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FIXED POINTS SET FUNCTION OF NONEXPANSIVE RANDOM MAPPING ON METRIC SPACES

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Abstract. We prove that the set of random fixed points of a nonexpansive random map on a uniformly convex complete separable metric space is nonempty. We also show that the fixed point set function of a nonexpansive random map is closed and convex valued.

1. Introduction

The last fifty years have seen a dramatic expansion in the theory of random operators. Random fixed point theorems are of fundamental importance in probabilistic functional analysis. Random techniques have become crucial in diverse areas, from pure mathematics to applied sciences including biology, physics, chemistry, engineering, and of course famously, random methods have revolutionised the financial markets. Recently many authors [2, 4, 5, 9, 13, 14, 15, 20, 21, 22, 23] had studied the existence and various applications of random fixed point theorems in Banach spaces. Takahashi [19] introduced the notion of convexity in metric spaces. Subsequently Guay, Singh and Whitfield [11], Naimpally, Singh and Whitfield [12], Beg et al. [3, 6], Ciric [10], Shimiziu and Takahashi [16] and many other authors have studied fixed point theorems on convex metric spaces. Recently Shimizu and Takahashi [17] and Beg [1] introduced the concept of uniform convexity in convex metric spaces and studied its properties. The aim of this paper is to prove the existence of random fixed point for nonexpansive random mapping defined on uniformly convex complete separable metric spaces. We also proved that the fixed point set function of a nonexpansive random map is closed and convex valued.

2. Preliminaries

Let (X,d) be a complete separable metric space and let (Ω,Σ) be a measurable space (i.e., Σ is a sigma algebra of subsets of Ω). A function $\xi:\Omega\to X$ is said to be Σ*-measurable* if for any open subset B of X, $\xi^{-1}(B)\in\Sigma$. A mapping $f:\Omega\times X\to X$ is said to be a *random map* if and only if for each fixed $x\in X$,

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the mapping $f(.,x) : \Omega \to X$ is measurable. The random map $f : \Omega \times X \to X$ is *continuous* if for each $\omega \in \Omega$, the mapping $f(\omega,.) : X \to X$ is continuous. A random mapping $f : \Omega \times X \to X$ is said to be *nonexpansive* (*asymptotically regular*) if $d(f(\omega,x),f(\omega,y)) \leqslant d(x,y)$ ($\lim_{n\to\infty} d\left(f^n(\omega,x), f^{n+1}(\omega,x)\right) = 0$) for every $x,y \in X$. A measurable mapping $\xi : \Omega \to X$ is a *random fixed point* of the random map $f : \Omega \times X \to X$ if and only if $f(\omega,\xi(\omega)) = \xi(\omega)$ for each $\omega \in \Omega$. We denote by $M(\Omega,X)$ and $RF(f)$, the set of measurable mappings from Ω into X and set of of random fixed points of the random map f respectively. For further details we refer to [4, 5, 8, 9].

DEFINITION 2.1. Let X be a metric space and $I = [0,1]$. A mapping $W : X \times X \times I \to X$ is said to be a convex structure on X if for each $(x,y,\lambda) \in X \times X \times I$ and $u \in X$,

$$d(u,W(x,y,\lambda)) \leqslant \lambda d(u,x) + (1-\lambda)d(u,y). \tag{1}$$

Metric space X together with the convex structure W is called a *convex metric space*. Obviously, $W(x,x,\lambda) = x$.

Let X be a convex metric space. A nonempty subset $K \subset X$ is said to be convex if $W(x,y,\lambda) \in K$ whenever $(x,y,\lambda) \in K \times K \times I$. Takahashi [19] has shown that open spheres $B(x,r) = \{y \in X : d(x,y) < r\}$ and closed spheres $B[x,r] = \{y \in X : d(x,y) \leqslant r\}$ are convex. Also, if $\{K_\alpha : \alpha \in A\}$ is a family of convex subsets of X, then $\cap\{K_\alpha = \alpha \in A\}$ is convex. For $A \subset X$, we denote by $ClCo(A)$ the intersection of all closed convex sets containing A.

