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SOME TWO–SIDED BOUNDING INEQUALITIES FOR THE BUTZER–FLOCKE–HAUSS OMEGA FUNCTION

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Abstract. A new integral representation is obtained for the Butzer-Flocke-Hauss complete real-argument Omega function $\Omega(x)$, which is closely associated with the complex-index Bernoulli function $B_\alpha(z)$ and with the complex-index Euler function $E_\alpha(z)$. Three two-sided bounding inequalities are given for this Omega function and their efficiency is also discussed.

1. Introduction, definitions and preliminaries

In the course of their investigation of the *complex-index* Euler function $E_\alpha(z)$, Butzer, Flocke and Hauss [4] introduced the following special function:

$$\Omega(w) = 2\int_{0+}^{\frac{1}{2}} \sinh(uw)\cot(\pi u)du \qquad (w \in \mathbb{C}), \tag{1}$$

which they called the *complete Omega function* (see also [2, Definition 7.1]). On the other hand, in view of the definition of the Hilbert transform, the complete Omega function $\Omega(w)$ is the Hilbert transform $\mathcal{H}(e^{-wx})_1(0)$ at 0 of the 1-periodic function $(e^{-xw})_1$ defined by the *periodic* continuation of the following exponential function [2, p. 67]:

$$e^{-xw} \qquad \left(x \in \left[-\frac{1}{2}, \frac{1}{2}\right];\, w \in \mathbb{C}\right),$$

that is,

$$\mathcal{H}(e^{-xw})_1(0) := \text{P.V.}\int_{-\frac{1}{2}}^{\frac{1}{2}} e^{wu}\cot(\pi u)\,du = \Omega(w) \qquad (w \in \mathbb{C}),$$

where the integral is to be understood in the sense of Cauchy's P.V. (*Principal Value*) at zero.

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Recently, Butzer *et al.* [5] gave an integral expression for the Omega function $\Omega(x)$ $(x \in \mathbb{R})$, which we recall here as follows (*cf.* [5, Theorem 2]):

$$\Omega(x) = \frac{2}{\pi} \sinh\left(\frac{x}{2}\right) \int_0^\infty \cos\left(\frac{xt}{2\pi}\right) \frac{dt}{e^t + 1} \qquad (x \in \mathbb{R}). \tag{2}$$

Additional links to the various applications of the Omega function $\Omega(w)$ $(w \in \mathbb{C})$ in generating-function descriptions and allied considerations of the complex-index Euler $E_\alpha(z)$ and the complex-index Bernoulli function $B_\alpha(z)$ include (for example) [2, 3, 4].

The main features of this paper are being listed below:

(i) A new integral representation for the real-argument $\Omega(x)$ and the consequent bounding inequality;

(ii) A bounding inequality based upon (2);

(iii) A bounding inequality by means of the Chaplygin Comparation Theorem applied to a certain linear ODE [5, Theorem 1] of which $\Omega(x)$ is a solution.

An appropriate efficiency analysis is also provided for each of the bounding inequalities derived here.

Throughout this paper, we use the familiar notation $[x]$ to denote the integer part of the real number x.

2. The first set of main results

We begin by considering the following partial-fraction expansion of the Omega function (see [2, Theorem 1.3] and [4]):

$$\frac{\pi\Omega(2\pi w)}{2\sinh(\pi w)} = \sum_{n=1}^\infty \frac{(-1)^{n-1}\, n}{n^2 + w^2} \qquad (w \in \mathbb{C}), \tag{3}$$

which, for $w = x$ $(x \in \mathbb{R})$, readily yields

$$\begin{aligned} \widetilde{S}_1(x) &:= \sum_{n=1}^\infty \frac{(-1)^{n-1}\, n}{n^2 + x^2} = \sum_{n=1}^\infty (-1)^{n-1}\, n \int_0^\infty e^{-(n^2+x^2)t} dt \\ &= \int_0^\infty e^{-x^2 t} \left(\sum_{n=1}^\infty (-1)^{n-1}\, n\, e^{-n^2 t} \right) dt. \end{aligned} \tag{4}$$

