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NONEXISTENCE OF GLOBAL SOLUTIONS OF A QUASILINEAR HYPERBOLIC EQUATION

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Abstract. In this work, the nonexistence of the global solutions to a class of initial boundary value problems with dissipative terms in the boundary conditions is considered for a quasilinear hyperbolic equation. The nonexistence proof is achieved by the use of a lemma due to O. Ladyzhenskaya and V. K. Kalantarov and by the usage of the so called concavity method. In this method one writes down a functional which reflects the properties of dissipative boundary conditions and represents the norm of the solution in some sense, then proves that this functional satisfies the hypotheses of Ladyzhenskaya-Kalantarov lemma. Hence from the conclusion of the lemma one concludes that in finite time t_2, this functional and hence the norm of the solution blows up.

1. Introduction

For two decades the needs of the contemporary technology, have stimulated the interest in blowing solutions and conditions for which this blowing solutions arise. Fortunately recent achievements in the field of differential equations enabled us to correspond these needs. In this context, initial-boundary value problems written for hyperbolic quasilinear partial differential equations emerged with several applications to the mechanics and engineering sciences. Natures of the solutions to these equations have been investigated by several means.

The nonexistence of global solutions of quasilinear hyperbolic equations with no dissipative terms in the boundary conditions are investigated by H. A. Levine [3], J. L. Lions [4], O. A. Ladyzhenskaya and V. K. Kalantarov [2], and R. T. Glassey [1].

However the methods used for the investigation of the initial-boundary value problems with no dissipative terms in the boundary conditions are quite insufficient to deal with the problems with the dissipative terms in the boundary conditions. The tool used in this work is a Lemma due to O. A. Ladyzhenskaya and V. K. Kalantarov. Part (b) of the Lemma was introduced also by H. A. Levine in [3]. From now on we'll call it the LK Lemma. The most crucial point in the application of this tool is to find a functional that represents the dissipation on the boundary and satisfies the conditions of the LK Lemma. This method is known as the "generalized convexity" method.

The initial-boundary value problem discussed in this paper is a problem written for a quasilinear hyperbolic equation and contains a dissipative term in the boundary conditions.

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Let us begin by stating LK Lemma [2] without proof.

LEMMA. *If a function*

$$\psi(t) \in C^2, \quad \psi(t) \geqslant 0$$

satisfies the inequality

$$\psi''(t)\psi(t) - (1+\gamma)[\psi'(t)]^2 \geqslant -2C_1\psi(t)\psi'(t) - C_2\psi(t)^2 \tag{1}$$

for some real numbers $\gamma > 0,\ C_1,\ C_2 \geqslant 0$*, then the following hold:*

a) If

$$\psi(0) > 0,\ \ \psi'(0) > -\gamma_2\gamma^{-1}\psi(0),\ \ C_1 + C_2 > 0 \tag{2}$$

where

$$\gamma_1 = -C_1 + \sqrt{C_1^2 + \gamma C_2},\ \ \gamma_2 = -C_1 - \sqrt{C_1^2 + \gamma C_2}, \tag{3}$$

then for the real number,

$$t_2 = \frac{1}{2\sqrt{C_1^2 + C_2^2}} \ln \frac{\gamma_1\psi(0) + \gamma\psi'(0)}{\gamma_2\psi(0) + \gamma\psi'(0)} \tag{4}$$

there exists a positive real number $t_1 < t_2$ *such that as* $t \longrightarrow t_1$

$$\psi(t) \longrightarrow +\infty \tag{5}$$

b) If $\psi(0) > 0,\ \psi'(0) > 0$ *and* $C_1 = C_2 = 0$ *then for the real number* $t_2 = \psi(0)/(\gamma\psi'(0))$*, there exists a positive real number* $t_1 \leqslant t_2$ *such that as* $t \longrightarrow t_1$

