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ON SOME CLASSES OF SEQUENCES DEFINED BY SEQUENCES OF ORLICZ FUNCTIONS

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(*communicated by H. M. Srivastava*)

Abstract. In this paper we introduce some new seminormed sequence spaces using a sequence of Orlicz functions and examine some properties of these sequence spaces. Furthermore we introduce $\Delta^m_{u_g}-$ statistical convergence and give a relation between $\Delta^m_{u_g}-$ statistical convergence and strongly $\Delta^m_{u_g}-$ Cesàro summable sequences with respect to an Orlicz function.

1. Introduction

Let w be the set of *all* sequences of real or complex numbers and ℓ_∞, c and c_0 be the linear spaces of *bounded*, *convergent* and *null* sequences $x=(x_k)$ with complex terms, respectively, normed by $\|x\|_\infty = \sup_k |x_k|$, where $k \in \mathbb{N} = \{1,2,...\}$, the set of positive integers. Throughout the article $w(X)$, $c(X)$, $c_0(X)$ and $\ell_\infty(X)$ will represent the spaces of *all*, *convergent*, *null* and *bounded* X valued sequence spaces. For $X=\mathbb{C}$, the field of complex numbers, these represent the corresponding scalar valued sequence spaces. The zero sequence is denoted by $\bar{\theta}=(\theta,\theta,...)$, where θ is the zero element of X.

The notion of statistical convergence was introduced by Fast [3] and Schoenberg [14], independently. Over the years and under different names statistical convergence has been discussed in the theory of Fourier analysis, ergodic theory and number theory. Later on it was further investigated from the sequence space point of view and linked with summability theory by Šalát [13] and Fridy [4]. The idea depends on the density of subsets of the set $\mathbb{N}$ of natural numbers. The density of E a subset of $\mathbb{N}$ is defined by $\delta(E)=\lim_{n\to\infty}\frac{1}{n}\sum_{k=1}^{n}\chi_E(k)$ provided the limit exists, where χ_E is the characteristic function of E. A sequence $x=(x_k)$ is called statistically convergent to a number L, if for every $\varepsilon>0$, $\delta\{k\in\mathbb{N}:|x_k-L|\geqslant\varepsilon\}=0$ (see [3],[4]).

Recall ([5], [7], [12]) that an Orlicz function is a function $M:[0,\infty)\to[0,\infty)$, which is continuous, non-decreasing and convex with $M(0)=0$, $M(x)>0$ for $x>0$ and $M(x)\to\infty$ as $x\to\infty$.

The study of Orlicz sequence spaces was initiated with a certain specific purpose in Banach space theory. Indeed, Lindberg [8] got interested in Orlicz spaces in connection

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with finding Banach spaces with symmetric Schauder bases having complementary subspaces isomorphic to c_0 or ℓ_p $(1 \leqslant p < \infty)$. Subsequently Lindenstrauss and Tzafriri [9] investigated these Orlicz sequence spaces in more detail, and they proved that every Orlicz sequence space ℓ_M contains a subspace isomorphic to ℓ_p $(1 \leqslant p < \infty)$. Later on, different classes of sequence spaces defined by Orlicz function were studied by Nung and Lee [11], Woo [16], Tripathy and Mahanta [15], Choudhary and Parashar [1] and many others. The Orlicz sequence spaces are the special cases of Orlicz spaces studied in Ref. [7].

The idea of difference sequence sets was introduced by Kizmaz [6] and this subject was generalized by Et and Colak [2] as follows:

$$X(\Delta^m) = \{x \in w : \Delta^m x \in X\},$$

for $X = \ell_\infty$, c and c_0, where $m \in \mathbb{N}$, $\Delta^m x_k = \Delta^{m-1}x_k - \Delta^{m-1}x_{k+1} = \sum_{v=0}^{m}(-1)^v \binom{m}{v} x_{k+v}$ and $\Delta^0 x_k = x_k$ for all $k \in \mathbb{N}$.

