < 1$ based on φ-mixing random variables. However, our conditions are different when dealing with the case of random coefficients. In specific, in Theorem 2.2, the moment condition $E|\varepsilon|\log(1+|\varepsilon|) < \infty$ is a little stronger but condition (2.1) is weaker than that in Chen et al. (2009).

By Theorem 2.2, we can obtain the following Kolmogorov type strong law of large numbers for linear processes of m-ANA random variables with random coefficients.

COROLLARY 2.2. *Under the conditions of Theorem 2.2, we have*

$$n^{-1} \sum_{t=1}^{k} X_t \to 0 \ a.s.$$

The following result considers the complete moment convergence for linear processes of m-ANA random variables with random coefficients.

THEOREM 2.3. *Suppose* $\theta \geqslant 1$, $r \geqslant 1$, $1 \leqslant p < 2$, *and* $X_t = \sum_{j=-\infty}^{\infty} A_j \varepsilon_{t-j}$ *is a linear process with random coefficients, where* $\{\varepsilon_n, n \in \mathbb{Z}\}$ *is a sequence of zero mean* m*-ANA random variables stochastically dominated by a random variable* ε, *and* $\{A_n, n \in \mathbb{Z}\}$ *is a sequence of random variables independent of* $\{\varepsilon_n, n \in \mathbb{Z}\}$ *such that (2.1) holds for some* $q > \max\{rp, 2p(r-1)/(2-p), \theta\}$. *If*

$$\begin{cases} E|\varepsilon|^{rp} < \infty, \ for \ \ \theta < rp, \\ E|\varepsilon|^{rp}\log(1+|\varepsilon|) < \infty, \ for \ \ \theta = rp, \\ E|\varepsilon|^{\theta} < \infty, \ for \ \ \theta > rp, \end{cases} \tag{2.4}$$

then for any $\varepsilon > 0$*,*

$$\sum_{n=1}^{\infty} n^{r-2-\theta/p} E\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| - \varepsilon n^{1/p}\right)_+^{\theta} < \infty. \tag{2.5}$$

REMARK 2.3. Let $\{\varepsilon_n, n\in\mathbb{Z}\}$ be NA and $\{A_n, n\in\mathbb{Z}\}$ be non-random, Theorem 2.3 degenerates to the corresponding result of Chen et al. (2008). However, the method used here is quite different from that of Chen et al. (2008). Hosseini and Nezakati (2019) and Lu and Wang (2020) obtained the complete moment convergence under extended negatively dependent and widely orthant dependent settings, respectively. However, only $\theta = 1$ were considered in their results.

REMARK 2.4. At the end of this section, we will show that (2.5) implies (2.2) and (2.3). To be specific, if $rp > 1$, for any $\theta < rp$, $E|\varepsilon|^{rp} < \infty$ implies from the Markov's inequality that

$$\begin{aligned}
&\sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| > \varepsilon n^{1/p}\right)\\
&= \sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| - \varepsilon n^{1/p}/2 > \varepsilon n^{1/p}/2\right)\\
&= \sum_{n=1}^{\infty} n^{r-2} P\left(\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| - \varepsilon n^{1/p}/2\right)_+ > \varepsilon n^{1/p}/2\right)\\
&\leqslant C\sum_{n=1}^{\infty} n^{r-2-\theta/p} E\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| - \varepsilon n^{1/p}\right)_+^{\theta} < \infty,
\end{aligned}$$

which verifies (2.2). If $r = p = 1$, then taking $\theta = 1$, (2.3) follows from (2.5) and by similar argument as above under the moment condition $E|\varepsilon|\log(1+|\varepsilon|) < \infty$.

3. Preliminary lemmas

In this section, we will provide some important lemmas, which are important to prove the main results of the paper. The following two lemmas can be seen in Wu et al. (2021).

