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A CONJECTURE ABOUT THE INERTIA OF HERMITIAN MATRICES

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(*communicated by R. A. Brualdi*)

Abstract. Let H be a Hermitian matrix which has been decomposed into m rows and m columns of blocks. Suppose further that we know the inertia of each diagonal block and a range of possible ranks for each off-diagonal block. What are the possible inertias of H?

In this paper, a conjecture on the inertias of Hermitian matrices with a prescribed 3×3 block decomposition is presented, based on several important works on the subject.

1. Introduction and preliminaries

Define the inertia of an $n \times n$ Hermitian matrix H as the triple $\mathrm{In}(H) = (\pi, \nu, \delta)$, where π, ν and $\delta = n - \pi - \nu$ are respectively the number of positive, negative, and zero eigenvalues. When n is given and we will not be using the symbol δ we write $\mathrm{In}(H)$ as $(\pi, \nu, *)$.

For $i, j = 1, \ldots, m$, suppose that H_{ij} is an $n_i \times n_j$ matrix with $H_{ji} = H_{ij}^*$ and $n = n_1 + \cdots + n_m$. We say that $(H_{ij})_{i,j=1,\ldots,m}$ is an $m \times m$ block decomposition of H if $H = (H_{ij})$. In the last decades the characterization of the inertias of Hermitian matrices with prescribed 2×2 and 3×3 block decompositions has been extensively investigated. In the first case, after the papers [18] and [2] in 1981, Cain and Marques de Sá established the following result.

THEOREM 1.1. ([3]) *Let us consider nonnegative integers n_i, π_i, ν_i such that $\pi_i + \nu_i \leqslant n_i$, for $i = 1, 2$, and let $0 \leqslant r \leqslant R \leqslant \min\{n_1, n_2\}$. Then the following conditions are equivalent:*

(I) For $i = 1, 2$, there exist $n_i \times n_i$ Hermitian matrices H_i and an $n_1 \times n_2$ matrix X such that $\mathrm{In}(H_i) = (\pi_i, \nu_i, *)$, $r \leqslant \mathrm{rank} X \leqslant R$ and

$$H = \begin{bmatrix} H_1 & X \\ X^* & H_2 \end{bmatrix}$$

has inertia $(\pi, \nu, *)$.

(II) Let $k \in \{1, 2\}$. Let W_k be any fixed Hermitian matrix of order n_k and inertia $(\pi_k, \nu_k, *)$. (I) holds with $H_k = W_{kk}$.

(III) Let W be any fixed $n_1 \times n_2$ matrix with $r \leqslant \mathrm{rank} W \leqslant R$. (I) holds with $X = W$.

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(IV) *For* $k = 1,2$*, let* W_{kk} *be any fixed* $n_k \times n_k$ *Hermitian matrix with inertia* $(\pi_k, \nu_k, *)$*. (I) holds with* $H_1 = W_{11}$ *and* $H_2 = W_{22}$*.*

(V) *The following inequalities hold:*

$$\begin{aligned} \pi &\geqslant \max\{\pi_1, \pi_2, r-\nu_1, r-\nu_2, \pi_1+\pi_2-R\}, \\ \nu &\geqslant \max\{\nu_1, \nu_2, r-\pi_1, r-\pi_2, \nu_1+\nu_2-R\}, \\ \pi &\leqslant \min\{n_1+\pi_2, \pi_1+n_2, \pi_1+\pi_2+R\}, \\ \nu &\leqslant \min\{n_1+\nu_2, \nu_1+n_2, \nu_1+\nu_2+R\}, \end{aligned}$$

$$\begin{aligned} \pi-\nu &\leqslant \pi_1+\pi_2, \\ \nu-\pi &\leqslant \nu_1+\nu_2, \end{aligned}$$

$$\begin{aligned} \pi+\nu &\geqslant \pi_1+\nu_1+\pi_2+\nu_2-R, \\ \pi+\nu &\leqslant \min\{n_1+n_2, \pi_1+\nu_1+n_2+R, n_1+\pi_2+\nu_2+R\}. \end{aligned}$$

In this important theorem we can see how much influence the pair H_1, H_2 of complementary submatrices and the off-diagonal block X have on the inertia of H.

