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SENSITIVITY ANALYSIS FOR PARAMETRIC COMPLETELY GENERALIZED STRONGLY NONLINEAR MIXED IMPLICIT QUASI–VARIATIONAL INCLUSIONS INVOLVING (H,η)–MONOTONE MAPPINGS

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Abstract. In this paper, by using a resolvent operator technique of (H,η)-monotone mappings and the property of a fixed-point set of set-valued contractive mappings, we study the behavior and sensitivity of the solutions of the parametric completely generalized strongly nonlinear mixed implicit quasi-variational inclusions in Hilbert space. Our results extend and improve some recent results in this field.

1. Introduction

It is well known that variational inequality theory and complementarity problem theory play an important and fundamental role in the study of a wide class of problems arising in differential equations, mechanics, physics, optimization and control, nonlinear programming, economics and transportation equilibrium, and engineering sciences. For this reason, various variational inclusions have been intensively studied in recent years.

Sensitivity analysis of a solution set for variational inequalities have been studied by many authors. Dafermos [16], Mukhherjee and Berma [17], Noor [18], Yen [19] used the projection technique to dealt with the sensitivity analysis of solutions for variational inequalities with single-valued mappings. Robinson [20] used the implicit function approach with normal mappings studied the sensitivity analysis of solutions for variational inequalities in finite-dimensional spaces. By using resolvent operator technique, Adly [21], Noor and Noor [22], Agrawal, Cho and Huang [23], Noor [24] studied the sensitivity analysis of solutions for the quasi-variational inclusions with single-valued mappings; Ding [25] studied the behavior and sensitivity analysis of solutions for generalized nonlinear implicit quasi-variational inclusions; Liu, Debnath, Kang and Ume [26] studied the behavior and sensitivity analysis of solutions for parametric completely generalized nonlinear implicit quasi-variational inclusions; Peng and Long [30] studied the behavior and sensitivity analysis of solutions for parametric completely generalized

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strongly nonlinear implicit quasi-variational inclusions; Ding [31] studied the behavior and sensitivity analysis of solutions for parametric completely generalized mixed implicit quasi-variational inclusions involving h-maximal monotone mappings.

Inspired and motivated by recent research works in this field, in this paper, by using implicit resolvent operator technique of (H,η)-monotone mappings and the property of fixed-point set of set-valued contractive mappings, we study the behavior and sensitivity analysis of solutions of a new class of parametric completely generalized strongly nonlinear mixed implicit quasi-variational inclusions with multi-valued and single-valued nonlinear mappings in Hilbert space. Our results extend, improve and unify the corresponding results in [17–31] and the reference therein.

2. Preliminaries and Definitions

Let $\mathscr{H}$ be a real Hilbert space with norm and inner product denoted by $\|\cdot\|$ and $\langle\cdot,\cdot\rangle$, respectively. Let $C(\mathscr{H})$ denote the families of all nonempty compact subsets of $\mathscr{H}$, and $\tilde{D}(\cdot,\cdot)$ denote the Hausdorff metric on $C(\mathscr{H})$ defined by

$$\tilde{D}(A,B) = max\{\sup_{a\in A} d(a,B), \sup_{b\in B} d(A,b)\}, \qquad \forall A,B \in C(\mathscr{H}),$$

where $d(a,B) = \inf_{b\in B}\|a-b\|$, $d(A,b) = \inf_{a\in A}\|a-b\|$. We first recall some definitions and lemmas later.

DEFINITION 2.1. ([32, 33]) Let $\eta : \mathscr{H}\times\mathscr{H} \longrightarrow \mathscr{H}$ and $H : \mathscr{H} \longrightarrow \mathscr{H}$ be two single-valued operators and $M : \mathscr{H} \longrightarrow 2^{\mathscr{H}}$ be a set-valued operator. M is said to be

(i) η-monotone if,

$$\langle x-y, \eta(u,v)\rangle \geqslant 0, \forall u,v \in \mathscr{H}, x\in Mu, y\in Mv.$$

(ii) (H,η)-monotone if M is η-monotone and $(H+\lambda M)(\mathscr{H}) = \mathscr{H}$, for all $\lambda > 0$.

REMARK 2.1. (1) If $\eta(u,v) = u-v$, then the definition of η-monotonicity is that of monotonicity and the definition of (H,η)-monotonicity becomes that of H-monotonicity. It is easy to know that if $H = I$ (the identity map on $\mathscr{H}$), then the definition of (I,η)-monotone operators is that of maximal η-monotone operators and the definition of I-monotone operators is that of maximal monotone operators. Hence, the class of (H,η)-monotone operators provides a unifying frameworks for classes of maximal monotone operators, maximal η-monotone operators, H-monotone operators.