LEMMA 3.1. *Increasing functions defined on disjoint subsets of an* m*-ANA sequence* $\{X_n, n\geqslant 1\}$ *with mixing coefficients* $\rho^-(s)$ *are also* m*-ANA with mixing coefficients not greater than* $\rho^-(s)$*.*

The following lemma is about the Rosenthal-type maximum inequality and Marcinkiewicz-Zygmund type maximum inequality for m-ANA random variables.

LEMMA 3.2. *Suppose that $\{X_n, n \geqslant 1\}$ is a sequence of m-ANA random variables with $EX_n = 0$, $E|X_n|^p < \infty$ for some $p > 1$. Then there exists a positive constant C depending only on m, p, and $\rho^-(\cdot)$ such that for all $n \geqslant 1$,*

$$E\left(\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} X_i\right|\right)^p \leqslant C\sum_{i=1}^{n} E|X_i|^p, \text{ for } 1<p<2 \tag{3.1}$$

and

$$E\left(\max_{1\leqslant j\leqslant n}\left|\sum_{i=1}^{j} X_i\right|\right)^p \leqslant C\left\{\sum_{i=1}^{n} E|X_i|^p + \left(\sum_{i=1}^{n} EX_i^2\right)^{p/2}\right\}, \text{ for } p\geqslant 2. \tag{3.2}$$

By Lemma 3.2, we can obtain the following inequalities, which are essential in proving the main results.

LEMMA 3.3. *Let $p \geqslant 1$, and $\{\varepsilon_n, n \in \mathbb{Z}\}$ be a sequence of m-ANA random variables with zero mean and $E|\varepsilon_n|^p < \infty$ for each n. If $\{A_n, n \in \mathbb{Z}\}$ is a sequence of random variables independent of $\{\varepsilon_n, n \in \mathbb{Z}\}$ with $E(\sum_{j=-\infty}^{\infty}|A_j|)^p < \infty$, then*

$$E\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty}\sum_{j=1-i}^{k-i} A_j\varepsilon_i\right|^p \leqslant C\sup_{j\in\mathbb{Z}}\sum_{i=1-j}^{n-j} E|\varepsilon_i|^p, \text{ if } 1\leqslant p<2 \tag{3.3}$$

and

$$E\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty}\sum_{j=1-i}^{k-i} A_j\varepsilon_i\right|^p \leqslant C\sup_{j\in\mathbb{Z}}\left\{\sum_{i=1-j}^{n-j} E|\varepsilon_i|^p + \left(\sum_{i=1-j}^{n-j} E\varepsilon_i^2\right)^{p/2}\right\}, \text{ if } p\geqslant 2. \tag{3.4}$$

Proof. We only give the proof of (3.4) since (3.3) is analogous. It follows from the Hölder's inequality and (3.2) that

$$\begin{aligned}
&E\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty}\sum_{j=1-i}^{k-i} A_j\varepsilon_i\right|^p = E\max_{1\leqslant k\leqslant n}\left|\sum_{j=-\infty}^{\infty} A_j\sum_{i=1-j}^{k-j}\varepsilon_i\right|^p \\
&\leqslant E\left(\sum_{j=-\infty}^{\infty}|A_j|\max_{1\leqslant k\leqslant n}\left|\sum_{i=1-j}^{k-j}\varepsilon_i\right|\right)^p \\
&= E\left(\sum_{j=-\infty}^{\infty}|A_j|^{1-1/p}\cdot|A_j|^{1/p}\max_{1\leqslant k\leqslant n}\left|\sum_{i=1-j}^{k-j}\varepsilon_i\right|\right)^p \\
&\leqslant E\left[\left(\sum_{j=-\infty}^{\infty}|A_j|\right)^{1-1/p}\left(\sum_{j=-\infty}^{\infty}|A_j|\max_{1\leqslant k\leqslant n}\left|\sum_{i=1-j}^{k-j}\varepsilon_i\right|^p\right)^{1/p}\right]^p \\
&= \sum_{s=1}^{\infty} E\left[\left(\sum_{j=-\infty}^{\infty}|A_j|\right)^{p-1}\left(\sum_{j=-\infty}^{\infty}|A_j|\max_{1\leqslant k\leqslant n}\left|\sum_{i=1-j}^{k-j}\varepsilon_i\right|^p\right) I\left(s-1\leqslant\sum_{j=-\infty}^{\infty}|A_j|<s\right)\right]
\end{aligned}$$

