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A CONDITION OF UNIFORM EXPONENTIAL STABILITY FOR SEMIGROUPS

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Abstract. The aim of this paper is to prove that the uniform exponential stability of a strongly continuous semigroup $\{T(t)\}_{t\geqslant 0}$ (acting on a complex Hilbert space H) can be derived as a consequence of the well behavior of its numerical range in a suitable Orlicz space. More precisely, assuming that there exists an Orlicz space $E=(L^{\Phi},\rho^{\Phi})$ over $\mathbf{R}_+$ such that

$$\liminf_{\alpha\downarrow 0}[\alpha||\exp_{-\alpha}||_{E^\star}]=0$$

and

$$\sup_{||x||\leqslant 1}\rho^{\Phi}(|\langle T(\cdot)x,x\rangle|)\leqslant M<\infty,$$

then the uniform growth bound ω_0 of the semigroup verifies an estimate of the form

$$\omega_0\leqslant M_\beta:=\beta-(2M||\exp_{-\beta}||_{E^\star})^{-1}<0$$

for some positive number β. As an application, the well posedness of an abstract infinite time Cauchy problem is discussed.

1. Introduction

Let H be a complex Hilbert space and let $1\leqslant p<\infty$. Recall that a semigroup $\mathbf{T}=\{T(t)\}_{t\geqslant 0}$ on H is called:

- *weakly-L^p-stable* if for every $x,y\in H$ we have

$$\int_0^\infty |\langle T(t)x,y\rangle|^p dt<\infty;$$

- *uniformly exponentially stable* if its uniform growth bound is negative, that is

$$\omega_0(\mathbf{T}):=\lim_{t\to\infty}\frac{\ln||T(t)||}{t}<0,$$

  or, equivalently, if

$$||T(t)||\leqslant Ne^{-\nu t}\qquad\text{for all } t\geqslant 0.$$

  for some positive constants N and ν.

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It is clear that each uniformly exponentially semigroup is weakly- L^p -stable. In 1983 A. J. Pritchard and J. Zabczyk [9] raised the problem whether every weakly- L^p -stable semigroup is uniformly exponentially stable. The answer is positive and a solution can be found in [3], [11]. In this note we extend their result to the more general framework of Orlicz spaces. In order to formulate our generalization we shall need a preparation on Orlicz spaces. For further details the reader is referred to [4], [5], [6], [1] and references therein.

The Orlicz spaces over $\mathbb{R}_+$ are attached to nondecreasing convex functions $\Phi : [0,\infty) \to [0,\infty]$ such that $\Phi(0) = \Phi(0+) = 0$ and Φ is not identically 0 or ∞ on $(0,\infty)$. We denote by L^Φ the set of all complex-valued measurable functions f defined on $\mathbb{R}_+$ for which there exists a positive λ such that $\int_0^\infty \Phi(\lambda|f(t)|)dt < \infty$. Clearly, L^Φ is a linear space with respect to the usual operations and we can turn L^Φ into an Orlicz space by considering on it the norm ρ^Φ, where

$$\rho^\Phi(f) := \inf\{k > 0 : \int_0^\infty \Phi(k^{-1}|f(t)|)dt \leqslant 1\}.$$

If Φ satisfies the Δ_2 -condition i.e., there exists a positive constant C such that

$$\Phi(2t) \leqslant C\Phi(t) \quad \text{for all} \quad t \geqslant 0,$$

then the dual space $\left(L^\Phi\right)^\star$ is also an Orlicz space. Moreover, in this case $\left(L^\Phi\right)^\star$ can be identified with $L^{\Phi^\star}$, where

$$\Phi^\star(t) := \sup_{s\geqslant 0}(ts - \Phi(s)), \quad t \geqslant 0$$

is the Legendre transform of Φ.

Clearly, all Lebesgue spaces $L^p(\mathbb{R}_+)$ (for $1 \leqslant p < \infty$) are examples of Orlicz spaces which satisfy the Δ_2 -condition.