In 1992, Cain and Marques de Sá ([3]) extended the methods given by Haynsworth and Ostrowski in [15], for estimating and computing the inertias of certain skew-triangular block matrices. Later this result was improved in [10], which can have the following block tridiagonal version.

THEOREM 1.2. ([10]) *Let us consider nonnegative integers* n_i, π_i, ν_i *such that* $\pi_i + \nu_i \leqslant n_i$*, for* $i = 1,2,3$*, and let* $0 \leqslant r_{i,i+1} \leqslant R_{i,i+1} \leqslant \min\{n_i, n_{i+1}\}$*, for* $i = 1,2$*. Then the following conditions are equivalent:*

(I) *For* $i = 1,2,3$*, and* $j = 1,2$*, there exist* $n_i \times n_i$ *Hermitian matrices* H_i *and* $n_j \times n_{j+1}$ *matrices* $X_{j,j+1}$ *such that* $\mathrm{In}(H_i) = (\pi_i, \nu_i, *)$*,* $r_{j,j+1} \leqslant \mathrm{rank} X_{j,j+1} \leqslant R_{j,j+1}$ *and*

$$H = \begin{bmatrix} H_1 & X_{12} & 0 \\ X_{12}^* & H_2 & X_{23} \\ 0 & X_{23}^* & H_3 \end{bmatrix}$$

has inertia $(\pi, \nu, *)$*.*

(II) *Let* $k \in \{1,2,3\}$*. Let* W_{kk} *be any fixed* $n_k \times n_k$ *Hermitian matrix with inertia* $(\pi_k, \nu_k, *)$*. (I) holds with* $H_k = W_{kk}$*.*

(III) *Let* $k \in \{1,2\}$*. Let* $W_{k,k+1}$ *be any fixed* $n_k \times n_{k+1}$ *matrix with* $r_{k,k+1} \leqslant \mathrm{rank} W_{k,k+1} \leqslant R_{k,k+1}$*. (I) holds with* $X_{k,k+1} = W_{k,k+1}$*.*

(IV) *For* $k = 1,2,3$ *let* W_{kk} *be any fixed* $n_k \times n_k$ *Hermitian matrix with inertia* $(\pi_k, \nu_k, *)$*. (I) holds with* $H_1 = W_{11}$*,* $H_2 = W_{22}$ *and* $H_3 = W_{33}$*.*

(V) *Let* $(i,j,k) = (1,2,3)$ *or* $(2,3,1)$*. Let* W_{kk} *be any fixed* $n_k \times n_k$ *Hermitian matrix with inertia* $(\pi_k, \nu_k, *)$ *and let* W_{ij} *be any fixed* $n_i \times n_j$ *matrix with* $r_{ij} \leqslant \mathrm{rank}\, W_{ij} \leqslant R_{ij}$*. (I) holds with* $H_k = W_{kk}$ *and* $X_{ij} = W_{ij}$*.*

(VI) The following inequalities hold:

$$\begin{aligned}
\pi \;&\geqslant \max\{\pi_2, r_{12}-\nu_2, r_{23}-\nu_2,\\
&\qquad \pi_1-\nu_2+r_{23}-R_{12}, \pi_1-\nu_3+r_{23},\\
&\qquad \pi_3-\nu_1+r_{12}, \pi_3-\nu_2+r_{12}-R_{23},\\
&\qquad \pi_1+\pi_2-R_{12}, \pi_1+\pi_3, \pi_2+\pi_3-R_{23},\\
&\qquad \pi_1+\pi_2+\pi_3-R_{12}-R_{23}\},\\
\nu \;&\geqslant \max\{\nu_2, r_{12}-\pi_2, r_{23}-\pi_2,\\
&\qquad \nu_1-\pi_2+r_{23}-R_{12}, \nu_1-\pi_3+r_{23},\\
&\qquad \nu_3-\pi_1+r_{12}, \nu_3-\pi_2+r_{12}-R_{23},\\
&\qquad \nu_1+\nu_2-R_{12}, \nu_1+\nu_3, \nu_2+\nu_3-R_{23},\\
&\qquad \nu_1+\nu_2+\nu_3-R_{12}-R_{23}\},\\
\pi \;&\leqslant \min\{n_1+\pi_2+n_3, \pi_1+\pi_2+\pi_3+R_{12}+R_{23},\\
&\qquad \pi_1+\pi_2+n_3+R_{12}, \pi_1+n_2+\pi_3, n_1+\pi_2+\pi_3+R_{23}\},\\
\nu \;&\leqslant \min\{n_1+\nu_2+n_3, \nu_1+\nu_2+\nu_3+R_{12}+R_{23},\\
&\qquad \nu_1+\nu_2+n_3+R_{12}, \nu_1+n_2+\nu_3, n_1+\nu_2+\nu_3+R_{23}\},\\
\pi+\nu \;&\geqslant \max\{\pi_1+\nu_1+\pi_2+\nu_2-R_{12}, \pi_2+\nu_2+\pi_3+\nu_3-R_{23},\\
&\qquad \pi_1+\nu_1+\pi_2+\nu_2+\pi_3+\nu_3-R_{12}-R_{23},\\
&\qquad \pi_1+\nu_1-\pi_2-\nu_2+2r_{23}-R_{12},\\
&\qquad \pi_3+\nu_3-\pi_2-\nu_2+2r_{12}-R_{23}\},\\
\pi+\nu \;&\leqslant \min\{n_1+n_2+n_3, \pi_1+\nu_1+n_2+n_3+R_{12},\\
&\qquad n_1+\pi_2+\nu_2+n_3+R_{12}+R_{23}, n_1+n_2+\pi_3+\nu_3+R_{23},\\
&\qquad \pi_1+\nu_1+\pi_2+\nu_2+n_3+2R_{12}+R_{23},\\
&\qquad \pi_1+\nu_1+n_2+\pi_3+\nu_3+R_{12}+R_{23},\\
&\qquad n_1+\pi_2+\nu_2+\pi_3+\nu_3+R_{12}+2R_{23}\}\\
\pi-\nu \;&\leqslant \min\{\pi_1+\pi_2+\pi_3,\\
&\qquad \pi_1+\pi_2+\pi_3-\nu_1+R_{12}, \pi_1+\pi_2+\pi_3-\nu_3+R_{23}\},\\
\nu-\pi \;&\leqslant \min\{\nu_1+\nu_2+\nu_3,\\
&\qquad \nu_1+\nu_2+\nu_3-\pi_1+R_{12}, \nu_1+\nu_2+\nu_3-\pi_3+R_{23}\}.
\end{aligned}$$

For $m > 2$ Cain (cf. [1]) characterized the inertias of Hermitian matrices with m-by-m block decompositions in terms of a system of linear inequalities involving the orders of the blocks and the inertias of the main diagonal blocks, extending the main results of [2]. Given the integers $x_1, \ldots, x_m$ and $y_1, \ldots, y_m$, Cain defines, for $k = 1, \ldots, m$, the set

$$L_k(m, x_*, y_*) = \min\left\{\sum_{i\notin I} x_i + \sum_{i\in I} y_i \mid I \subset \{1, \ldots, m\} \text{ and } |I| = k\right\},$$

and establishes the theorem:

THEOREM 1.3. ([1]) *Let* m, n_i, π_i, ν_i *be nonnegative integers and* $\pi_i + \nu_i \leqslant n_i$, *for* $i = 1, \ldots, m$, *and set* $n = n_1 + \cdots + n_m$. *The following conditions are equivalent:*

(I) There exists an $n \times n$ *Hermitian matrix* $H = [H_{ij}]$ *where* H_{ij} *are* $n_i \times n_j$ *blocks, satisfying* $\mathrm{In}(H_{ii}) = (\pi_i, \nu_i, *)$ *and* $\mathrm{In}(H) = (\pi, \nu, *)$.

(II) Given Hermitian matrices X_i *of order* n_i, *with* $\mathrm{In}(X_i) = (\pi_i, \nu_i, *)$ *for* $i = 1, \ldots, m$, *there exists an* $n \times n$ *Hermitian matrix* $H = [H_{ij}]$, *where* H_{ij} *are* $n_i \times n_j$ *blocks, satisfying* $X_i = H_{ii}$ *and* $\mathrm{In}(H) = (\pi, \nu, *)$.