The *inner* alternating Dirichlet series $\widetilde{\mathcal{D}}_{\mathbb{N}}(t)$ in (4) can be expressed as a Laplace integral as follows:

$$\widetilde{\mathcal{D}}_{\mathbb{N}}(t) = \sum_{n=1}^\infty (-1)^{n-1}\, n\, e^{-n^2 t} = t \int_0^\infty e^{-tu} \left(\sum_{\ell \in \mathbb{N}:\, \ell^2 \leqq u} (-1)^{\ell-1}\, \ell \right) du. \tag{5}$$

But the finite sum $\widetilde{\mathcal{A}}(u)$ in (5) is given by

$$\begin{aligned}\widetilde{\mathcal{A}}(u) &= \sum_{\ell=1}^{[\sqrt{u}]}(-1)^{\ell-1}\,\ell = \begin{cases} \frac{1}{2}\left([\sqrt{u}]+1\right) & ([\sqrt{u}] \quad \text{odd}) \\ -\frac{1}{2}[\sqrt{u}] & ([\sqrt{u}] \quad \text{even}) \end{cases} \\ &= \frac{[\sqrt{u}]+1}{2}\sin^2\left(\frac{\pi[\sqrt{u}]}{2}\right) - \frac{[\sqrt{u}]}{2}\cos^2\left(\frac{\pi[\sqrt{u}]}{2}\right) \\ &= \frac{1}{2}\sin^2\left(\frac{\pi[\sqrt{u}]}{2}\right) - \frac{[\sqrt{u}]}{2}\cos\left(\pi[\sqrt{u}]\right).\end{aligned} \tag{6}$$

It follows from (6) that

$$\widetilde{\mathcal{A}}(u) \equiv 0 \qquad (0 \leqq u < 1).$$

Therefore, since

$$\widetilde{\mathcal{D}}_{\mathbb{N}}(t) = \frac{t}{2}\int_1^\infty e^{-tu}\left(\sin^2\left(\frac{\pi[\sqrt{u}]}{2}\right) - [\sqrt{u}]\cos\left(\pi[\sqrt{u}]\right)\right)du,$$

by collecting all these expressions together, we conclude that

$$\begin{aligned}\widetilde{S}_1(x) &= \frac{1}{2}\int_0^\infty\int_1^\infty t\,e^{-x^2t}\left(\sin^2\left(\frac{\pi[\sqrt{u}]}{2}\right) - [\sqrt{u}]\cos\left(\pi[\sqrt{u}]\right)\right)dt\,du \\ &= \frac{1}{2}\int_1^\infty\left(\sin^2\left(\frac{\pi[\sqrt{u}]}{2}\right) - [\sqrt{u}]\cos\left(\pi[\sqrt{u}]\right)\right)\left(\int_0^\infty t\,e^{-(u+x^2)t}\,dt\right)du \\ &= \frac{1}{2}\int_1^\infty \frac{\sin^2\left(\frac{1}{2}\pi[\sqrt{u}]\right) - [\sqrt{u}]\cos\left(\pi[\sqrt{u}]\right)}{(u+x^2)^2}\,du.\end{aligned} \tag{7}$$

Some straightforward steps would now lead us from (3) to our first main result asserted by Theorem 1 below.

THEOREM 1. *For all* $x \in \mathbb{R}$, *the following integral representation holds true for* $\Omega(x)$:

$$\Omega(x) = \frac{1}{\pi}\sinh\left(\frac{x}{2}\right)\int_1^\infty \frac{\sin^2\left(\frac{1}{2}\pi[\sqrt{u}]\right) - [\sqrt{u}]\cos\left(\pi[\sqrt{u}]\right)}{\left(u + x^2/(4\pi^2)\right)^2}\,du. \tag{8}$$

We are now interested in a one-sided bounding inequality for the Omega function $\Omega(x)$, which is derived *via* the integral representation (8).