All normed spaces and their convex subsets are convex metric spaces. But there are many examples of convex metric spaces which are not embedded in any normed space (see Takahashi [19]).

DEFINITION 2.2. A convex metric space X is said to have *property* (B) if:

$$d(W(x,x_1,t),\ \ W(x,x_2,t)) \leqslant (1-t)d(x_1,x_2)$$

for all $x,x_1,x_2 \in X$ and $t \in (0,1)$.

Property (B) is a convex metric space analogue of the condition (I) for starshaped metric space of Guay, Singh and Whitfield [11, Definition 3.2].

DEFINITION 2.3. A convex metric space X is said to be *uniformly convex* if for any $\epsilon > 0$, there exists $\alpha = \alpha(\epsilon) > 0$ such that for all $r > 0$ and $x,y,z \in X$ with $d(z,x) \leqslant r,\ d(z,y) \leqslant r$ and $d(x,y) \geqslant r\epsilon$,

$$d\left(z, W\left(x,y,\frac{1}{2}\right)\right) \leqslant r(1-\alpha(\epsilon)) < r. \tag{2}$$

REMARK 2.4. (i) (Beg, [1].) The function α is increasing on $[0,2r]$ and continuous on $[0,2r)$. Also $\alpha(0) = 0$ and $\alpha(2r) = 1$.

(ii) If (x_n) and (y_n) are sequences in a uniformly convex metric space X such that for some $z \in X$, $\lim_{n\to\infty} d(z,x_n) = \lim_{n\to\infty} d(z,y_n) = r$, and $\lim_{n\to\infty} d(z,W(x_n,y_n,s)) = r$, then $\lim_{n\to\infty} d(x_n,y_n) = 0$.

(iii) Let X be an uniformly convex metric space. If $d(x,z) = r_1$, $d(y,z) = r_2$ and $d(x,y) = r_1 + r_2$ then $z = W\left(x, y, \frac{r_2}{r_1+r_2}\right)$.

For more details see [1, 17, 19].

3. Random fixed points

THEOREM 3.1. *Let X be a convex complete separable metric space having property (B) and C be a closed convex and bounded subset of X.*

If $f : \Omega \times C \to C$ is a nonexpansive random mapping then for each $\omega \in \Omega$, $\inf\{d(\xi(\omega), f(\omega, \xi(\omega))) : \xi \in M(\Omega, C)\} = 0$.

Proof. Let $z \in C$ be fixed. Consider the mapping $f_n : \Omega \times C \to C$ defined by

$$f_n(\omega, \xi(\omega)) = W\left(z, f(\omega, \xi(\omega)), \frac{1}{n}\right),$$

for all natural numbers n. Since X is separable space, therefore each f_n is a measurable map. The mapping f_n is a random contraction, since

$$\begin{aligned} d\left(f_n(\omega, \xi(\omega)), f_n(\omega, \eta(\omega))\right) &= d\left(W\left(z, f(\omega, \xi(\omega)), \frac{1}{n}\right), W\left(z, f(\omega, \eta(\omega)), \frac{1}{n}\right)\right) \\ &\leqslant \left(1 - \frac{1}{n}\right) d(f(\omega, \xi(\omega)), f(\omega, \eta(\omega))) \quad \text{(by property (B))} \\ &\leqslant \left(1 - \frac{1}{n}\right) d(\xi(\omega), \eta(\omega)) < \alpha_n d(\xi(\omega), \eta(\omega)), \end{aligned}$$

where $0 < \alpha_n = 1 - \frac{1}{n} < 1$. Random Contraction Principle [8, theorem 6] further implies that there exists $\xi_n \in M(\Omega, X)$ for which $f_n(\omega, \xi_n(\omega)) = \xi_n(\omega)$ for each $\omega \in \Omega$. Hence

$$\begin{aligned} d\left(\xi_n(\omega), f(\omega, \xi_n(\omega))\right) &= d\left(f_n(\omega, \xi_n(\omega)), f(\omega, \xi_n(\omega))\right) \\ &= d\left(W\left(z, f(\omega, \xi_n(\omega)), \frac{1}{n}\right), f(\omega, \xi_n(\omega))\right) \\ &\leqslant \frac{1}{n} d\left(z, f(\omega, \xi_n(\omega))\right) \leqslant \frac{1}{n}(\text{diam.}C) \end{aligned}$$

where $\text{diam.}C$ stands for the diameter of C.