$$
\begin{aligned}
&\leqslant \sum_{s=1}^{\infty}\sum_{j=-\infty}^{\infty} s^{p-1}E|A_j|I\left(s-1\leqslant \sum_{j=-\infty}^{\infty}|A_j|<s\right)E\max_{1\leqslant k\leqslant n}\left|\sum_{i=1-j}^{k-j}\varepsilon_i\right|^p\\
&\leqslant \sum_{s=1}^{\infty}\sum_{j=-\infty}^{\infty} s^{p-1}E|A_j|I\left(s-1\leqslant \sum_{j=-\infty}^{\infty}|A_j|<s\right)\left\{\sum_{i=1-j}^{n-j}E|\varepsilon_i|^p+\left(\sum_{i=1-j}^{n-j}E\varepsilon_i^2\right)^{p/2}\right\}\\
&\leqslant C\sup_{j\in\mathbb{Z}}\left\{\sum_{i=1-j}^{n-j}E|\varepsilon_i|^p+\left(\sum_{i=1-j}^{n-j}E\varepsilon_i^2\right)^{p/2}\right\}\\
&\quad\times\left[1+2^{p-1}\sum_{s=2}^{\infty}E\left(\sum_{j=-\infty}^{\infty}|A_j|\right)^p I\left(s-1\leqslant \sum_{j=-\infty}^{\infty}|A_j|<s\right)\right]\\
&\leqslant C\sup_{j\in\mathbb{Z}}\left\{\sum_{i=1-j}^{n-j}E|\varepsilon_i|^p+\left(\sum_{i=1-j}^{n-j}E\varepsilon_i^2\right)^{p/2}\right\}.
\end{aligned}
$$

This completes the proof of the lemma. □

The next lemma provides the basic properties of stochastic domination. The first inequality can be found in Adler and Rosalsky (1987) while the second one is due to Adler et al. (1989).

LEMMA 3.4. *Let $\{\varepsilon_n, n\geqslant 1\}$ be a sequence of random variables stochastically dominated by a random variable ε, that is, $\sup_{n\geqslant 1}P(|\varepsilon_n|>t)\leqslant CP(|\varepsilon|>t)$ for all $t\geqslant 0$. Then, for all $n\geqslant 1$, $a>0$ and $b>0$, the following inequalities hold:*

$$E|\varepsilon_n|^a I(|\varepsilon_n|\leqslant b)\leqslant C\{E|\varepsilon|^a I(|\varepsilon|\leqslant b)+b^a P(|\varepsilon|>b)\},$$

$$E|\varepsilon_n|^a I(|\varepsilon_n|>b)\leqslant CE|\varepsilon|^a I(|\varepsilon|>b).$$

The last one is a general moment inequality, which was obtained by Wu et al. (2017).

LEMMA 3.5. *Let $\{Y_n, n\geqslant 1\}$ and $\{Z_n, n\geqslant 1\}$ be two sequences of random variables. Then for any $q>r>0$, $\varepsilon>0$, and $a>0$, the following inequality holds:*

$$
\begin{aligned}
&E\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=1}^{k}(Y_i+Z_i)\right|-\varepsilon a\right)_+^r\\
&\leqslant C_r\left(\varepsilon^{-q}+\frac{r}{q-r}\right)a^{r-q}E\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=1}^{k}Y_i\right|^q\right)+C_rE\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=1}^{k}Z_i\right|^r\right),
\end{aligned}
$$

where $C_r=1$ if $0<r\leqslant 1$ or $C_r=2^{r-1}$ if $r>1$.