$$\psi(t) \longrightarrow +\infty \tag{6}$$

2. The IBV problem

Let us consider the initial-boundary value problem

$$u_{tt} + \Delta^2 u = f(u) \quad (t,x) \in (0,T)\times\Omega \tag{7}$$

$$u = 0,\ \ \Delta u + \alpha(x)\frac{\partial u_t}{\partial \nu} = 0\ \ (t,x) \in (0,T)\times\Gamma \ \text{ with }\ \alpha(x) \geqslant 0\ \ x \in \Gamma \tag{8}$$

$$u(0,x) = u_0(x),\ \ u_t(0,x) = u_1(x)\ \ x \in \Omega \tag{9}$$

where Ω is a bounded domain in R^n with a sufficiently smooth boundary Γ, $T > 0$ is an arbitrary real number, ν is the outward normal to the boundary, and $\alpha(x)$ is a smooth nonnegative function given on the boundary of the domain Ω.

Let $f(u)$ with the primitive

$$\mathscr{F}(u) = \int_0^u f(\xi)d\xi \tag{10}$$

satisfy the inequality

$$f(u)\cdot u \geqslant 2(2\gamma+1)\mathscr{F}(u) - C_0 \tag{11}$$

for some real numbers $\gamma > 0,\ C_0 > 0$ and for all $u \in R$. Then we can prove the following theorem about the nonexistence of global solutions of the initial-boundary value problem (7)–(9):

THEOREM. *Suppose problem (7-9) has a global classical solution. Let* $u_0(x)$ *and* $u_1(x)$ *be smooth functions such that*

$$\int_\Omega u_0^2 dx > 0,$$

$$2(2\gamma+1)\left(\int_\Omega \mathscr{F}(u_0)dx - \frac{1}{2}\int_\Omega u_1^2 dx - \frac{1}{2}\int_\Omega (\Delta u_0)^2 dx\right) - C_0|\Omega| \geqslant 0 \tag{12}$$

$$2\int_\Omega u_0 u_1 dx \; >$$

$$2\gamma^{-1}(1+\gamma)\left[\int_\Omega u_0^2 dx + \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx\right] - \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx. \tag{13}$$

Let

$$\gamma_1 = 0, \; \gamma_2 = -2(1+\gamma), \tag{14}$$

$$A = \int_\Omega u_0^2 dx + \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \tag{15}$$

$$B = 2\gamma\int_\Omega u_0 u_1 dx + \gamma\int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \tag{16}$$

and

$$t_2 = \frac{1}{2(1+\gamma)}\ln\left(\frac{B}{\gamma_2 A + B}\right). \tag{17}$$

Then there exists a real number $t_1 < t_2$ *such that*

$$\lim_{t\to t_1}\left\{\int_\Omega u^2 dx + \int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt\right\} = +\infty. \tag{18}$$

Proof. To prove the theorem, it suffices to show that the function

$$\psi(t) = \int_\Omega u^2 dx + \int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt + \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \tag{19}$$

satisfies the hypotheses of the Ladyzhenskaya-Kalantarov Lemma. To achieve this goal let us observe

$$2\int_0^t\int_\Gamma \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu}dxdt = \int_0^t\int_\Gamma \alpha(x)\frac{\partial}{\partial t}\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt$$

$$= \int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dx - \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx.$$

Hence

$$\int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dx = 2\int_0^t\int_\Gamma \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu}dxdt + \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx.$$

Let us compute the derivatives $\psi'(t)$ and $\psi''(t)$. Thus one has

$$\psi'(t) = 2\int_{\Omega} uu_t dx + 2\int_0^t \int_{\Gamma} \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu} dxdt + \int_{\Gamma} \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \tag{20}$$

and

$$\psi''(t) = 2\int_{\Omega}(u_t)^2 dx + 2\int_{\Omega} u_{tt} u dx + 2\int_{\Gamma} \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu} dx. \tag{21}$$

It is clear that using (7) one can write

$$\int_{\Omega} u_{tt} u dx = \int_{\Omega} f(u) u dx - \int_{\Omega} (\Delta^2 u) u dx. \tag{22}$$