(III) The following inequalities hold:

$$\begin{aligned} \max\{\pi_1, \ldots, \pi_m\} &\leqslant \pi, \\ \max\{\nu_1, \ldots, \nu_m\} &\leqslant \nu, \\ \pi - (k-1)\nu &\leqslant L_k(m, n_*, \pi_*), \quad \text{for } k = 1, \ldots, m, \\ \nu - (k-1)\pi &\leqslant L_k(m, n_*, \nu_*), \quad \text{for } k = 1, \ldots, m, \\ \pi + \nu &\leqslant n. \end{aligned}$$

In the next section a conjecture on inertias of 3×3 partitioned Hermitian matrices is given.

2. A conjecture

As we have seen in previous section the effort to characterize the inertias of Hermitian matrices with a prescribed 2×2 or 3×3 block decomposition has produced many results. While the case of 2×2 block decompositions is solved the case 3×3 seems to be far from decided.

All these theorems state that a number of conditions are equivalent. In each theorem the last of the conditions is a large collection of inequalities. Although we are not certain what the general theorem for 3×3 block decompositions should say, here it is without that last complicated condition:

THEOREM 2.1. *Let us assume that the quantities* π_i, ν_i, n_i, *for* $i = 1, 2, 3$, *are nonnegative and*

$$\pi_i + \nu_i \leqslant n_i,$$

and

$$0 \leqslant r_{ij} \leqslant R_{ij} \leqslant \min\{n_i, n_j\}, \quad 1 \leqslant i < j \leqslant 3.$$

Then the following conditions are equivalent:

(I) For $i = 1, 2, 3$, *and* $j = 2, 3$, *there exist* $n_i \times n_i$ *Hermitian matrices* H_i *and* $n_i \times n_j$ *matrices* X_{ij} *such that* $\mathrm{In}(H_i) = (\pi_i, \nu_i, *)$, $r_{ij} \leqslant \operatorname{rank} X_{ij} \leqslant R_{ij}$ *when* $i < j$ *and*

$$H = \begin{bmatrix} H_1 & X_{12} & X_{13} \\ X_{12}^* & H_2 & X_{23} \\ X_{13}^* & X_{23}^* & H_3 \end{bmatrix}$$

has inertia $(\pi, \nu, *)$.

(II) Let $k \in \{1, 2, 3\}$. *Let* W_{kk} *be any fixed Hermitian matrix of order* n_k *with inertia* $(\pi_k, \nu_k, *)$. *(I) holds with* $H_k = W_{kk}$.

(III) *Let* $j,k \in \{1,2,3\}$ *such that* $j < k$. *Let* W_{jk} *be any fixed* $n_j \times n_k$ *matrix with* $r_{jk} \leqslant \operatorname{rank} W_{jk} \leqslant R_{jk}$. *(I) holds with* $X_{jk} = W_{jk}$.

(IV) *For* $k = 1,2,3$ *let* W_{kk} *be any fixed* $n_k \times n_k$ *Hermitian matrix with inertia* $(\pi_k, \nu_k, *)$. *(I) holds with* $H_1 = W_{11}$, $H_2 = W_{22}$ *and* $H_3 = W_{33}$.

(V) *Let* $(i,j,k) = (1,2,3)$, $(1,3,2)$, *or* $(2,3,1)$. *Let* W_{kk} *be any fixed* $n_k \times n_k$ *Hermitian matrix with inertia* $(\pi_k, \nu_k, *)$ *and let* W_{ij} *be any fixed* $n_i \times n_j$ *matrix with* $r_{ij} \leqslant \operatorname{rank} W_{ij} \leqslant R_{ij}$. *(I) holds with* $H_k = W_{kk}$ *and* $X_{ij} = W_{ij}$.