We can now state our main result:

THEOREM 1. *Let* $\mathbf{T} = \{T(t)\}_{t\geqslant 0}$ *be a strongly continuous semigroup acting on a complex Hilbert space* H. *Then* $\mathbf{T}$ *is uniformly exponentially stable if (and only if) it verifies the following condition*

$$M = \sup_{||x||\leqslant 1} \rho^\Phi(|\langle T(\cdot)x, x\rangle|) < \infty, \tag{1.1}$$

with respect to an Orlicz space $E = \left(L^\Phi, \rho^\Phi\right)$ *whose dual space* $E^\star$ *has the property that*

$$\liminf_{\alpha\downarrow 0}[\alpha|| \exp_{-\alpha} ||_{E^\star}] = 0. \tag{1.2}$$

The necessity of the condition (1.1) is straightforward. In fact, if the semigroup $\mathbf{T}$ is uniformly exponentially stable, then (1.1) works for all Orlicz spaces. The sufficiency part is detailed in the next section.

As shows the case where $\mathbf{T}$ is the left translation semigroup on $H = L^2(\mathbb{R})$ and $E = L^\infty(\mathbb{R}_+)$, the condition (1.2) is essential for the validity of Theorem 1.

In the special case where $\Phi(t) = t^p$ (for $1 \leqslant p < \infty$), the result of Theorem 1 was first proved by G. Weiss [11]. Clearly, in that case the condition (1.2) is automatically fulfilled. Our result covers more general Orlicz functions Φ which satisfy the Δ_2-condition and $\lim_{t\to 0+} t\rho^{\Phi^*}(\exp_{-t}) = 0$ such as $\Phi(t) = e^t - t - 1$. In this case

$$\Phi^*(t) = (t+1)\ln(t+1) - t$$

and

$$\begin{aligned}\rho^{\Phi^*}(\exp_{-\alpha}) &= \inf\left\{k>0 : \int_0^\infty \Phi^*\Big(\frac{e^{-\alpha t}}{k}\Big)dt \leqslant 1\right\}\\ &= \inf\left\{k>0 : \frac{1}{\alpha}\int_0^{1/k} \frac{u+1}{u}\ln(u+1)du - \frac{1}{k\alpha} \leqslant 1\right\}\\ &= \sup\left\{b>0 : \int_0^{b} \frac{u+1}{u}\ln(u+1)du \leqslant b+\alpha\right\} = b_0,\end{aligned}$$

where b_0 is the unique solution of the following equation (in variable x),

$$\int_0^x \frac{u+1}{u}\ln(u+1)du = x+\alpha.$$

The map $\alpha : x \to \int_0^x \frac{u+1}{u}\ln(u+1)du - x$ (from $[0,\infty)$ into $[0,\infty)$) is surjective and also increasing, so that its inverse is continuous. Consequently α^{-1} is bounded on $[0,1]$, which yields $\lim_{t\to 0+} t\rho^{\Phi^*}(\exp_{-t}) = 0$.

J.M.A.M. van Neerven [7] has noticed that any bounded strongly continuous semigroup (acting on a complex Hilbert space H) is uniformly exponentially stable if there exists a nondecreasing function $\varphi : \mathbb{R}_+ \to \mathbb{R}_+$ such that $\varphi(t) > 0$ for $t > 0$ and

$$\int_0^\infty \varphi(|\langle T(t)x,y\rangle|)dt < \infty, \quad \text{for all } x,y \in H.$$

We leave open the question whether the boundedness condition can be dropped.