It further implies that $\lim_{n\to\infty} d(\xi_n(\omega), f(\omega, \xi_n(\omega))) = 0$ for each $\omega \in \Omega$. Hence for each $\omega \in \Omega$,

$$\inf\{d(\xi(\omega), f(\omega, \xi(\omega))) : \xi \in M(\Omega, C)\} = 0.$$

COROLLARY 3.2. *Let X be a convex complete separable metric space having property (B) and C be a compact and convex subset of X. If $f : \Omega \times C \to C$ is a nonexpansive random mapping then f has a random fixed point.*

Proof. Since C is compact, the continuous function $d(\xi(\omega), f(w, \xi(\omega)))$ attains its minimum value zero. Therefore Theorem 3.1 implies that f must have a random fixed point.

Corollary 3.2. is also a consequence of a generalization of random Schauder's fixed point theorem (see [7]). Such a random fixed point is no longer unique. If C is not compact, then f need not to have a random fixed point.

THEOREM 3.3. *Let X be a uniformly convex complete separable metric space having property (B) and C be a closed convex and bounded subset of X. If f : $\Omega \times C \to C$ is a nonexpansive random mapping then $RF(f) \neq \phi$.*

Proof. For any $\epsilon > 0$, define $G_\epsilon : \Omega \to C$ by

$$G_\epsilon(\omega) = \{\xi(\omega) \in C : \xi \in M(\Omega, C) \quad \text{and} \quad d(\xi(\omega), f(\omega, \xi(\omega))) \leqslant \epsilon\}. \tag{3}$$

Theorem 3.1 implies that G_ϵ is nonempty and closed valued multifunction and separablity of X implies that G_ϵ is measurable. If $\eta(\omega)$ and $\xi(\omega)$ belong to $G_\epsilon(\omega)$ for each $\omega \in \Omega$, and $z = W(\xi(\omega), \eta(\omega), \frac{1}{2})$, then

$$\begin{aligned} d(f(\omega, z), \xi(\omega)) &\leqslant d(f(\omega, z), f(\omega, \xi(\omega))) + d(f(\omega, \xi(\omega)), \xi(\omega)) \\ &\leqslant d(z, \xi(\omega)) + \epsilon \\ &= d\left(W\left(\xi(\omega), \eta(\omega), \frac{1}{2}\right), \xi(\omega)\right) + \epsilon \\ &\leqslant \frac{1}{2} d(\eta(\omega), \xi(\omega)) + \epsilon. \end{aligned}$$

Similarly $d(f(\omega, z), \eta(\omega)) \leqslant \frac{1}{2} d(\eta(\omega), \xi(\omega)) + \epsilon$. Uniform convexity (Definition 2.3) of X implies,

$$\begin{aligned} d(f(\omega, z), z) &= d\left(f(\omega, z), W\left(\xi(\omega), \eta(\omega), \frac{1}{2}\right)\right) \\ &\leqslant \left(1 - \alpha\left(\frac{d(\xi(\omega), \eta(\omega))}{\frac{1}{2} d(\xi(\omega), \eta(\omega)) + \epsilon}\right)\right)\left[\frac{1}{2} d(\xi(\omega), \eta(\omega)) + \epsilon\right]. \end{aligned} \tag{4}$$