The main purpose of this paper is to introduce the following sequence spaces and examine some properties of the resulting sequence spaces. We define the following sequence spaces:

DEFINITION 1. Let $p = (p_n)$ and $\bar{q} = (\bar{q}_n)$ denote the sequences of positive real numbers and the sequence $\bar{Q} = (\bar{Q}_n)$ is such that $\bar{q}_1 > 0$ and $\bar{Q}_n = \bar{q}_1 + \bar{q}_2 + ... + \bar{q}_n$, for all $n \in \mathbb{N}$. Let $M = (M_k)$ be a sequence of Orlicz functions, X be a locally convex Hausdorff topological linear space whose topology is determined by a set Q of continuous seminorm g and $u = (u_k)$ be any fixed sequence of non-zero complex numbers u_k. Then

$$w_1(\mathbf{M}, \Delta^m, \bar{q}, p, u, g) = \{x \in w(X) : \frac{1}{\bar{Q}_n} \sum_{k=1}^{n} \bar{q}_k [M_k(g(\frac{u_k \Delta^m x_k - L}{\rho}))]^{p_k} \to 0,$$
$$\text{as } n \to \infty, \text{ for some } \rho > 0 \text{ and } L \in X\},$$

$$w_0(\mathbf{M}, \Delta^m, \bar{q}, p, u, g) = \{x \in w(X) : \frac{1}{\bar{Q}_n} \sum_{k=1}^{n} \bar{q}_k [M_k(g(\frac{u_k \Delta^m x_k}{\rho}))]^{p_k} \to 0,$$
$$\text{as } n \to \infty, \text{ for some } \rho > 0\},$$

$$w_\infty(\mathbf{M}, \Delta^m, \bar{q}, p, u, g) = \{x \in w(X) : \sup_n \frac{1}{\bar{Q}_n} \sum_{k=1}^{n} \bar{q}_k [M_k(g(\frac{u_k \Delta^m x_k}{\rho}))]^{p_k} < \infty,$$
$$\text{for some } \rho > 0\}.$$

Throughout the paper Y will denote any one of the notation $0, 1$ or ∞. When $\bar{q}_n = 1$ $(\bar{Q}_n = n)$ and $M_n = M$ for all $n \in \mathbb{N}$, we denote the above spaces by $w_Y(M, \Delta^m, p, u, g)$ for $Y = 0, 1, \infty$. If $x \in w_1(M, \Delta^m, \bar{q}, p, u, g)$, we say that x is strongly $\Delta^m_{u_g}$ – Cesàro summable with respect to the Orlicz function M.

2. Main results

In this section we prove the main results of this paper involving $w_1(\mathbf{M}, \Delta^m, \bar{q}, p, u, g)$, $w_0(\mathbf{M}, \Delta^m, \bar{q}, p, u, g)$ and $w_\infty(\mathbf{M}, \Delta^m, \bar{q}, p, u, g)$ and obtain some inclusion relations involving these sequence spaces.

THEOREM 2.1. *Let the sequence* (p_k) *be bounded. Then* $w_1(\mathbf{M},\Delta^m,\bar{q},p,u,g)$, $w_0(\mathbf{M},\Delta^m,\bar{q},p,u,g)$ *and* $w_\infty(\mathbf{M},\Delta^m,\bar{q},p,u,g)$ *are linear spaces.*

Proof. Linearity is easy to check and thus omitted.

THEOREM 2.2. *For any sequence* $M=(M_k)$ *of Orlicz functions and a bounded sequence* $p=(p_k)$ *of strictly positive real numbers,* $w_0(\mathbf{M},\Delta^m,\bar{q},p,u,g)$ *is paranormed space (not necessarily total paranormed) with*

$$g_\Delta(x)=\sum_{i=1}^{m} g(x_i)+\inf_{\rho>0\, n\geqslant 1}\{\rho^{\frac{p_n}{H}}:\sup_k\{[M_k(g(\tfrac{u_k\Delta^m x_k}{\rho}))]\}\leqslant 1,\rho>0,n\in\mathbb{N}\},$$

where $H=\max(1,\sup_k p_k)$.