4. Proofs of the main results

Proof of Theorem 2.1. Since $E\varepsilon_i = 0$, we can decompose ε_i for all $i \in \mathbb{Z}$ and for each $n \geqslant 1$ by

$$\varepsilon_i = \varepsilon_{ni}(1) - E\varepsilon_{ni}(1) + \varepsilon_{ni}(2) - E\varepsilon_{ni}(2),$$

where

$$\begin{aligned}
\varepsilon_{ni}(1) &= \varepsilon_i I(|\varepsilon_i| \leqslant n^{1/p}) + n^{1/p} I(\varepsilon_i > n^{1/p}) - n^{1/p} I(\varepsilon_i < -n^{1/p}),\\
\varepsilon_{ni}(2) &= \varepsilon_i - \varepsilon_{ni}(1) = (\varepsilon_i - n^{1/p}) I(\varepsilon_i > n^{1/p}) + (\varepsilon_i + n^{1/p}) I(\varepsilon_i < -n^{1/p}).
\end{aligned}$$

By Lemma 3.1, $\{\varepsilon_{ni}(1) - E\varepsilon_{ni}(1), i \in \mathbb{Z}\}$ and $\{\varepsilon_{ni}(2) - E\varepsilon_{ni}(2), i \in \mathbb{Z}\}$ are both m-ANA random variables with mean zero. Further, it is easy to see that

$$|\varepsilon_{ni}(1)| = |\varepsilon_i| I(|\varepsilon_i| \leqslant n^{1/p}) + n^{1/p} I(|\varepsilon_i| > n^{1/p}), \tag{4.1}$$

and

$$|\varepsilon_{ni}(2)| = (|\varepsilon_i| - n^{1/p}) I(|\varepsilon_i| > n^{1/p}). \tag{4.2}$$

Therefore, for each $1 \leqslant k \leqslant n$, $n \geqslant 1$, we have

$$\sum_{t=1}^{k} X_t = \sum_{t=1}^{k} \sum_{j=-\infty}^{\infty} A_j \varepsilon_{t-j} = \sum_{i=-\infty}^{\infty} \sum_{j=1-i}^{k-i} A_j \varepsilon_i = \sum_{l=1}^{2} \sum_{i=-\infty}^{\infty} \sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(l) - E\varepsilon_{ni}(l)).$$

Hence,

$$\begin{aligned}
&\sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| > \varepsilon n^{1/p}\right)\\
&\leqslant \sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty} \sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(1) - E\varepsilon_{ni}(1))\right| > \varepsilon n^{1/p}/2\right)\\
&\quad + \sum_{n=1}^{\infty} n^{r-2} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty} \sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(2) - E\varepsilon_{ni}(2))\right| > \varepsilon n^{1/p}/2\right)\\
&=: I_1 + I_2.
\end{aligned}$$

Firstly, we will prove $I_1 < \infty$. By (2.1) and Jensen's inequality, we have $E\left(\sum_{j=-\infty}^{\infty} |A_j|\right)^s < \infty$ for any $0 < s < q$. Thus, if $rp < 2$, we may also assume that $rp < q \leqslant 2$ without losing generality. With respect to the Markov's inequality, (3.3), Lemma 3.4 and (4.1), we have that if $rp < 2$,

$$\begin{aligned}
I_1 &\leqslant C \sum_{n=1}^{\infty} n^{r-2-q/p} E \max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty} \sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(1) - E\varepsilon_{ni}(1))\right|^q\\
&\leqslant C \sum_{n=1}^{\infty} n^{r-2-q/p} \sup_{j\in\mathbb{Z}} \sum_{i=1-j}^{n-j} E|\varepsilon_{ni}(1)|^q\\
&\leqslant C \sum_{n=1}^{\infty} n^{r-2-q/p} \sup_{j\in\mathbb{Z}} \sum_{i=1-j}^{n-j} \left\{E|\varepsilon_i|^q I(|\varepsilon_i| \leqslant n^{1/p}) + n^{q/p} P(|\varepsilon_i| > n^{1/p})\right\}
\end{aligned}$$