Case (i). If $d(\xi(\omega), \eta(\omega)) \leqslant \sqrt{\epsilon}$ for each $\omega \in \Omega$, then (4) implies that

$$d(f(\omega, z), z) \leqslant \frac{\sqrt{\epsilon}}{2} + \epsilon.$$

Case (ii). In case $d(\xi(\omega), \eta(\omega)) > \sqrt{\epsilon}$ for some $\omega \in \Omega$, then

$$d(f(\omega, z), z) \leqslant \left(1 - \alpha\left(\frac{\sqrt{\epsilon}}{\frac{1}{2}\sqrt{\epsilon} + \epsilon}\right)\right)\left[\frac{1}{2}(\text{diam.} C) + \epsilon\right]$$

$$= \left(1-\alpha\left(\frac{2}{1+2\sqrt{\epsilon}}\right)\right)\left[\frac{1}{2}\,(\text{diam}.C)+\epsilon\right].$$

In both cases, we may conclude that $z \in G_{\beta(\epsilon)}$, where $\lim_{\epsilon\to\infty} \beta(\epsilon) = 0$

Now let $\xi \in M(\Omega, C)$, $r_\epsilon = \inf_{\omega\in\Omega} d(\xi(\omega), G_\epsilon(\omega))$ and $r = \lim_{\epsilon\to 0} r_\epsilon$. Also let $\delta(\epsilon)$ be a positive function, that decreases to zero as $\epsilon \to 0$. Let $D_\epsilon(\omega) = \{x \in G_\epsilon(\omega) : d(x, \xi(\omega)) \leqslant r + \delta(\epsilon)\}$.

If x and y are two points in $D_\epsilon(\omega)$ such that $d(x,y) > (\text{diam}.D_\epsilon(\omega)) - \delta(\epsilon)$, then

$$\begin{aligned} r_{\beta(\epsilon)} &\leqslant d\left(\xi(\omega),\ W\left(x,y,\frac{1}{2}\right)\right) \\ &\leqslant \left(1-\alpha\left\{\frac{(\text{diam}.D_\epsilon(\omega))-\delta(\epsilon)}{r+\delta(\epsilon)}\right\}\right)\{r+\delta(\epsilon)\}. \end{aligned}$$

It further imlies that $\lim_{\epsilon\to 0} \text{diam}.D_\epsilon(\omega) = 0$, otherwise we have a contradiction. Cantor's Intersection Theorem (see Simmon [18]) further implies $\cap\{D_\epsilon(\omega) : \epsilon > 0\} \neq \phi$. It further implies $\cap\{G_\epsilon(\omega) : \epsilon > 0\} \neq \phi$. Moreover,

$$\{\xi(\omega) : \xi \in RF(f)\} = \cap\{G_\epsilon(\omega) : \epsilon > 0\}.$$

Hence $RF(f) \neq \phi$.

4. Fixed points set function

Let $f : \Omega \times C \to C$ be a random mapping on a convex subset C of a convex complete separable metric space X. We denote by $F(\omega)$ the fixed point set of $f(\omega, .)$ (i.e. $F(\omega) = \{x \in C : x \in f(\omega, x)\}$). We do not assume the existence of fixed points for the deterministic mapping $f(\omega, .) : C \to C$, $F(\omega)$ may be empty. For s in $(0,1)$ define the measurable function $f_s : \Omega \times C \to C$ by $f_s(\omega, x) = W(x, f(\omega, x), s)$. If the set of random fixed points of f is nonempty then f_s has same random fixed points as f. Indeed, let ξ be a random fixed point of f, then

$$f_s(\omega, \xi(\omega)) = W(\xi(\omega), f(\omega, \xi(\omega)), s) = W(\xi(\omega), \xi(\omega), s) = \xi(\omega).$$

Thus ξ is a random fixed point of f_s. Conversely, if ξ is a random fixed point of f_s, then

$$f_s(w, \xi(\omega)) = W(\xi(\omega), f(\omega, \xi(\omega)), s) = \xi(\omega),$$

which gives us $f(\omega, \xi(\omega)) = \xi(\omega)$.