Proof. Clearly $g_\Delta(x)=g_\Delta(-x)$. Since $M_k(0)=0$, for all $k\in\mathbb{N}$ we get $\inf\{\rho^{\frac{p_n}{H}}\}=0$ for $x=\theta$. Now let $x,y\in w_0(\mathbf{M},\Delta^m,\bar{q},p,u,g)$ and let us choose $\rho_1>0$ and $\rho_2>0$ such that

$$\sup_k[M_k(g(\frac{u_k\Delta^m x_k}{\rho_1}))]\leqslant 1$$

and

$$\sup_k[M_k(g(\frac{u_k\Delta^m x_k}{\rho_2}))]\leqslant 1.$$

Let $\rho=\rho_1+\rho_2$. Then we get the triangle inequality from the following inequality

$$\sup_k[M_k(g(\frac{u_k\Delta^m(x_k+y_k)}{\rho}))]\leqslant(\frac{\rho_1}{\rho_1+\rho_2})\sup_k[M_k(g(\frac{u_k\Delta^m x_k}{\rho_1+\rho_2}))] + (\frac{\rho_2}{\rho_1+\rho_2})\sup_k[M_k(g(\frac{u_k\Delta^m y_k}{\rho_1+\rho_2}))]\leqslant 1.$$

Finally let λ be a given non-zero scalar, then the continuity of the scalar multiplication follows from the following equality

$$\begin{aligned} g_\Delta(\lambda x)=&\sum_{i=1}^{m} g(\lambda x_i)+\inf\{\rho^{\frac{p_n}{H}}:\sup_k\{[M_k(g(\frac{u_k\Delta^m(\lambda x_k)}{\rho}))]\}\leqslant 1\}\\ =&|\lambda|\sum_{i=1}^{m} g(x_i)+\inf\{(|\lambda|s)^{\frac{p_n}{H}}:\sup_k\{[M_k(g(\frac{u_k\Delta^m(x_k)}{s}))]\}\leqslant 1\},\end{aligned}$$

where $s=\frac{\rho}{|\lambda|}$. This completes the proof.

The proof of the following results are easy and thus omitted.

THEOREM 2.3. *Let* (M_k) *and* (T_k) *be sequences of Orlicz functions. For any two sequences* $p=(p_k)$ *and* $t=(t_k)$ *of positive real numbers and for any two seminorms* g_1 *and* g_2 *we have*

(*i*) *If* g_1 *is stronger than* g_2, *then* $w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g_1)\subset w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g_2)$,

(*ii*) $w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g_1)\cap w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g_2)\subset w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g_1+g_2)$,

(*iii*) $w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g)\cap w_Y(\mathbf{T},\Delta^m,\bar{q},p,u,g)\subset w_Y(\mathbf{M}+\mathbf{T},\Delta^m,\bar{q},p,u,g)$,

(*iv*) $w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g_1)\cap w_Y(\mathbf{M},\Delta^m,\bar{q},t,u,g_2)\neq\emptyset$,

(v) *The inclusions* $w_Y(\mathbf{M},\Delta^{m-1},\bar{q},g) \subset w_Y(\mathbf{M},\Delta^m,\bar{q},g)$ *are strict. In general* $w_Y(\mathbf{M},\Delta^i,\bar{q},g) \subset w_Y(\mathbf{M},\Delta^m,\bar{q},g)$ *for* $i = 1,2,...,m-1$ *and the inclusion is strict.*

THEOREM 2.4. (i) *The sequence spaces* $w_0(\mathbf{M},\bar{q},p,u,g)$ *and* $w_\infty(\mathbf{M},\bar{q},p,u,g)$ *are monotone.*

(ii) *The space* $w_1(\mathbf{M},\bar{q},p,u,g)$ *is not monotone as such is not solid.*

Proof. The proof of (i) is trivial and hence omitted. We refer [5] for the definition.

(ii) The space $w_1(\mathbf{M},\bar{q},p,u,g)$ is not monotone follows from the following example.