$$\begin{aligned}
&\leqslant C\sum_{n=1}^{\infty} n^{r-1-q/p}\left\{E|\varepsilon|^q I(|\varepsilon|\leqslant n^{1/p})+n^{q/p}P(|\varepsilon|>n^{1/p})\right\}\\
&= C\sum_{n=1}^{\infty} n^{r-1-q/p}\sum_{j=1}^{n} E|\varepsilon|^q I(j-1<|\varepsilon|^p\leqslant j)+C\sum_{n=1}^{\infty} n^{r-1}P(|\varepsilon|>n^{1/p})\\
&\leqslant C\sum_{j=1}^{\infty} j^{r-q/p}E|\varepsilon|^q I(j-1<|\varepsilon|^p\leqslant j)+CE|\varepsilon|^{rp}\\
&\leqslant CE|\varepsilon|^{rp}<\infty;
\end{aligned}$$

and if $rp\geqslant 2$, we have by the Markov's inequality, (3.4), (4.1), Lemma 3.4, $q>2p(r-1)/(2-p)$ and similar argument as the case $rp<2$ that

$$\begin{aligned}
I_1 &\leqslant C\sum_{n=1}^{\infty} n^{r-2-q/p}\sup_{j\in\mathbb{Z}}\left\{\sum_{i=1-j}^{n-j} E|\varepsilon_{ni}(1)|^q+\left(\sum_{i=1-j}^{n-j} E|\varepsilon_{ni}(1)|^2\right)^{q/2}\right\}\\
&\leqslant CE|\varepsilon|^{rp}+C\sum_{n=1}^{\infty} n^{r-2-q/p}\sup_{j\in\mathbb{Z}}\left\{\sum_{i=1-j}^{n-j}\left[E\varepsilon_i^2 I(|\varepsilon_i|\leqslant n^{1/p})+n^{2/p}P(|\varepsilon_i|>n^{1/p})\right]\right\}^{q/2}\\
&\leqslant C\sum_{n=1}^{\infty} n^{r-2-q/p+q/2}\left\{E\varepsilon^2 I(|\varepsilon|\leqslant n^{1/p})+n^{2/p}P(|\varepsilon|>n^{1/p})\right\}^{q/2}\\
&\leqslant C\sum_{n=1}^{\infty} n^{r-2-q/p+q/2}(E\varepsilon^2)^{q/2}<\infty.
\end{aligned}$$

Now we prove $I_2<\infty$. Using the Markov's inequality, (3.3), Lemma 3.4 and (4.2), and noting that $E\left(\sum_{j=-\infty}^{\infty}|A_j|\right)<\infty$, we obtain

$$\begin{aligned}
I_2 &\leqslant C\sum_{n=1}^{\infty} n^{r-2-1/p}E\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty}\sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(2)-E\varepsilon_{ni}(2))\right|\\
&\leqslant C\sum_{n=1}^{\infty} n^{r-2-1/p}\sup_{j\in\mathbb{Z}}\sum_{i=1-j}^{n-j} E|\varepsilon_{ni}(2)|\\
&\leqslant C\sum_{n=1}^{\infty} n^{r-1-1/p}E|\varepsilon|I(|\varepsilon|>n^{1/p})\\
&= C\sum_{n=1}^{\infty} n^{r-1-1/p}\sum_{j=n}^{\infty} E|\varepsilon|I(j<|\varepsilon|^p\leqslant j+1)\\
&= C\sum_{j=1}^{\infty} E|\varepsilon|I(j<|\varepsilon|^p\leqslant j+1)\sum_{n=1}^{j} n^{r-1-1/p}\\
&\leqslant C\sum_{j=1}^{\infty} j^{r-1/p}E|\varepsilon|I(j<|\varepsilon|^p\leqslant j+1)\\
&\leqslant CE|\varepsilon|^{rp}<\infty.
\end{aligned}$$