If the inequality (11) is satisfied by $f(u)$, from the boundary conditions of the problem one obtains,

$$\begin{aligned}\psi''(t) &= 2\int_{\Omega}(u_t)^2 dx + 2\int_{\Omega} f(u) u dx - 2\int_{\Omega} \Delta^2 u u dx + 2\int_{\Gamma} \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu} dx \\ &= 2\int_{\Omega}(u_t)^2 dx + 2\int_{\Omega} f(u) u dx - 2\int_{\Omega}(\Delta u)^2 dx \\ &\geqslant 2\int_{\Omega}(u_t)^2 dx + 4(2\gamma+1)\int_{\Omega}\mathscr{F}(u) dx - 2C_0|\Omega| - 2\int_{\Omega}(\Delta u)^2 dx.\end{aligned} \tag{23}$$

In the above, to replace the term $\int_{\Omega} \Delta^2 u u dx$ the Green's identity

$$\int_{\Omega}(u\,\Delta v - v\,\Delta u) dx = \int_{\Gamma}\left(u\frac{\partial v}{\partial \nu} - v\frac{\partial u}{\partial \nu}\right) dx$$

is used.

Multiplying both sides of the Equation (7) by u_t and integrating over the domain Ω one has,

$$\frac{1}{2}\frac{d}{dt}\int_{\Omega}(u_t)^2 dx + \frac{1}{2}\frac{d}{dt}\int_{\Omega}(\Delta u)^2 dx + \int_{\Gamma}\alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dx - \frac{d}{dt}\int_{\Omega}\mathscr{F}(u) dx = 0. \tag{24}$$

If we integrate both sides of the Equation (24) over the interval $[0,t]$ with respect to t, with the definition of $E(t)$ given below one has

$$\begin{aligned}E(t) &= \frac{1}{2}\int_{\Omega}(u_t)^2 dx + \frac{1}{2}\int_{\Omega}(\Delta u)^2 dx - \int_{\Omega}\mathscr{F}(u) dx \\ &= E(0) - \int_0^t \int_{\Gamma}\alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt.\end{aligned} \tag{25}$$

Hence using the function $E(t)$, a lower bound for $\psi''(t)$ can be found:

$$\begin{aligned}\psi''(t) &\geqslant 4(2\gamma+1)\left[\int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt - E(0)\right]\\ &\quad + 4\gamma\int_\Omega (\Delta u)^2 dx + 4(\gamma+1)\int_\Omega (u_t)^2 dx - 2C_0|\Omega|\\ &= 4\gamma\int_\Omega (\Delta u)^2 dx + 4(\gamma+1)\left[\int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt + \int_\Omega (u_t)^2 dx\right]\\ &\quad + 4\gamma\int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt - 4(2\gamma+1)E(0) - 2C_0|\Omega|. \end{aligned} \tag{26}$$

Hence one obtains,

$$\begin{aligned}\psi''(t) \geqslant 4(\gamma+1)&\left[\int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt + \int_\Omega (u_t)^2 dx\right]\\ &-4(2\gamma+1)E(0) - 2C_0|\Omega|. \end{aligned} \tag{27}$$

If the conditions (12) and the definition of $E(t)$ in (25) are used we also obtain,

$$\psi''(t) \geqslant 4(\gamma+1)\left[\int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt + \int_\Omega (u_t)^2 dx\right]. \tag{28}$$

Hence $\psi''(t) \geqslant 0$ for all $t \geqslant 0$ and by assumption (13) we have

$$\psi'(0) = 2\int_\Omega u_0 u_1 dx + \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \geqslant 0.$$

Therefore $\psi'(t) \geqslant 0$ for all $t \geqslant 0$.