2. Proof of Theorem 1

Proof. We already noticed that only the sufficiency part needs an argument. For this, we need the remark that the condition of boundedness (1.1) yields

$$N = \sup_{||x||,||y||\leqslant 1} \rho^\Phi(|\langle T(\cdot)x,y\rangle|) \leqslant 2M < \infty,$$

as a consequence of the polarization identity

$$|\langle T(t)x,y\rangle| = \frac{1}{4}\sum_{k=0}^{3} i^k\langle T(t)(x+i^ky), x+i^ky\rangle\,.$$

The next step is to motivate the existence of the improper integral

$$\int_0^\infty u^\star(t)T(t)xdt := \lim_{s\to\infty}\int_0^s u^\star(t)T(t)xdt,$$

for all $u^\star \in E^\star$ and $x \in H.$ In terms of series, this limit means the convergence of

$$\sum_{n=0}^{\infty}\int_{s_n}^{s_{n+1}} u^\star(t)T(t)xdt \tag{2.1}$$

for all positive sequences $(s_n)_n,$ with $s_0 = 0,$ which are increasing to $\infty.$ This can be derived from a classical result due to Orlicz-Pettis, which asserts that every weakly unconditionally convergent series (in a Banach space) is also unconditionally convergent. In fact,

$$\begin{aligned}
\sum_{n=0}^{N}\left|\left\langle\int_{s_n}^{s_{n+1}} u^\star(t)T(t)xdt, y\right\rangle\right| &= \sum_{n=0}^{N} e^{i\lambda_n}\left\langle\int_{s_n}^{s_{n+1}} u^\star(t)T(t)xdt, y\right\rangle\\
&= \left\langle\sum_{n=0}^{N}\int_{s_n}^{s_{n+1}} e^{i\lambda_n}u^\star(t)T(t)xdt, y\right\rangle\\
&= \left\langle\int_0^{s_{N+1}}\left(\sum_{n=0}^{N} e^{i\lambda_n}\chi_{[s_n,s_{n+1})}\right)u^\star(t)T(t)xdt, y\right\rangle\\
&\leqslant M||u^\star||_{E^\star}||x||\,\|y\|,
\end{aligned}$$

for all $x, y \in H$ and $N \in \mathbb{N},$ which yields the weak unconditional convergence of the series (2.1).

Since

$$\left\|\int_0^s u^\star(t)T(t)xdt\right\| = \sup_{||y||\leqslant 1}\left|\left\langle\int_0^s u^\star(t)T(t)xdt, y\right\rangle\right|$$

we get also the inequality

$$\left\|\int_0^\infty u^\star(t)T(t)xdt\right\| \leqslant M||x||||u^\star||_{E^\star}.$$

As well known, the dual space of any Orlicz space is a rearrangement invariant Banach function space which contains the space $L^1(\mathbb{R}_+)\cap L^\infty(\mathbb{R}_+).$ See [1], [5], [6]. Thus for each $\beta > 0$ the function $\exp_{-\beta}$ belongs to $E^\star.$ Moreover, if $\lambda \in \mathbb{C}$ and $\operatorname{Re}\lambda > 0,$ then the improper integral $\int_0^\infty e^{-\lambda t}T(t)xdt$ exists for all $x \in X;$ necessarily, every such λ belongs to $\rho(A)$ and the formula $R(\lambda, A)x = \int_0^\infty e^{-\lambda t}T(t)xdt$ holds.

By our hypothesis (1.2), we can choose a $z_0 \in \mathbb{C}$ such that $\beta = \operatorname{Re} z_0 > 0$ and

$$M_{z_0} := \beta - (2M||\exp_{-\beta}||_{E^\star})^{-1} < 0.$$

Then for every $\lambda \in \mathbb{C}$ with $M_{z_0} < \operatorname{Re}\lambda < 0$ the point $\lambda_0 = \operatorname{Re} z_0 + i\ \operatorname{Im}\lambda$ belongs to $\rho(A)$. Since

$$\begin{aligned} |\lambda - \lambda_0| &= \operatorname{Re}\lambda_0 - \operatorname{Re}\lambda < (2M||\exp_{-\beta}||_{E^\star})^{-1} \\ &\leqslant \frac{1}{2||R(\lambda_0, A)||} < \frac{1}{||R(\lambda_0, A)||}, \end{aligned}$$

this yields that λ also belongs to $\rho(A)$ and

$$||R(\lambda, A)|| \leqslant \frac{||R(\lambda_0, A)||}{1 - |\lambda - \lambda_0|||R(\lambda_0, A)||} \leqslant 2M||\exp_{-\beta}||_{E^\star}.$$