Proof. (See also [3, 10]) It is straightforward that each of (II)–(V) implies (I). Suppose now that H satisfies (I). Let M be a block diagonal matrix $M_1 \oplus M_2 \oplus M_3$, where each M_i denotes an $n_i \times n_i$ invertible matrix. For $i = 1,2,3$, and $j = 2,3$ set $Y_{ii} = M_i^* H_i M_i$, $Y_{ij} = M_i^* X_{ij} M_j$ when $i < j$. We have $Y = (Y_{ij})_{i,j} = M^* H M$. Then $\operatorname{rank} Y_{1j} = \operatorname{rank} X_{1j}$, and by Sylvester's Theorem $\operatorname{In}(Y) = \operatorname{In}(H)$ and $\operatorname{In}(Y_{ii}) = \operatorname{In}(H_i)$. Thus Y has all the rank and inertia properties required in (II)-(V). In each of these cases the only additional requirement is that, for certain $i < j,k$, $M_i^* X_{ij} M_j = W_{ij}$ and $M_k^* H_k M_k = W_{kk}$. Such M_p's can always be founded ([17]). □

Note that the set of linear inequalities in each theorem is self-$\pi\nu$-dual in the sense that it remains invariant after we transform each inequality into its $\pi\nu$-dual, i.e., the set is the same after the substitution in each inequality of the symbols ν, π, ν_i, π_i for π, ν, π_i, ν_i, respectively. Also, when r_{13} and R_{13} are not zero they must show up in the inequalities in a way that respects the symmetries and the patterns revealed in [10].

Carefully reflecting on the results reviewed above leads us to conjecture:

CONJECTURE 2.2. *Under the conditions of the Theorem 2.1, (I) (and, therefore, (II)–(V)) is equivalent to the following system of linear inequalities:*

$$
\begin{aligned}
\pi \ \geqslant \max\{&\pi_1, \pi_2, \pi_3,\\
&r_{12} - \nu_1, r_{13} - \nu_1, r_{12} - \nu_2, r_{23} - \nu_2, r_{13} - \nu_3, r_{23} - \nu_3,\\
&\pi_1 - \nu_2 + r_{23} - R_{12}, \pi_1 - \nu_3 + r_{23} - R_{13},\\
&\pi_2 - \nu_1 + r_{13} - R_{12}, \pi_2 - \nu_3 + r_{13} - R_{23},\\
&\pi_3 - \nu_1 + r_{12} - R_{13}, \pi_3 - \nu_2 + r_{12} - R_{23},\\
&\pi_1 + \pi_2 - R_{12}, \pi_1 + \pi_3 - R_{13}, \pi_2 + \pi_3 - R_{23},\\
&\pi_1 + \pi_2 + \pi_3 - R_{12} - R_{13} - R_{23}\ \},\\
\nu \ \geqslant \max\{&\nu_1, \nu_2, \nu_3,\\
&r_{12} - \pi_1, r_{13} - \pi_1, r_{12} - \pi_2, r_{23} - \pi_2, r_{13} - \pi_3, r_{23} - \pi_3,\\
&\nu_1 - \pi_2 + r_{23} - R_{12}, \nu_1 - \pi_3 + r_{23} - R_{13},\\
&\nu_2 - \pi_1 + r_{13} - R_{12}, \nu_2 - \pi_3 + r_{13} - R_{23},\\
&\nu_3 - \pi_1 + r_{12} - R_{13}, \nu_3 - \pi_2 + r_{12} - R_{23},\\
&\nu_1 + \nu_2 - R_{12}, \nu_1 + \nu_3 - R_{13}, \nu_2 + \nu_3 - R_{23},\\
&\nu_1 + \nu_2 + \nu_3 - R_{12} - R_{13} - R_{23}\ \},
\end{aligned}
$$