Now we search for a lower bound for the functional

$$\chi(t) = \psi''(t)\psi(t) - (1+\gamma)[\psi'(t)]^2. \tag{29}$$

If we use the equivalents in (19), (20) and (28) for $\psi(t)$ and its derivatives, we have

$$\begin{aligned}\chi(t) \geqslant 4(\gamma+1)\Bigg\{&\left[\int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt + \int_\Omega (u_t)^2 dx\right]\\ &\times\left[\int_\Omega u^2 dx + \int_0^t\int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt + \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx\right]\\ &-\left[\int_\Omega u u_t dx + \int_0^t\int_\Gamma \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu} dxdt\right]^2\\ &-\left[\int_\Omega u u_t dx + \int_0^t\int_\Gamma \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu} dxdt\right]\times\int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \end{aligned}$$

$$
\begin{aligned}
&-\left[\frac{1}{2}\int_{\Gamma}\alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx\right]^2\Bigg\}\\
&=4(\gamma+1)\Bigg\{\left[\int_0^t\int_{\Gamma}\alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt+\int_{\Omega}(u_t)^2dx\right]\\
&\quad\times\left[\int_{\Omega}u^2dx+\int_0^t\int_{\Gamma}\alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt+\int_{\Gamma}\alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx\right]\\
&\quad-\left[\int_{\Omega}uu_tdx+\int_0^t\int_{\Gamma}\alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu}dxdt\right]^2\\
&\quad-\frac{1}{2}\int_{\Gamma}\alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx\;\;\psi(t)'\Bigg\}. \qquad (30)
\end{aligned}
$$

We are going to show that the sum of the two terms appear in the first three lines of the inequality (30) above is nonnegative.

According to the Cauchy-Schwarz inequality

$$
\int |u\cdot v|dx \leqslant \left(\int u^2dx\right)^{1/2}\cdot\left(\int v^2dx\right)^{1/2} \qquad (31)
$$

we have

$$
\int u\cdot u_tdx \leqslant \left(\int u^2dx\right)^{1/2}\cdot\left(\int u_t^2dx\right)^{1/2},
$$

and

$$
\int_0^t\int_{\Gamma}\alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu}dxdt \leqslant
$$

$$
\left[\int_0^t\int_{\Gamma}\left(\alpha(x)^{\frac{1}{2}}\frac{\partial u}{\partial \nu}\right)^2 dx\,dt\right]^{1/2}\cdot\left[\int_0^t\int_{\Gamma}\left(\alpha(x)^{\frac{1}{2}}\frac{\partial u_t}{\partial \nu}\right)^2 dx\,dt\right]^{1/2}.
$$

Adding the two sides of the above two inequality and taking the squares of the both sides of the resulting inequality one has

$$
\begin{aligned}
&\left[\int_{\Omega}u\cdot u_tdx+\int_0^t\int_{\Gamma}\alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu}dxdt\right]^2\leqslant\Bigg\{\left(\int_{\Omega}u^2dx\right)^{1/2}\cdot\left(\int_{\Omega}u_t^2dx\right)^{1/2}\\
&\quad+\left[\int_0^t\int_{\Gamma}\left(\alpha(x)^{\frac{1}{2}}\frac{\partial u}{\partial \nu}\right)^2 dx\,dt\right]^{1/2}\cdot\left[\int_0^t\int_{\Gamma}\left(\alpha(x)^{\frac{1}{2}}\frac{\partial u_t}{\partial \nu}\right)^2 dx\,dt\right]^{1/2}\Bigg\}^2\\
&\leqslant\left[\int_{\Omega}u^2dx+\int_0^t\int_{\Gamma}\alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dx\,dt\right]\times\left[\int_{\Omega}u_t^2dx+\int_0^t\int_{\Gamma}\alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dx\,dt\right]
\end{aligned}
$$

where the last part of the inequality above is obtaind using the inequality

$$
(a_1b_1+a_2b_2)^2\leqslant\left(a_1^2+a_2^2\right)\left(b_1^2+b_2^2\right)
$$

in real numbers.