Finally, the Gearhart-Prüss Theorem (see [2], [10]) allows us to conclude that $\omega_0(\mathbf{T}) \leqslant M_{z_0} < 0$. □

3. Applications

In this section we consider a linear operator $A : D(A) \subset H \to H$ acting on the complex Hilbert space H, that generates a strongly continuous semigroup $\mathbf{T} = \{T(t)\}_{t \geqslant 0}$.

THEOREM 2. *Under the above assumptions on* A, *if moreover*

(i) Φ *verifies the* Δ_2*-condition and* $2M = \sup_{||x|| \leqslant 1} \rho^{\Phi}(|\langle T(\cdot)x, x\rangle|) < \infty$;

(ii) *the corresponding dual function* Φ^* *is strictly increasing on* $[0, \infty)$ *and*

$$\liminf_{\alpha \downarrow 0} [\alpha||\exp_{-\alpha}||_{L^{\Phi^*}}] = 0,$$

then for each $b \in H$ *and each* $u^*(\cdot)$ *in* L^{Φ^*}, *the following infinite time Cauchy Problem*

$$(A, b, -\infty, 0) : \left\{ \begin{array}{rcl} \dot{x}(t) &=& Ax(t) + bu^*(-t) \quad \textit{for } t \leqslant 0 \\ x(-\infty) &=& \lim_{t \to -\infty} x(t) = 0, \end{array} \right.$$

has a unique solution on $(-\infty, 0]$.

Proof. First we shall prove that the function ϕ given by the improper integral

$$\phi(t) = \int_{-\infty}^{t} T(t-\tau)u^*(-\tau)bd\tau = \lim_{s \to -\infty} \int_{s}^{t} T(t-\tau)u^*(-\tau)bd\tau,$$

is correctly defined on $(-\infty, 0]$. In fact, using the Hölder inequality, for all $t_1 < t_2$ in $(-\infty, t]$, we get

$$\begin{aligned} \left\| \int_{t_1}^{t_2} T(t-\tau)u^*(-\tau)bd\tau \right\| &\leqslant \sup_{||y|| \leqslant 1} \int_{t_1}^{t_2} |\langle T(t-\tau)b, y\rangle| \cdot |u^*(-\tau)|d\tau \\ &\leqslant \sup_{||y|| \leqslant 1} \int_{t-t_2}^{t-t_1} |\langle T(\rho)b, y\rangle| \cdot |u^*(\rho - t)|d\rho \\ &\leqslant \sup_{||y|| \leqslant 1} \int_{0}^{\infty} 1_{[t-t_2, t-t_1]}(\rho)|\langle T(\rho)b, y\rangle| \cdot |u^*(\rho - t)|d\rho \\ &\leqslant M||b||\rho^{\Phi^*}(1_{[t-t_2, t-t_1]}(\cdot)|u^*(\cdot - t)|). \end{aligned}$$

Taking into account that L^{Φ^*} is rearrangement invariant, we have the relations

$$\begin{aligned}\rho^{\Phi^*}(1_{[t-t_2,t-t_1]}(\cdot)|u^*(\cdot-t)|) &= \rho^{\Phi^*}(1_{[-t_2,-t_1]}(t+\cdot)|u^*(\cdot)|)\\ &= \rho^{\Phi^*}(1_{[-t-t_2,-t-t_1]}(\cdot)|u^*(\cdot)|).\end{aligned}$$

Put $s_1=-t-t_2$ and $s_2=-t-t_1$. Then $0\leqslant -t\leqslant s_1<s_2<\infty$ and, conversely, all such pairs s_1,s_2 come this way.