THEOREM 2. *The following one-sided bounding inequality holds true for* $\Omega(x)$:

$$|\Omega(x)| \leqq \pi\left|\sinh\left(\frac{x}{2}\right)\right|\left(\frac{4}{x^2+4\pi^2} + \frac{1}{|x|}\right) \qquad (x \in \mathbb{R}\setminus\{0\}). \tag{9}$$

Proof. By setting

$$\alpha := \frac{x^2}{4\pi^2},$$

the formula (8) gives us the following obvious consequence:

$$|\Omega(x)| \leqq \frac{1}{\pi}\left|\sinh\left(\frac{x}{2}\right)\right| \left(\underbrace{\int_1^\infty \frac{du}{(u+\alpha)^2}}_{I_1} + \underbrace{\int_1^\infty \frac{[\sqrt{u}]}{(u+\alpha)^2}\,du}_{I_2} \right). \tag{10}$$

It is not difficult to observe that

$$I_1 = \frac{4\pi^2}{x^2+4\pi^2} \tag{11}$$

and

$$\begin{aligned} I_2 &= \int_0^\infty \frac{[\sqrt{u}\,]}{(u+\alpha)^2}\,du \leqq \int_0^\infty \frac{\sqrt{u}}{(u+\alpha)^2}\,du \\ &= \frac{1}{\sqrt{\alpha}} \int_0^\infty \frac{\sqrt{u}}{(1+u)^2}\,du = \frac{1}{\sqrt{\alpha}}\,B\left(\frac{3}{2},\frac{1}{2}\right) = \frac{\pi^2}{|x|}, \end{aligned} \tag{12}$$

where $B(\lambda,\mu)$ denotes the familiar Beta function given by

$$B(\lambda,\mu) := \int_0^1 t^{\lambda-1}(1-t)^{\mu-1}\,dt = B(\mu,\lambda) \qquad (\min\{\Re(\lambda),\Re(\mu)\} > 0)$$

or, equivalently, by

$$B(\lambda,\mu) = \frac{\Gamma(\lambda)\Gamma(\mu)}{\Gamma(\lambda+\mu)}$$

in terms of the classical Gamma function.

Upon substituting from (11) and (12) into (10), we deduce the inequality (9) asserted by Theorem 2.

3. Two-sided bounding inequalities for $\Omega(x)$

Butzer *et al.* [5] showed that the real-argument Butzer-Flocke-Hauss complete Omega function $\Omega(x)$ is a particular solution of the following linear ODE (*cf.* [5, Theorem 1]):

$$\frac{dy}{dx} = \frac{1}{2}\coth\left(\frac{x}{2}\right)y - \frac{x}{\pi^3}\sinh\left(\frac{x}{2}\right)\widetilde{S}\left(\frac{x}{2\pi}\right) \qquad (x \in \mathbb{R}), \tag{13}$$

where

$$\widetilde{S}(w) = \begin{cases} \dfrac{1}{2w}\displaystyle\int_0^\infty \frac{t\sin(wt)}{e^t+1}\,dt & (w \neq 0) \\ \eta(3) & (w = 0) \end{cases} \tag{14}$$

and

$$\eta(s) := \sum_{n=1}^\infty \frac{(-1)^{n-1}}{n^s} =: (1-2^{1-s})\,\zeta(s) \qquad (\Re(s) > 0;\ s \neq 1)$$

denotes the Dirichlet Eta function, $\zeta(s)$ being the Riemann Zeta function.

Our aim in this section is first to derive a two-sided bounding inequality for $\Omega(x)$ with the help of the linear first-order ODE (13) and the Chaplygin Comparison Theorem associated with it (see, for details, [1, Section 15] and [6, Section I.1]).