Now we are ready to make an estimation for the sum of the two terms which appear in the first three lines of inequality (30) above:

$$\begin{aligned}
& \left[\int_0^t \int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt + \int_\Omega (u_t)^2 dx\right] \\
& \times \left[\int_\Omega u^2 dx + \int_0^t \int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt + \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx\right] \\
& - \left[\int_\Omega u u_t dx + \int_0^t \int_\Gamma \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu} dxdt\right]^2 \\
= \quad & \left[\int_0^t \int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt + \int_\Omega (u_t)^2 dx\right] \times \left[\int_\Omega u^2 dx + \int_0^t \int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt\right] \\
& - \left[\int_\Omega u u_t dx + \int_0^t \int_\Gamma \alpha(x)\frac{\partial u}{\partial \nu}\frac{\partial u_t}{\partial \nu} dxdt\right]^2 \\
& + \left[\int_0^t \int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt + \int_\Omega (u_t)^2 dx\right] \times \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \\
\geqslant \quad & \left[\int_0^t \int_\Gamma \alpha(x)\left(\frac{\partial u_t}{\partial \nu}\right)^2 dxdt + \int_\Omega (u_t)^2 dx\right] \times \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \\
\geqslant \quad & 0
\end{aligned}$$

Hence one has

$$\chi(t) \geqslant -2(\gamma+1)\int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx \;\; \psi'(t) \geqslant$$

$$-2(\gamma+1)\left[\int_0^t \int_\Gamma \alpha(x)\left(\frac{\partial u}{\partial \nu}\right)^2 dxdt + \int_\Omega u^2 dx + \int_\Gamma \alpha(x)\left(\frac{\partial u_0}{\partial \nu}\right)^2 dx\right]\psi'(t)$$

$$= -2(\gamma+1)\psi(t)\psi'(t). \tag{32}$$

Therefore finally for $\chi(t)$ one obtains the lower bound

$$\chi(t) = \psi''(t)\psi(t) - (1+\gamma)[\psi'(t)]^2 \geqslant -2(\gamma+1)\psi(t)\psi'(t). \tag{33}$$

Hence we see that the hypotheses of the Lemma are fulfilled with $C_1 = 1+\gamma$, $C_2 = 0$, and the conclusion of the Lemma gives us (18) and the proof of the theorem is complete. □

3. An Example

As an example to the problem above, let us consider the initial boundary value problem

$$u_{tt} + u_{xxxx} = u^3 \quad (t,x) \in (0,T) \times [0,1] \tag{34}$$

$$u = 0, \;\; u_{xx} + \frac{\partial u_t}{\partial x} = 0 \;\; t \in (0,T), \;\; x = 0,1 \tag{35}$$

$$u(0,x) = u_0(x), \;\; u_t(0,x) = u_1(x) \;\; x \in [0,1] \tag{36}$$

given in one space dimension, where $\Omega \subset R$ is the interval $[0,1]$.

In the above

$$f(u) = u^3, \;\; \mathscr{F}(u) = \frac{1}{4}u^4. \tag{37}$$

Hence for the real numbers $\gamma = \frac{1}{2}$, $C_0 = 1$ and for all $u \in R^1$, the inequality (11) is satisfied with $C_1 = 3/2$ and $C_2 = 0$.

Let $u_0(x)$ and $u_1(x)$ be two functions such that

$$\int_0^1 u_0^2 dx > 0, \;\; \int_\Omega u_0^4 dx - 2\int_0^1 u_1^2 dx - 2\int_0^1 \left(\frac{d^2u_0}{dx^2}\right) dx - 1 \geqslant 0 \tag{38}$$

$$2\int_0^1 u_0 u_1 dx > 6\int_0^1 u_0^2 dx. \tag{39}$$

For example the functions $u_0 = 20\sin \pi x$, $u_1 = 100$ satisfy these conditions.

Hence, for the nonexistence of the global solutions of the initial-boundary value problem (34)-(36), the Theorem supplies a sufficient condition. In this example

$$\gamma_1 = 0, \; \gamma_2 = -3, \tag{40}$$

$$t_2 = \frac{1}{3} \times \ln \left[\frac{\int_0^1 u_0 u_1 dx}{-3\int_0^1 u_0^2 dx + \int_0^1 u_0 u_1 dx}\right] \approx 0.212406, \tag{41}$$

and there exists a real number $t_1 < 0.212406$ such that

$$\lim_{t \to t_1} \int_0^1 u^2 dx = +\infty. \tag{42}$$

There is a numerical evidence that

$$t_1 \approx 0.097\,. \tag{43}$$

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