EXAMPLE 1. Let $p_k = 1+\frac{1}{k}$, $u_k = 1$, $\bar{q}_k = (1,2,1,2,...)$ for all $k \in \mathbb{N}$, $M_k(x) = x^p$, for all $k \in \mathbb{N}$ and for some $p \geqslant 1$, and $X = \ell_\infty$, $g((x^i)) = \sup_i |x^i|$, for $(x^i) \in \ell_\infty$. Consider the sequence (x_k) defined by $x_k = e = (1,1,1,1,...)$ for all $k \in \mathbb{N}$. Consider the J^{th} stepspace E_J for a sequence space E defined as, for $(x_k) \in E$, (y_k) is the J^{th} canonical preimage of (x_k) *i.e.* $(y_k) \in E_J$ implies $y_k = x_k$, if k is odd and $y_k = \theta$, otherwise. Then $(y_k) \notin E$. Hence the space $w_1(M,\bar{q},p,u,g)$ is not monotone.

THEOREM 2.5. *Let* $0 < p_k \leqslant r_k$ *for all* $k \in \mathbb{N}$ *and* $(\frac{r_k}{p_k})$ *be bounded, then* $w_Y(\mathbf{M},\Delta^m,\bar{q},r,u,g) \subseteq w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g)$.

Proof. If we take $w_k = [M_k(q(\frac{u_k\Delta^m x_k - L}{\rho}))]^{r_k}$ for all $k \in \mathbb{N}$, then following the technique applied for proving Theorem 5 by Maddox [10], the Theorem can be easily proved.

THEOREM 2.6. (i) *Let* (M_k) *be a sequence of Orlicz functions. Then* $w_0(\mathbf{M},\Delta^m,\bar{q},p,u,g) \subset w_1(\mathbf{M},\Delta^m,\bar{q},p,u,g) \subset w_\infty(\mathbf{M},\Delta^m,\bar{q},p,u,g)$ *and the inclusions are strict,*

(ii) *If* $|u_k| \leqslant 1$ *then* $w_Y(\mathbf{M},\Delta^m,\bar{q},p,g) \subset w_Y(\mathbf{M},\Delta^m,\bar{q},p,u,g)$ *for* $Y = 0$ or ∞.

Proof. (i) The inclusion $w_0(\mathbf{M},\Delta^m,\bar{q},p,u,g) \subset w_1(\mathbf{M},\Delta^m,\bar{q},p,u,g)$ is obvious. Now let $x \in w_1(\mathbf{M},\Delta^m,\bar{q},p,u,g)$. Then there exists some positive number ρ such that

$$\frac{1}{\bar{Q}_n}\sum_{k=1}^{n}\bar{q}_k[M(g(\frac{u_k\Delta^m x_k - L}{\rho}))]^{p_k} \to 0.$$

Since M_k is non decreasing and convex, we have

$$\frac{1}{\bar{Q}_n}\sum_{k=1}^{n}\bar{q}_k[M(g(\frac{u_k\Delta^m x_k}{\rho}))]^{p_k} \leqslant \frac{D}{\bar{Q}_n}\sum_{k=1}^{n}\bar{q}_k[M(g(\frac{u_k\Delta^m x_k - L}{\rho}))]^{p_k} + D\max[1,M(\frac{g(L)}{\rho})]^H,$$

where $\sup_k p_k = G$ and $D = \max(1,2^{G-1})$. Thus $x \in w_\infty(\mathbf{M},\Delta^m,\bar{q},p,u,g)$. To show that the inclusions are strict consider the following example.

EXAMPLE 2. Let $p_k = 2$, for k odd and $p_k = 3$ for k even. Let $m \geqslant 0$ be given. Let $u_k = 1$, $\bar{q}_k = 1$ for all $k \in \mathbb{N}$, $M_k(x) = x$, $X = \ell_\infty$ and $g((x^i)) = \sup_i |x^i|$, where

$(x^i) \in \ell_\infty$. Then the sequence $x = (k^m, k^m, k^m, ...)$ belongs to $w_1(\mathbf{M}, \Delta^m, \bar{q}, p, u, g)$ but does not belong to $w_0(\mathbf{M}, \Delta^m, \bar{q}, p, u, g)$.