Given $0<\eta\leqslant 1$, the function $\frac{1}{\eta}u^*(\cdot)$ belongs to L^{Φ^*}, which yields $\int_0^\infty \Phi^*\left(\frac{1}{\eta}|u^*(\tau)|\right)d\tau<\infty$. Therefore there exists $\delta>0$ such that for all $\delta\leqslant s_1<s_2<\infty$ we have

$$\int_{s_1}^{s_2}\Phi^*\left(\frac{1}{\eta}|u^*(\tau)|\right)d\tau=\int_0^\infty \Phi^*\left(1_{[s_1,s_2]}(\tau)\frac{1}{\eta}|u^*(\tau)|\right)d\tau\leqslant\eta\leqslant 1.$$

This gives us

$$\rho^{\Phi^*}(1_{[-t-t_2,-t-t_1]}(\cdot)|u^*(\cdot)|)\leqslant\eta,$$

whenever $t_1<t_2<-\delta$. In fact,

$$\eta\in\{k>0:\int_{s_1}^{s_2}\Phi^*\left(\frac{1}{k}|u^*(\tau)|\right)d\tau=\int_0^\infty\Phi^*\left(1_{[s_1,s_2]}\frac{1}{k}|u^*(\tau)|\right)d\tau\leqslant 1\}.$$

Clearly, ϕ verifies the integral equation:

$$x(t)=T(t-s)x(s)+\int_s^t T(t-\tau)u^*(-\tau)bd\tau,\quad s\leqslant t\leqslant 0.$$

Moreover, for each $t<0$ we have

$$\begin{aligned}\|\phi(t)\| &= \left\|\int_{-\infty}^t T(t-\tau)u^*(-\tau)bd\tau\right\|\\ &= \sup_{\|y\|\leqslant 1}\int_0^\infty |\langle T(\rho)b,y\rangle|\cdot|u^*(\rho-t)|d\rho\\ &\leqslant M\|b\|\rho^{\Phi^*}(|u^*(\cdot-t)|)=\rho^{\Phi^*}(1_{[-t,\infty)}(\cdot)|u^*(\cdot)|).\end{aligned}$$

On the other hand $\rho^{\Phi^*}(1_{[-t,\infty)}(\cdot)|u^*(\cdot)|)\to 0$ as $t\to-\infty$. Indeed, for $1\geqslant\varepsilon>0$ arbitrarily fixed and $t<0$ sufficiently small, we have

$$\int_0^\infty\Phi^*\left(1_{[-t,\infty)}(s)\frac{|u^*(s)|}{\varepsilon}\right)ds=\int_0^\infty\Phi^*\left(\frac{1}{\varepsilon}|u^*(s)|\right)ds<\varepsilon.$$

Then $\lim_{t\to-\infty}\phi(t)=0$, which ends the proof of the fact that ϕ is a solution of the problem $(A,b,-\infty,0)$. □

THEOREM 3. *Assume that Φ satisfies the condition (1.2). If for each $b\in H$ and each $u^*(\cdot)\in(L^\Phi)^*$ the infinite time Cauchy Problem $(A,b,-\infty,0)$ has a unique solution, then the semigroup generated by A is uniformly exponentially stable.*

Proof. Let E the set of all H-valued bounded and continuous functions g defined on $(-\infty, 0]$. Endowed with the norm $|g|_E := \sup_{t \leqslant 0} |g(t)|$, the set E becomes a Banach space. Let $b \in H$ and $h > 0$ be fixed and denote by x_{u^*} the unique solution of $(A, b, -\infty, 0)$. We will consider the bounded linear operators P and Q, defined by:

$$u^* \mapsto Qu^* := x_{u^*} : (L^\Phi)^* \to E \quad \text{and} \quad g \mapsto Pg := g(0) : E \to H.$$