Consider the Cauchy problem given by

$$\frac{dy}{dx} = f(x,y) \qquad \big(y(x_0) = y_0\big). \tag{15}$$

For a given interval $\mathbb{I}$ in $\mathbb{R}$, let $x_0 \in \mathbb{I}$ and let the functions $\varphi, \psi \in C^1(\mathbb{I})$. We say that φ and ψ are the *lower* and the *upper functions*, respectively, if

$$\varphi'(x) \leqq f\big(x, \varphi(x)\big) \quad \text{and} \quad \psi'(x) \geqq f\big(x, \psi(x)\big)$$

$$\big(x \in \mathbb{I};\ \varphi(x_0) = \psi(x_0) = y_0\big).$$

Suppose also that the function $f(x,y)$ is continuous on some domain $\mathbb{D}$ in the (x,y)-plane containing the interval $\mathbb{I}$ with the lower and upper functions φ and ψ, respectively. Then the solution $y(x)$ of the Cauchy problem (15) satisfies the following two-sided inequality:

$$\varphi(x) \leqq y(x) \leqq \psi(x) \qquad (x \in \mathbb{I}). \tag{16}$$

This is actually the so-called *Chaplygin type Differential Inequality* or the *Chaplygin type Comparison Theorem* (see [1, p. 202] and [6, pp. 3-4]).

REMARK. The two-sided bounding inequalities (17) below were stated (*without* proof) by Butzer *et al.* [5, Theorem 3]. Here we take the opportunity to give a *complete* proof of this interesting result.

THEOREM 3. *For all* $x \geqq 0$, *the following two-sided bounding inequalities hold true*:

$$\frac{1}{\pi}\sinh\left(\frac{x}{2}\right)\ln\left(\frac{\zeta(3)x^2+8\pi^2}{3x^2+2\pi^2}\right) \leqq \Omega(x) \leqq \frac{1}{\pi}\sinh\left(\frac{x}{2}\right)\ln\left(\frac{3x^2+8\pi^2}{\zeta(3)x^2+2\pi^2}\right). \tag{17}$$

Moreover, *for* $x < 0$, *the opposite inequalities hold true.*

Proof. Consider the following two-sided bounding inequalities for the *alternating* Mathieu series $\widetilde{S}(x)$ [7, Proposition 2] (see also [8]):

$$\frac{4\zeta(3)-3}{(3x^2+4)\big(\zeta(3)x^2+1\big)} < \widetilde{S}(x) < \frac{12-\zeta(3)}{(3x^2+1)\big(\zeta(3)x^2+4\big)} \qquad (x \neq 0) \tag{18}$$

and suppose that $\mathbb{I} = [0,\infty)$. Applying the bounds (18) to the ODE (13), for the Cauchy problem given by

$$\Omega'(x) = \frac{1}{2}\coth\left(\frac{x}{2}\right)\Omega(x) - \frac{x}{\pi^3}\sinh\left(\frac{x}{2}\right)\widetilde{S}\left(\frac{x}{2\pi}\right) \qquad \big(\Omega(0) = 0\big)$$

we deduce the following two inequalities:

$$\varphi'(x) \geqq \frac{1}{2}\coth\left(\frac{x}{2}\right)\varphi(x) + \frac{4\pi\big(\zeta(3)-12\big)}{(3x^2+2\pi^2)\big(\zeta(3)x^2+8\pi^2\big)}\,x\sinh\left(\frac{x}{2}\right) \tag{19}$$

and

$$\psi'(x) \leqq \frac{1}{2}\coth\left(\frac{x}{2}\right)\psi(x) + \frac{4\pi\big(3-4\zeta(3)\big)}{(3x^2+8\pi^2)\big(\zeta(3)x^2+2\pi^2\big)}\, x\sinh\left(\frac{x}{2}\right). \tag{20}$$