The proof of (ii) is trivial, hence omitted.

The proof of the following result follows from Theorem 2.6 (i).

COROLLARY 2.7. *$w_0(\mathbf{M}, \Delta^m, \bar{q}, p, u, g)$ and $w_1(\mathbf{M}, \Delta^m, \bar{q}, p, u, g)$ are nowhere dense subset of $w_\infty(\mathbf{M}, \Delta^m, \bar{q}, p, u, g)$.*

THEOREM 2.8. *Let $M = (M_k)$ and $T = (T_k)$ be two sequences of Orlicz functions. If M_k and T_k are equivalent for each $k \in \mathbb{N}$, then $w_Y(\mathbf{M}, \Delta^m, \bar{q}, p, u, g) = w_Y(\mathbf{T}, \Delta^m, \bar{q}, p, u, g)$.*

Proof. The proof is trivial and thus omitted.

3. $\Delta^m_{u_g}-$ statistical convergence

In this section we introduce $\Delta^m_{u_g}-$ *statistically convergent* sequences and give some inclusion relations between $\Delta^m_{u_g}-$ statistically convergent sequences and $w_1(M, \Delta^m, u, g)-$ summable sequences with respect to an Orlicz function. We also show that the spaces $S^g_u(\Delta^m) \cap \ell_\infty(u_g, \Delta^m)$ may be represented as $w_1(M, \Delta^m, u, g) \cap \ell_\infty(u_g, \Delta^m)$ spaces.

DEFINITION 2. A sequence $x = (x_k)$ in X is said to be $\Delta^m_{u_g}-$ *statistically convergent* to L if for every $g \in Q$ and $\varepsilon > 0$,

$$\delta\{k \in \mathbb{N} : g(u_k \Delta^m x_k - L) \geqslant \varepsilon\} = 0.$$

In this case we write $x_k \to L(S^g_u(\Delta^m))$, where $S^g_u(\Delta^m)$ is the set of all $\Delta^m_{u_g}-$ statistically convergent sequences.

If $X = \mathbb{C}$, $g(x) = |x|$, we shall write $S_u(\Delta^m)$ instead of $S^g_u(\Delta^m)$, and if $u = e$ we shall write $S(\Delta^m)$ instead of $S_u(\Delta^m)$. In the special case $L = 0$, we denote by $S^g_{0u}(\Delta^m)$.

THEOREM 3.1. *Let $0 < p < \infty$. Then*

(i) If $x_k \to L(w^g_u(\Delta^m))$, then $x_k \to L(S^g_u(\Delta^m))$,

(ii) If $x \in \ell_\infty(u_g, \Delta^m)$ and $x_k \to L(S^g_u(\Delta^m))$, then $x_k \to L(w^g_u(\Delta^m))$, where $\ell_\infty(u_g, \Delta^m) = \{x \in w(X) : \sup_k g(u_k \Delta^m x_k) < \infty\}$,

$$w^g_u(\Delta^m) = \{x \in w(X) : \frac{1}{n}\sum_{k=1}^{n}[g(u_k \Delta^m x_k - L)]^p \to 0, n \to \infty, \text{ for some } L\}.$$

Proof. (i) Let $x \in w^g_u(\Delta^m)$ and $\varepsilon > 0$, then we have

$$\sum_{k=1}^{n}[g(u_k \Delta^m x_k - L)]^p \geqslant \varepsilon^p |\{k \leqslant n : g(u_k \Delta^m x_k - L) \geqslant \varepsilon\}|.$$

Hence $x \in S^g_u(\Delta^m)$.