Since PQ is bounded we infer the existence of a positive constant K_b such that

$$\left\| \int_{-\infty}^{0} T(-\tau) u^*(-\tau) d\tau \right\| \leqslant K_b ||u^*||_{(L^\Phi)^*} \quad \text{for all } u^* \in (L^\Phi)^*.$$

Then for each $u^* \in (L^\Phi)^*$ with $u^*(s) = 0$ for all $s > h$, we have that

$$\left| \int_0^T \langle T(\tau)b, y \rangle \, u^*(\tau) d\tau \right| \leqslant K_b ||u^*||_{(L^\Phi)^*} \text{ for all } y \in H, \quad ||y|| \leqslant 1,$$

and because $(L^\Phi)^*$ is a Banach function space, the previous inequality actually works for all $u^* \in (L^\Phi)^*$. Equivalently,

$$\left| \int_0^\infty 1_{[0,h]}(\tau) \langle T(\tau)b, y \rangle \, u^*(\tau) d\tau \right| \leqslant K_b ||u^*||_{(L^\Phi)^*},$$

for all $y \in H$, $||y|| \leqslant 1$, and all $u^* \in (L^\Phi)^*$. Now it is easy to see that

$$\rho^\Phi(1_{[0,h]}(\cdot) |\langle T(\cdot)b, y \rangle|) \leqslant K_b, \quad \text{for all } y \in H, \quad ||y|| \leqslant 1.$$

Therefore

$$\rho^\Phi(|\langle T(\cdot)b, y \rangle|) \leqslant K_b, \quad \text{for all } y \in H, \quad ||y|| \leqslant 1,$$

and from Theorem 1 we can conclude that the semigroup $\mathbf{T}$ is uniformly exponentially stable. □

Assume that for each $x, y \in H$ the map $\langle T(\cdot)x, y \rangle$ defines an element of L^Φ. Then the map given by the formula

$$(x, y) \mapsto \langle T(\cdot)x, y \rangle : H \times H \to L^\Phi$$

is a continuous sesquilinear function (linear in the first variable and anti-linear in the second one). By the Closed Graph Theorem we get the existence of a positive constant M such that

$$\rho^\Phi(|\langle T(\cdot)x, y \rangle|) \leqslant M ||x|| \cdot ||y|| \quad \text{for all } x, y \in H.$$

This shows that the condition (1.1) can be replaced by the following one,

$$\int_0^\infty \Phi(|\langle T(t)x, y \rangle|) dt < \infty, \quad \text{for all } x, y \in H. \tag{3.1}$$

We conclude our paper with an example.

Let $H = L^2[0, \pi]$ and $A : D(A) \subset H \to H$ given by $Ax = \frac{d^2x}{d\xi^2}$, where the domain $D(A)$ consists of all absolutely continuous functions $x(\cdot)$ defined on $[0, \pi]$, which verify the following three conditions: $i)$ $x(0) = x(\pi) = 0$; $ii)$ the first derivative $\frac{dx}{d\xi}$ is absolutely continuous on $[0, \pi]$; $iii)$ the second derivative $\frac{d^2x}{d\xi^2}$ belongs to H. With the above notations, for each $u^*(\cdot) \in (L^\Phi)^*$ and each $b(\cdot) \in H$, the infinite Cauchy Problem

$$\begin{aligned} &\frac{\partial y(t,\xi)}{\partial t} = \frac{\partial^2 y(t,\xi)}{\partial \xi^2} + u^*(-t)b(\xi) \quad \text{for } t \in (-\infty, 0],\ \xi \in (0, \pi) \\ &\lim_{t \to -\infty} \int_0^\pi |y(t,\xi)|^2 d\xi = 0 \end{aligned}$$

has a unique solution. Indeed, the uniform growth bound $\omega_0(\mathbf{T})$ of the semigroup $\mathbf{T}$ generated by A is equal to -1 and condition (3.1) applies (due to the fact that Φ is a convex function).

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