THEOREM 4.1. *Let $f : \Omega \times C \to C$ be a nonexpansive random mapping on a closed convex and bounded subset C of a uniformly convex complete separable metric space X having property (B). Then f_s is asymptotically regular random map.*

Proof. Let ξ_0 be an arbitrary in $M(\Omega, C)$ and consider the sequence (ξ_n), $\xi_n(\omega) = f_s^n(\omega, \xi_0(\omega))$. Let ξ' be a random fixed point of f (Theorem 3.3), then for each

$\omega \in \Omega$,

$$\begin{aligned} d(\xi'(\omega), f_s^{n+1}(\omega, \xi_0(\omega))) \\ &= d(f(\omega, \xi'(\omega)), f_s^{n+1}(\omega, \xi_0(\omega))) \\ &= d(f(\omega, \xi'(\omega)), W(f_s^n(\omega, \xi_0(\omega)), f(\omega, f_s^n(\omega, \xi_0(\omega))), s)) \\ &\leqslant s\, d\big(f(\omega, \xi'(\omega)),\, f_s^n(\omega, \xi_0(\omega))\big) \\ &\quad +(1-s)\, d\big(f(\omega, \xi'(\omega)),\, f(\omega, f_s^n(\omega, \xi_0(\omega)))\big) \\ &\leqslant s\, d\big(\xi'(\omega),\, f_s^n(\omega, \xi_0(\omega))\big) + (1-s)\, d\big(\xi'(\omega), f_s^n(\omega, \xi_0(\omega))\big) \\ &= d\big(\xi'(\omega),\, f_s^n(\omega, \xi_0(\omega))\big) \end{aligned}$$

Thus for each $\omega \in \Omega$ the sequence $(\beta_n(\omega))$ defined by $\beta_n(\omega) = d(\xi'(\omega), f_s^n(\omega, \xi_0(\omega)))$ is monotone decreasing sequence of positive real numbers. Therefore it is convergent to $\beta(\omega)$ (say) i.e.

$$\lim_{n\to\infty} d\big(\xi'(\omega),\, f_s^n(\omega, \xi_0(\omega))\big) = \beta(\omega). \tag{5}$$

If $\beta(\omega) = 0$ for each $\omega \in \Omega$, then

$$\begin{aligned} \lim_{n\to\infty} d\big(f_s^{n+1}(\omega, \xi_0(\omega)),\, f_s^n(\omega, \xi_0(\omega))\big) \\ &\leqslant \lim_{n\to\infty} d\big(f_s^{n+1}(\omega, \xi_0(\omega)),\, \xi'(\omega)\big) + \lim_{n\to\infty} d\big(\xi'(\omega),\, f_s^n(\omega, \xi_0(\omega))\big) \\ &= 0. \end{aligned}$$

Thus

$$\lim_{n\to\infty} d\big(f_s^{n+1}(\omega, \xi_0(\omega)),\, f_s^n(\omega, \xi_0(\omega))\big) = 0 \tag{6}$$

and the assertion of the theorem is proved.

If $\beta(\omega) > 0$ for some $\omega \in \Omega$, then

$$\lim_{n\to\infty} d\big(\xi'(\omega),\, f_s^n(\omega, \xi_0(\omega))\big) = \beta(\omega) \tag{5'}$$

and

$$\lim_{n\to\infty} d\big(\xi'(\omega),\, f_s^{n+1}(\omega, \xi_0(\omega))\big) = \beta(\omega). \tag{7}$$

Also since ξ' is a random fixed point of f, therefore

$$\lim_{n\to\infty} d\big(\xi'(\omega), f(\omega,\, f_s^n(\omega, \xi_0(\omega)))\big) = \beta(\omega). \tag{8}$$

Since X is a uniformly convex complete separable metric space, therefore Remark 2.4 together with equation $(5')$, (7) and (8) imply

$$\lim_{n\to\infty} d(f_s^n(\omega, \xi_0(\omega)),\, f(\omega, f_s^n(\omega, \xi_0(\omega)))) = 0. \tag{9}$$