We point out that the initial condition $\Omega(0) = 0$ is chosen in accordance with the behaviour of the Omega function $\Omega(x)$ given by

$$\Omega(x) = \frac{2}{\pi}\sinh\left(\frac{x}{2}\right)\widetilde{S}_1\left(\frac{x}{2\pi}\right) \sim \frac{2}{\pi}\,\eta(1)\sinh\left(\frac{x}{2}\right) = o(x) \qquad (x \to 0)\,, \tag{21}$$

provided by the partial-fraction expansions (3) and (4). The solutions φ and ψ of the lower and upper ODEs (which appear in (19) and (20) *with* equalities) are given by

$$\varphi(x) = \sinh\left(\frac{x}{2}\right)\left\{C_\varphi + \frac{1}{\pi}\ln\left(\frac{\zeta(3)x^2+8\pi^2}{3x^2+2\pi^2}\right)\right\} \tag{22}$$

and

$$\psi(x) = \sinh\left(\frac{x}{2}\right)\left\{C_\psi + \frac{1}{\pi}\ln\left(\frac{3x^2+8\pi^2}{\zeta(3)x^2+2\pi^2}\right)\right\}, \tag{23}$$

respectively, C_φ and C_ψ being arbitrary constants of integration.

According to the above-stated definitions of the lower and upper functions, and keeping (16) in mind, we now find the values of the integration constants C_φ and C_ψ. Indeed, since

$$\varphi(x) \sim \sinh\left(\frac{x}{2}\right)\left(C_\varphi + \frac{2\ln 2}{\pi}\right) \qquad (x \to 0)$$

and

$$\psi(x) \sim \sinh\left(\frac{x}{2}\right)\left(C_\psi + \frac{2\ln 2}{\pi}\right) \qquad (x \to 0),$$

it follows by the constraint (21) that

$$\begin{aligned}\varphi(x) &\sim \sinh\left(\frac{x}{2}\right)\left(C_\varphi + \frac{2\ln 2}{\pi}\right)\\ &\leqq \frac{2}{\pi}\,\eta(1)\sinh\left(\frac{x}{2}\right)\\ &\leqq \sinh\left(\frac{x}{2}\right)\left(C_\psi + \frac{2\ln 2}{\pi}\right) \sim \psi(x) \qquad (x \to 0),\end{aligned}$$

that is, that

$$C_\varphi = C_\psi \equiv 0.$$

Finally, since $\Omega(x)$ is an odd function, we arrive immediately at the second assertion of Theorem 3.

4. Comparation and further analysis of the derived bounds

In this section, our main goal is to give the sharpest bounds for $\Omega(x)$ by *further* analyzing the bounding inequalities (9) and (17) in light of the known integral representation (2). Our first observation from (2) is that

$$\Omega(x) \geqq 0 \qquad (x \geqq 0)\,. \tag{24}$$

Secondly, if we estimate $\Omega(x)$ in (2) by using the fact that

$$|\cos(x)| \leqq 1 \qquad (x \in \mathbb{R}),$$

we get

$$|\Omega(x)| \leqq \left(\frac{\ln 4}{\pi}\right) \left|\sinh\left(\frac{x}{2}\right)\right| \qquad (x \in \mathbb{R}).$$

At the first sight, it is clear that

$$\ln\left(\frac{3x^2+8\pi^2}{\zeta(3)x^2+2\pi^2}\right) \leqq \ln 4 \qquad (x \in \mathbb{R})\,,$$

where the equality is attained only at $x = 0$. Therefore, we favour (9) as the most efficient of all the upper bounds for the Omega function $\Omega(x)$, which are presented here so far.