This completes the proof of the theorem. □

Proof of Corollary 2.1. Let $r=1$ in Theorem 2.1. It follows from (2.2) that

$$\begin{aligned}\infty &> \sum_{n=1}^{\infty} n^{-1} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| > \varepsilon n^{1/p}\right)\\ &= \sum_{s=0}^{\infty}\sum_{2^s\leqslant n<2^{s+1}} n^{-1} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| > \varepsilon n^{1/p}\right)\\ &\geqslant \frac{1}{2}\sum_{s=0}^{\infty} P\left(\max_{1\leqslant k\leqslant 2^s}\left|\sum_{t=1}^{k} X_t\right| > \varepsilon (2^{s+1})^{1/p}\right),\end{aligned}$$

which together with the Borel-Cantelli lemma implies that as $s\to\infty$,

$$\frac{1}{(2^{s+1})^{1/p}}\max_{1\leqslant k\leqslant 2^s}\left|\sum_{t=1}^{k} X_t\right| \to 0 \text{ a.s.}$$

On the other hand, for any fixed n, there always exists a nonegative integer s such that $2^s\leqslant n<2^{s+1}$. Hence, it follows that as $n\to\infty$,

$$n^{-1/p}\left|\sum_{t=1}^{n} X_t\right| \leqslant \frac{1}{(2^s)^{1/p}}\max_{1\leqslant k\leqslant 2^{s+1}}\left|\sum_{t=1}^{k} X_t\right| \to 0 \text{ a.s.}$$

This completes the proof. □

Proof of Theorem 2.2. The proof is similar to that of Theorem 2.1. Denote for all $i\in\mathbb{Z}$ and for each $n\geqslant 1$ that

$$\begin{aligned}\varepsilon_{ni}(3) &= \varepsilon_i I(|\varepsilon_i|\leqslant n) + nI(\varepsilon_i>n) - nI(\varepsilon_i<-n),\\ \varepsilon_{ni}(4) &= \varepsilon_i - \varepsilon_{ni}(3) = (\varepsilon_i-n)I(\varepsilon_i>n) + (\varepsilon_i+n)I(\varepsilon_i<-n).\end{aligned}$$

By Lemma 3.1 again, $\{\varepsilon_{ni}(3)-E\varepsilon_{ni}(3), i\in\mathbb{Z}\}$ and $\{\varepsilon_{ni}(4)-E\varepsilon_{ni}(4), i\in\mathbb{Z}\}$ are also m-ANA random variables with mean zero. Hence, we can obtain that

$$\begin{aligned}&\sum_{n=1}^{\infty} n^{-1} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| > \varepsilon n\right)\\ \leqslant\ &\sum_{n=1}^{\infty} n^{-1} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty}\sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(3)-E\varepsilon_{ni}(3))\right| > \varepsilon n/2\right)\\ &\quad+\sum_{n=1}^{\infty} n^{-1} P\left(\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty}\sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(4)-E\varepsilon_{ni}(4))\right| > \varepsilon n/2\right)\\ =:\ &I_3+I_4.\end{aligned}$$

We may also assume without loss of generality that $1 < q \leqslant 2$. Using the Markov's inequality, (3.3), and Lemma 3.4, we have that