(*ii*) Suppose that $x \in \ell_\infty(u_g, \Delta^m)$, $x_k \to L(S_u^g(\Delta^m))$ and set $g(u_k\Delta^m x_k) + g(L) = K$. Let $\varepsilon > 0$ be given and $n_0(\varepsilon) \in \mathbb{N}$ such that

$$\frac{1}{n}|\{k \leqslant n : g(u_k\Delta^m x_k - L) \geqslant (\frac{\varepsilon}{2})^{\frac{1}{p}}\}| < \frac{\varepsilon}{2K^p}$$

for all $n > n_0$. Let $K_n = \{k \leqslant n : g(u_k\Delta^m x_k - L) \geqslant (\frac{\varepsilon}{2})^{\frac{1}{p}}\}$.

Now for all $n > n_0$ we have

$$\begin{aligned}\frac{1}{n}\sum_{k=1}^{n}[g(u_k\Delta^m x_k - L)]^p &= \frac{1}{n}\sum_{k\in K_n}[g(u_k\Delta^m x_k - L)]^p + \frac{1}{n}\sum_{k\notin K_n}[g(u_k\Delta^m x_k - L)]^p \\ &< \frac{1}{n}(\frac{n\varepsilon}{2K^p})K^p + \frac{1}{n}n(\frac{\varepsilon}{2}) = \varepsilon.\end{aligned}$$

Hence $x_k \to L(w_u^g(\Delta^m))$.

THEOREM 3.2. *Let M be an Orlicz function. Then $w_1(M, \Delta^m, p, u, g) \subset S_u^g(\Delta^m)$.*

Proof. Let $x \in w_1(M, \Delta^m, u, g)$, $\varepsilon > 0$ and let Σ_1 denote the sum over $k \leqslant n$ such that $g(u_k\Delta^m x_k - L) \geqslant \varepsilon$ and Σ_2 denote the sum over $k \leqslant n$ such that $g(u_k\Delta^m x_k - L) < \varepsilon$. Then

$$\begin{aligned}\frac{1}{n}\sum_{k=1}^{n}[M(g(\frac{u_k\Delta^m x_k - L}{\rho}))]^{p_k} &= \frac{1}{n}\sum_{1}[M(g(\frac{u_k\Delta^m x_k - L}{\rho}))]^{p_k} + \frac{1}{n}\sum_{2}[M(g(\frac{u_k\Delta^m x_k - L}{\rho}))]^{p_k} \\ &\geqslant \frac{1}{n}\{k \leqslant n : g(u_k\Delta^m x_k - L) \geqslant \varepsilon\} \min([M(\varepsilon_1)]^{\inf p_k}, [M(\varepsilon_1)]^G),\end{aligned}$$

it follows that $x \in S_u^g(\Delta^m)$.

THEOREM 3.3. $\ell_\infty(u_g, \Delta^m) \cap S_u^g(\Delta^m) = \ell_\infty(u_g, \Delta^m) \cap w_1(M, \Delta^m, u, g)$.

Proof. By Theorem 3.2, we need only to show that $\ell_\infty(u_g, \Delta^m) \cap S_u^g(\Delta^m) \subset \ell_\infty(u_g, \Delta^m) \cap w_1(M, \Delta^m, u, g)$. Let $t_k = u_k\Delta^m x_k - L \to \theta(S^g)$. Let Σ_1 and Σ_2 be as in the previous result. Since $x \in \ell_\infty(u_g, \Delta^m)$, there exists $K > 0$ such that $[M(g(\frac{t_k}{\rho}))] \leqslant K$ for all t_k. Then for a given $\varepsilon > 0$ and for each $n \in \mathbb{N}$, we have

$$\begin{aligned}\frac{1}{n}\sum_{k=1}^{n}[M(g(\frac{t_k}{\rho}))] &\leqslant \frac{1}{n}\sum_{1} M(g(\frac{t_k}{\rho}))] + \frac{1}{n}\sum_{2}[M(g(\frac{t_k}{\rho}))] \\ &\leqslant \frac{K}{n}|\{k \leqslant n : g(t_k) \geqslant \varepsilon\rho\}| + M(\frac{\varepsilon}{\rho}).\end{aligned}$$

Hence $x \in w_1(M, \Delta^m, u, g)$.