Next we put

$$\psi_\Omega(x) := \begin{cases} \dfrac{\Omega(x)}{\sinh\left(\frac{x}{2}\right)} & (x \neq 0) \\ 0 & (x = 0), \end{cases}$$

where we have set $\psi_\Omega(0) = 0$ by continuity consideration [see Equation (21)]. We assume that $x > 0$ (the opposite case can easily be handled in a similar way). Then, upon dividing (9) by $\sinh\left(\frac{x}{2}\right)$ and (17) by π, we deduce the following two-sided bounding inequalities for $\psi_\Omega(x)$:

$$-\pi\left(\frac{4}{x^2+4\pi^2}+\frac{1}{x}\right) \leqq \psi_\Omega(x) \leqq \pi\left(\frac{4}{x^2+4\pi^2}+\frac{1}{x}\right) =: a(x) \tag{25}$$

and

$$b(x) := \frac{1}{\pi}\ln\left(\frac{\zeta(3)x^2+8\pi^2}{3x^2+2\pi^2}\right) \leqq \psi_\Omega(x) \leqq \frac{1}{\pi}\ln\left(\frac{3x^2+8\pi^2}{\zeta(3)x^2+2\pi^2}\right) =: c(x)\,. \tag{26}$$

So, in view of (24), we conclude that

$$b^+(x) := \max\{b(x), 0\} \leqq \psi_\Omega(x) \leqq \min\{a(x), c(x)\}\,. \tag{27}$$

Let

$$\Delta(\psi_\Omega) := \left\{x \in \mathbb{R}_+ : \, b^+(x) \leqq \psi_\Omega(x) \leqq \min\{a(x), c(x)\}\right\}.$$

It is not difficult to see that

$$x_0 = \frac{\pi\sqrt{6}}{\sqrt{3-\zeta(3)}}$$

is the unique zero of $b(x)$ and the only one intersection of the two functional upper bounds in (9), namely (25) and (26), occurs at

$$x_1 \approx 12.7639,$$

which is found *numerically* by using $\mathcal{M}$*athematica* 5.0 (see Figure 1). Thus, by means of (27), we arrive at the sharpest possible bounds for $\Omega(x)$ based upon the various bounding inequalities presented in this paper.

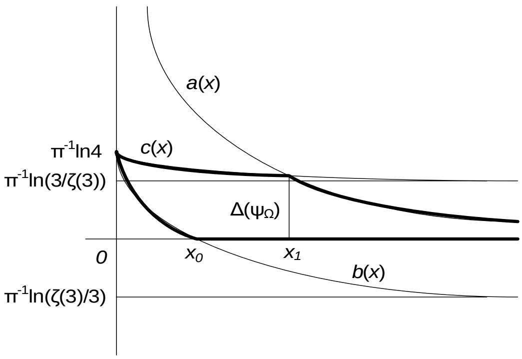

Figure 1. Bounding domain $\Delta(\psi_\Omega)$ *of* $\psi_\Omega(x) \quad (x \geqq 0)$

THEOREM 4. *The following two-sided bounding inequalities hold true for the Butzer-Flocke-Hauss complete Omega function* $\Omega(x)$:

$$L_\Omega(x) \leqq \Omega(x) \leqq U_\Omega(x) \qquad (x \geqq 0), \tag{28}$$

where

$$L_\Omega(x) = \begin{cases} \dfrac{1}{\pi}\sinh\left(\dfrac{x}{2}\right)\ln\left(\dfrac{\zeta(3)x^2+8\pi^2}{3x^2+2\pi^2}\right) & \left(0 \leqq x \leqq x_0 = \dfrac{\pi\sqrt{6}}{\sqrt{3-\zeta(3)}}\right) \\ 0 & \left(x > x_0 = \dfrac{\pi\sqrt{6}}{\sqrt{3-\zeta(3)}}\right) \end{cases} \tag{29}$$

and

$$U_\Omega(x) = \begin{cases} \dfrac{1}{\pi}\sinh\left(\dfrac{x}{2}\right)\ln\left(\dfrac{3x^2+8\pi^2}{\zeta(3)x^2+2\pi^2}\right) & (0 \leqq x \leqq x_1 \approx 12.7639) \\ \pi\sinh\left(\dfrac{x}{2}\right)\left(\dfrac{4}{x^2+4\pi^2}+\dfrac{1}{x}\right) & (x > x_1 \approx 12.7639). \end{cases} \tag{30}$$

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