Now

$$\begin{aligned} \lim_{n\to\infty} d\big(f_s^n(\omega, \xi_0(\omega)), f_s^{n+1}(\omega, \xi_0(\omega))\big) \\ &= \lim_{n\to\infty} d(f_s^n(\omega, \xi_0(\omega)),\, W(f_s^n(\omega, \xi_0(\omega)), f(\omega, f_s^n(\omega, \xi_0(\omega))), s)) \\ &\leqslant (1-s) \lim_{n\to\infty} d(f_s^n(\omega, \xi_0(\omega)),\, f(\omega, f_s^n(\omega, \xi_0(\omega)))) \text{ (by def. 2.1)} \\ &= 0 \text{ (by equality 9)}. \end{aligned}$$

Hence f_s is asymptotically regular random map on C.

THEOREM 4.2. *Let $f : \Omega \times C \to C$ be a nonexpansive random map on a closed, convex and bounded subset C of a uniformly convex complete separable metric space X having property (B). Then the fixed point set function F of f is nonemty closed convex valued i.e. $F(\omega)$ is nonempty closed convex set for each $\omega \in \Omega$.*

Proof. Theorem 3.3 implies $F(\omega) \neq \phi$ for each $\omega \in \Omega$. The mapping $F(\omega)$ is closed because f is continuous. For $x_1, x_2 \in F(\omega)$ and $t \in (0,1)$,

$$\begin{aligned} d(x_1,x_2) &\leqslant d\left(x_1, f\left(\omega, W\left(x_1,x_2,t\right)\right)\right) + d\left(f\left(\omega, W\left(x_1,x_2,t\right)\right), x_2\right) \\ &= d\left(f\left(\omega,x_1\right), f\left(\omega, W\left(x_1,x_2,t\right)\right)\right) + d\left(f\left(\omega, W\left(x_1,x_2,t\right)\right), f\left(\omega,x_2\right)\right) \\ &\leqslant d\left(x_1, W\left(x_1,x_2,t\right)\right) + d\left(W\left(x_1,x_2,t\right), x_2\right) \\ &\leqslant (1-t)\, d(x_1,x_2) + t\, d(x_1,x_2) \\ &= d(x_1,x_2). \end{aligned}$$

It implies that,

$$\begin{aligned} & d\left(x_1, f\left(\omega, W\left(x_1,x_2,t\right)\right)\right) + d\left(f\left(\omega, W\left(x_1,x_2,t\right)\right), x_2\right) \\ &\qquad = d\left(x_1, W\left(x_1,x_2,t\right)\right) + d\left(W\left(x_1,x_2,t\right), x_2\right) = d(x_1,x_2). \end{aligned} \tag{10}$$

Since f is nonexpansive random map and $x_1, x_2 \in F(\omega)$, it further implies that,

$$d\left(x_1, f\left(\omega, W\left(x_1,x_2,t\right)\right)\right) = d\left(x_1, W\left(x_1,x_2,t\right)\right) = r_1 \text{ (say)} \tag{11}$$

and

$$d\left(x_2, f\left(\omega, W\left(x_1,x_2,t\right)\right)\right) = d\left(x_2, W\left(x_1,x_2,t\right)\right) = r_2 \text{ (say)}. \tag{12}$$

Now using (10), we obtain

$$\begin{aligned} \frac{r_2}{r_1+r_2} &= \frac{d\left(x_2, W\left(x_1,x_2,t\right)\right)}{d\left(x_1, W\left(x_1,x_2,t\right)\right) + d\left(x_2, W\left(x_1,x_2,t\right)\right)} \\ &= \frac{t\, d(x_1,x_2)}{d(x_1,x_2)} = t. \end{aligned} \tag{13}$$

Equalities (10), (11), (12) and (13) together with Remark 2.4 imply that,

$$f\left(\omega, W\left(x_1,x_2,t\right)\right) = W\left(x_1,x_2,\frac{r_2}{r_1+r_2}\right) = W\left(x_1,x_2,t\right).$$

Therefore $W(x_1,x_2,t) \in F(\omega)$. Hence $F(\omega)$ is convex for each $\omega \in \Omega$.

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