$$
\begin{aligned}
\pi &\leqslant \min\{\pi_1+n_2+n_3, n_1+\pi_2+n_3, n_1+n_2+\pi_3,\\
&\qquad \pi_1+\pi_2+n_3+R_{12}, \pi_1+n_2+\pi_3+R_{13}, n_1+\pi_2+\pi_3+R_{23},\\
&\qquad \pi_1+\pi_2+\pi_3+R_{12}+R_{13}+R_{23}\},\\
\nu &\leqslant \min\{\nu_1+n_2+n_3, n_1+\nu_2+n_3, n_1+n_2+\nu_3,\\
&\qquad \nu_1+\nu_2+n_3+R_{12}, \nu_1+n_2+\nu_3+R_{13}, n_1+\nu_2+\nu_3+R_{23},\\
&\qquad \nu_1+\nu_2+\nu_3+R_{12}+R_{13}+R_{23}\},\\
\pi+\nu &\geqslant \max\{\pi_1+\nu_1+\pi_2+\nu_2-R_{12},\\
&\qquad \pi_1+\nu_1+\pi_3+\nu_3-R_{13},\\
&\qquad \pi_2+\nu_2+\pi_3+\nu_3-R_{23},\\
&\qquad \pi_1+\nu_1+\pi_2+\nu_2+\pi_3+\nu_3-R_{12}-R_{13}-R_{23},\\
&\qquad r_{12}+r_{13}-R_{23}, r_{12}+r_{23}-R_{13}, r_{13}+r_{23}-R_{12},\\
&\qquad \pi_1+\nu_1-\pi_2-\nu_2+2r_{23}-R_{12}, \pi_1+\nu_1-\pi_3-\nu_3+2r_{23}-R_{13},\\
&\qquad \pi_2+\nu_2-\pi_1-\nu_1+2r_{13}-R_{12}, \pi_2+\nu_2-\pi_3-\nu_3+2r_{13}-R_{23},\\
&\qquad \pi_3+\nu_3-\pi_1-\nu_1+2r_{12}-R_{13}, \pi_3+\nu_3-\pi_2-\nu_2+2r_{12}-R_{23}\},\\
\pi+\nu &\leqslant \min\{n_1+n_2+n_3,\\
&\qquad \pi_1+\nu_1+n_2+n_3+R_{12}+R_{13},\\
&\qquad n_1+\pi_2+\nu_2+n_3+R_{12}+R_{23},\\
&\qquad n_1+n_2+\pi_3+\nu_3+R_{13}+R_{23},\\
&\qquad \pi_1+\nu_1+\pi_2+\nu_2+n_3+2R_{12}+R_{13}+R_{23},\\
&\qquad \pi_1+\nu_1+n_2+\pi_3+\nu_3+R_{12}+2R_{13}+R_{23},\\
&\qquad n_1+\pi_2+\nu_2+\pi_3+\nu_3+R_{12}+R_{13}+2R_{23}\}\\
\pi-\nu &\leqslant \min\{n_1+\pi_2+\pi_3, \pi_1+n_2+\pi_3, \pi_1+\pi_2+n_3,\\
&\qquad \pi_1+\pi_2+\pi_3+R_{12}, \pi_1+\pi_2+\pi_3+R_{13}, \pi_1+\pi_2+\pi_3+R_{23},\\
&\qquad \pi_1+\pi_2+\pi_3-\nu_1+R_{12}+R_{13},\\
&\qquad \pi_1+\pi_2+\pi_3-\nu_2+R_{12}+R_{23},\\
&\qquad \pi_1+\pi_2+\pi_3-\nu_3+R_{13}+R_{23}\},\\
\nu-\pi &\leqslant \min\{n_1+\nu_2+\nu_3, \nu_1+n_2+\nu_3, \nu_1+\nu_2+n_3,\\
&\qquad \nu_1+\nu_2+\nu_3+R_{12}, \nu_1+\nu_2+\nu_3+R_{13}, \nu_1+\nu_2+\nu_3+R_{23},\\
&\qquad \nu_1+\nu_2+\nu_3-\pi_1+R_{12}+R_{13},\\
&\qquad \nu_1+\nu_2+\nu_3-\pi_2+R_{12}+R_{23},\\
&\qquad \nu_1+\nu_2+\nu_3-\pi_3+R_{13}+R_{23}\},\\
&-R_{12}-R_{13}-R_{23} \leqslant \pi-2\nu \leqslant \pi_1+\pi_2+\pi_3,\\
&-R_{12}-R_{13}-R_{23} \leqslant \nu-2\pi \leqslant \nu_1+\nu_2+\nu_3.
\end{aligned}
$$

It is not difficult to generalize the Theorem 2.1 to $m\times m$ block decompositions, but it is not clear how to generalize Conjecture 2.2 to the $m\times m$ case. Cain's result, Theorem 1.3, suggests new inequalities involving $\pi-3\nu, \nu-3\pi, \dots, \pi-(m-1)\nu, \nu-(m-1)\pi$.

The correct generalization of Conjecture 2.2 could depend on combinatorial and qualitative features of the decomposed Hermitian matrix, a subject which in the recent years has been quite thoroughly investigated (e.g. [13]). The inertially symmetric pentagons of Dancis and Cohen could play a role.

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