THEOREM 3.4. *The spaces $w_Y(M, \Delta^m, \bar{q}, p, u, g)$, $S_u^g(\Delta^m)$ and $S_{0u}^g(\Delta^m)$ are not solid for $m > 0$.*

Proof. To show that the spaces are not solid in general, consider the following example.

EXAMPLE 3. Let $p_k = 1$, $u_k = 1$, $\bar{q}_k = 1$ for all $k \in \mathbb{N}$, $M(x) = x$, $g(x) = |x|$ and $X = \mathbb{C}$. Then $x = (k^m) \in w_1(M,\Delta^m,\bar{q},p,u,g)$, $w_\infty(M,\Delta^m,\bar{q},p,u,g)$ and $S_u^g(\Delta^m)$. Let $\alpha_k = (-1)^k$ for all $k \in \mathbb{N}$, then $\alpha x \notin w_1(M,\Delta^m,\bar{q},p,u,g)$, $w_\infty(M,\Delta^m,\bar{q},p,u,g)$ and $S_u^g(\Delta^m)$. Hence $w_1(M,\Delta^m,\bar{q},p,u,g)$, $w_\infty(M,\Delta^m,\bar{q},p,u,g)$ and $S_u^g(\Delta^m)$ are not solid for $m > 0$. To show that $w_0(M,\Delta^m,\bar{q},p,u,g)$ and $S_{0u}^g(\Delta^m)$ are not solid, consider the sequence $(x_k) = (k^{m-1})$ and $\alpha_k = (-1)^k$ for all $k \in \mathbb{N}$.

THEOREM 3.5. *The spaces* $w_Y(M,\Delta^m,\bar{q},p,u,g)$, $S_u^g(\Delta^m)$ and $S_{0u}^g(\Delta^m)$ are not symmetric for $m > 0$.

Proof. To show that the spaces are not symmetric, consider the following example.

EXAMPLE 4. Under the restriction on p, u, $\bar{q}$, M, g and X as in the Example 3, consider $x = (k^m)$. Then the sequence $x = (k^m) \in w_1(M,\Delta^m,\bar{q},p,u,g)$, $w_\infty(M,\Delta^m,\bar{q},p,u,g)$ and $S_u^g(\Delta^m)$. Let $(y_k) = \{x_1,x_2,x_4,x_3,x_9,x_5,x_{16},x_6,x_{25},x_7, x_{36},x_8,x_{49},x_{10},...\}$. Then $y \notin w_1(M,\Delta^m,\bar{q},p,u,g)$, $w_\infty(M,\Delta^m,\bar{q},p,u,g)$ and $S_u^g(\Delta^m)$.

Now let us consider the sequence $x = (x_k)$ defined by

$$x_k = \begin{cases} 1, & \text{if } (2i-1)^2 \leqslant k < (2i)^2, \ i = 1,2,... \\ 4, & \text{otherwise.} \end{cases}$$

and let (y_k) be the same as above. Then $x \in S_{0u}^g(\Delta)$ but $y \notin S_{0u}^g(\Delta)$ for $m = 1$.

THEOREM 3.6. *The sequence spaces* $w_Y(M,\Delta^m,\bar{q},p,u,g)$, $S_u^g(\Delta^m)$ *and* $S_{0u}^g(\Delta^m)$ are not sequence algebra for $m > 0$.

Proof. Under the restriction on p, u, $\bar{q}$, M, g and X as in the Example 3, consider $x = (k^{m-1})$ and $y = (k^{m-1})$, then $x,y \in w_Y(M,\Delta^m,\bar{q},p,u,g)$, $S_u^g(\Delta^m)$ and $S_{0u}^g(\Delta^m)$, but $x.y \notin w_Y(M,\Delta^m,\bar{q},p,u,g)$, $S_u^g(\Delta^m)$ and $S_{0u}^g(\Delta^m)$.

COROLLARY 3.7. *The sequence spaces* $w_Y(M,\Delta^m,\bar{q},p,u,g)$, $S_u^g(\Delta^m)$ *and* $S_{0u}^g(\Delta^m)$ *are not perfect for* $m > 0$.

Proof. The proof follows from Theorem 3.4 and the definition.

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