$$
\begin{aligned}
I_3 &\leqslant C \sum_{n=1}^{\infty} n^{-1-q} E \max_{1 \leqslant k \leqslant n} \left| \sum_{i=-\infty}^{\infty} \sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(3) - E\varepsilon_{ni}(3)) \right|^q \\
&\leqslant C \sum_{n=1}^{\infty} n^{-1-q} \sup_{j \in \mathbb{Z}} \sum_{i=1-j}^{n-j} E|\varepsilon_{ni}(3)|^q \\
&\leqslant C \sum_{n=1}^{\infty} n^{-1-q} \sup_{j \in \mathbb{Z}} \sum_{i=1-j}^{n-j} \{E|\varepsilon_i|^q I(|\varepsilon_i| \leqslant n) + n^q P(|\varepsilon_i| > n)\} \\
&\leqslant C \sum_{n=1}^{\infty} n^{-q} \{E|\varepsilon|^q I(|\varepsilon| \leqslant n) + n^q P(|\varepsilon| > n)\} \\
&= C \sum_{n=1}^{\infty} n^{-q} \sum_{j=1}^{n} E|\varepsilon|^q I(j-1 < |\varepsilon| \leqslant j) + C \sum_{n=1}^{\infty} P(|\varepsilon| > n) \\
&\leqslant C \sum_{j=1}^{\infty} j^{1-q} E|\varepsilon|^q I(j-1 < |\varepsilon| \leqslant j) + CE|\varepsilon| \\
&\leqslant CE|\varepsilon| < \infty.
\end{aligned}
$$

Now we prove $I_4 < \infty$. By the Markov's inequality, (3.3), and Lemma 3.4, we obtain

$$
\begin{aligned}
I_4 &\leqslant C \sum_{n=1}^{\infty} n^{-2} E \max_{1 \leqslant k \leqslant n} \left| \sum_{i=-\infty}^{\infty} \sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(4) - E\varepsilon_{ni}(4)) \right| \\
&\leqslant C \sum_{n=1}^{\infty} n^{-2} \sup_{j \in \mathbb{Z}} \sum_{i=1-j}^{n-j} E|\varepsilon_{ni}(4)| \\
&\leqslant C \sum_{n=1}^{\infty} n^{-1} E|\varepsilon| I(|\varepsilon| > n) \\
&= C \sum_{n=1}^{\infty} n^{-1} \sum_{j=n}^{\infty} E|\varepsilon| I(j < |\varepsilon| \leqslant j+1) \\
&= C \sum_{j=1}^{\infty} E|\varepsilon| I(j < |\varepsilon| \leqslant j+1) \sum_{n=1}^{j} n^{-1} \\
&\leqslant C \sum_{j=1}^{\infty} \log j E|\varepsilon| I(j < |\varepsilon|^p \leqslant j+1) \\
&\leqslant CE|\varepsilon| \log(1 + |\varepsilon|) < \infty.
\end{aligned}
$$

This completes the proof of the theorem. □

Proof of Corollary 2.2. Taking $p = 1$ in the proof of Corollary 2.1, we can finish the proof of Corollary 2.2 immediately. □

Proof of Theorem 2.3. We use the same notations as those in the proof of Theorem 2.1. It follows from Lemma 3.5 that

$$\begin{aligned}
&\sum_{n=1}^{\infty} n^{r-2-\theta/p} E\left(\max_{1\leqslant k\leqslant n}\left|\sum_{t=1}^{k} X_t\right| - \varepsilon n^{1/p}\right)_+^{\theta} \\
\leqslant\ & C\sum_{n=1}^{\infty} n^{r-2-q/p} E\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty}\sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(1)-E\varepsilon_{ni}(1))\right|^q \\
&+C\sum_{n=1}^{\infty} n^{r-2-\theta/p} E\max_{1\leqslant k\leqslant n}\left|\sum_{i=-\infty}^{\infty}\sum_{j=1-i}^{k-i} A_j(\varepsilon_{ni}(2)-E\varepsilon_{ni}(2))\right|^{\theta} \\
=:\ & I_5+I_6.
\end{aligned}$$

Similar to the proof of $I_1<\infty$ in Theorem 2.1, we obtain $I_5<\infty$, where the two cases $rp<2$ and $rp\geqslant 2$ should be replaced by $\max\{rp,\theta\}<2$ and $\max\{rp,\theta\}\geqslant 2$, respectively. The proof of $I_6<\infty$ is conducted under the following two cases.

Case 1. $\theta\leqslant 2$. We have by (3.3), (4.2), Lemma 3.4 and (2.4) that

$$\begin{aligned}
I_6 &\leqslant C\sum_{n=1}^{\infty} n^{r-2-\theta/p}\sup_{j\in\mathbb{Z}}\sum_{i=1-j}^{n-j} E|\varepsilon_{ni}(2)|^{\theta} \\
&\leqslant C\sum_{n=1}^{\infty} n^{r-1-\theta/p} E|\varepsilon|^{\theta} I(|\varepsilon|>n^{1/p}) \\
&= C\sum_{j=1}^{\infty} E|\varepsilon|^{\theta} I(j<|\varepsilon|^p\leqslant j+1)\sum_{n=1}^{j} n^{r-1-\theta/p} \\
&\leqslant \begin{cases} C\sum_{j=1}^{\infty} j^{r-\theta/p}E|\varepsilon|^{\theta}I(j<|\varepsilon|^p\leqslant j+1), \text{ for } \theta<rp, \\ C\sum_{j=1}^{\infty}\log jE|\varepsilon|^{\theta}I(j<|\varepsilon|^p\leqslant j+1), \text{ for } \theta=rp, \\ C\sum_{j=1}^{\infty}E|\varepsilon|^{\theta}I(j<|\varepsilon|^p\leqslant j+1), \text{ for } \theta>rp \end{cases} \\
&\leqslant \begin{cases} E|\varepsilon|^{rp}<\infty, \text{ for } \theta<rp, \\ E|\varepsilon|^{rp}\log(1+|\varepsilon|)<\infty, \text{ for } \theta=rp, \\ E|\varepsilon|^{\theta}<\infty, \text{ for } \theta>rp \end{cases} \\
&<\infty.
\end{aligned}$$

Case 2. $\theta>2$. We have by (3.4), (4.2) and Lemma 3.4 that

$$\begin{aligned}
I_6 &\leqslant C\sum_{n=1}^{\infty} n^{r-2-\theta/p}\sup_{j\in\mathbb{Z}}\left\{\sum_{i=1-j}^{n-j}E|\varepsilon_{ni}(2)|^{\theta}+\left(\sum_{i=1-j}^{n-j}E|\varepsilon_{ni}(2)|^2\right)^{\theta/2}\right\} \\
&\leqslant C\sum_{n=1}^{\infty} n^{r-1-\theta/p}E|\varepsilon|^{\theta}I(|\varepsilon|>n^{1/p})+C\sum_{n=1}^{\infty} n^{r-2-\theta/p}\left[nE\varepsilon^2 I(|\varepsilon|>n^{1/p})\right]^{\theta/2} \\
&=: I_6'+I_6''.
\end{aligned}$$

Similar to the argument of Case 1, we obtain $I_6' < \infty$. So in order to complete the proof, it is enough to show $I_6'' < \infty$. Actually, if $\theta > rp$, we have

$$r-2+\theta/2-\theta^2/2p = r-2+\theta(1-\theta/p)/2 < r-2+(1-\theta/p) < -1,$$

and thus

$$I_6'' \leqslant C\sum_{n=1}^{\infty} n^{r-2-\theta/p}\left[n^{1+(2-\theta)/p}E|\varepsilon|^{\theta}I(|\varepsilon|>n^{1/p})\right]^{\theta/2} \leqslant C\sum_{n=1}^{\infty} n^{r-2+\theta/2-\theta^2/2p} < \infty;$$

and if $\theta \leqslant rp$ (under this case, it always holds $r > 1$), we also have

$$\begin{aligned} I_6'' &\leqslant C\sum_{n=1}^{\infty} n^{r-2-\theta/p}\left[n^{1+(2-rp)/p}E|\varepsilon|^{rp}I(|\varepsilon|>n^{1/p})\right]^{\theta/2} \\ &\leqslant C\sum_{n=1}^{\infty} n^{-1-(r-1)(\theta/2-1)} < \infty. \end{aligned}$$

Hence, the proof is completed. □

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