Throughout this paper, unless otherwise stated, we always suppose that Γ is a nonempty open subset of $\mathscr{H}$ in which the parameter λ takes values, $N : \mathscr{H}\times\mathscr{H}\times\mathscr{H}\times\Gamma \longrightarrow \mathscr{H}$, $W : \mathscr{H}\times\mathscr{H}\times\Gamma \longrightarrow \mathscr{H}$, $m,n,i,j : \mathscr{H}\times\Gamma \longrightarrow \mathscr{H}$ and $H : \mathscr{H} \longrightarrow \mathscr{H}$, $\eta : \mathscr{H}\times\mathscr{H} \longrightarrow \mathscr{H}$ be single-valued mappings and $A,B,C,D,E,F,G,S : \mathscr{H}\times\Gamma \longrightarrow C(\mathscr{H})$ are multi-valued mappings, and $M : \mathscr{H}\times\mathscr{H}\times\Gamma \longrightarrow 2^{\mathscr{H}}$ is a set-valued mapping such that for each given $(z,\lambda)\in\mathscr{H}\times\Gamma$, $M(\cdot,z,\lambda) : \mathscr{H} \longrightarrow 2^{\mathscr{H}}$ is an (H,η)-monotone mapping with $(G(\mathscr{H},\lambda)-$

$m(\mathscr{H},\lambda))\cap domM(\cdot,z,\lambda)\neq\emptyset$. We consider the following parametric completely generalized strongly nonlinear mixed implicit quasi-variational inclusion problem (in short, PCGSNMIQVIP):

For each fixed $\lambda\in\Gamma$ and $w\in\mathscr{H}$, find $x(\lambda)\in\mathscr{H}$,

$$\begin{gathered} a(\lambda)\in A(i(x(\lambda),\lambda),\lambda),\ b(\lambda)\in B(x(\lambda),\lambda),\ c(\lambda)\in C(x(\lambda),\lambda),\\ d(\lambda)\in D(x(\lambda),\lambda),\ e(\lambda)\in E(x(\lambda),\lambda),\ f(\lambda)\in F(x(\lambda),\lambda),\\ g(\lambda)\in G(x(\lambda),\lambda),\ s(\lambda)\in S(x(\lambda),\lambda) \end{gathered} \tag{2.1}$$

such that $w\in W(n(e(\lambda),\lambda),a(\lambda),\lambda)-N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)+M(g(\lambda)-m(s(\lambda),\lambda),f(\lambda),\lambda)$.

Special Cases

Case 1. If $j(x,\lambda)=x$ and $S(x,\lambda)=\{x\}$ for all $(x,\lambda)\in\mathscr{H}\times\Gamma$, and $\eta(x,y)=x-y$ for all $x,y\in\mathscr{H}$, then for all $(z,\lambda)\in\mathscr{H}\times\Gamma$, $M(\cdot,z,\lambda):\mathscr{H}\longrightarrow 2^{\mathscr{H}}$ is an H-monotone mapping, and the (PCGSNMIQVIP) (2.1) collapses to the following parametric completely generalized mixed implicit quasi-variational inclusion problem:

For each fixed $\lambda\in\Gamma$ and $w\in\mathscr{H}$, find $x(\lambda)\in\mathscr{H}$,

$$\begin{gathered} a(\lambda)\in A(i(x(\lambda),\lambda),\lambda),\ b(\lambda)\in B(x(\lambda),\lambda),\ c(\lambda)\in C(x(\lambda),\lambda),\\ d(\lambda)\in D(x(\lambda),\lambda),\ e(\lambda)\in E(x(\lambda),\lambda),\\ f(\lambda)\in F(x(\lambda),\lambda),\ g(\lambda)\in G(x(\lambda),\lambda) \end{gathered} \tag{2.2}$$

such that $w\in W(n(e(\lambda),\lambda),a(\lambda),\lambda)-N(b(\lambda),c(\lambda),d(\lambda),\lambda)+M(g(\lambda)-m(x(\lambda),\lambda),f(\lambda),\lambda)$.

The problem (2.2) was introduced and studied by Ding [31].

Case 2. If $H=I$, and for all $(x,y,z,\lambda)\in\mathscr{H}\times\mathscr{H}\times\mathscr{H}\times\Gamma$, $n(x,\lambda)=x$, $i(x,\lambda)=x$, and $N(x,y,z,\lambda)=N(y,z,\lambda)$, $\eta(x,y)=x-y$, then $M(\cdot,z,\lambda):\mathscr{H}\longrightarrow 2^{\mathscr{H}}$ is an maximal monotone mapping, and the (PCGSNMIQVIP) (2.1) collapses to the following parametric completely generalized strongly nonlinear implicit quasi-variational inclusion problem:

For each fixed $\lambda\in\Gamma$ and $w\in\mathscr{H}$, find $x(\lambda)\in\mathscr{H}$,

$$\begin{gathered} a(\lambda)\in A(x(\lambda),\lambda),\ c(\lambda)\in C(x(\lambda),\lambda),\ d(\lambda)\in D(x(\lambda),\lambda),\\ e(\lambda)\in E(x(\lambda),\lambda),\ f(\lambda)\in F(x(\lambda),\lambda),\\ g(\lambda)\in G(x(\lambda),\lambda),\ s(\lambda)\in S(x(\lambda),\lambda) \end{gathered} \tag{2.3}$$

such that $w\in W(e(\lambda),a(\lambda),\lambda)-N(c(\lambda),d(\lambda),\lambda)+M(g(\lambda)-m(s(\lambda),\lambda),f(\lambda),\lambda)$.

The problem (2.3) was introduced and studied by Peng and Long [30].

It follows from [30,31] that problem (3.2) and (3.3) contains many mathematical models in [17–26] as special cases.

Now, for each fixed $\lambda \in \Gamma$, the solution set of the (PCGSNMIQVIP) (2.1) is denoted as

$$\begin{aligned} S(\lambda) = \{x(\lambda) \in \mathscr{H} : \exists a(\lambda) \in A(i(x(\lambda),\lambda),\lambda),\ b(\lambda) \in B(x(\lambda),\lambda), \\ c(\lambda) \in C(x(\lambda),\lambda),\ d(\lambda) \in D(x(\lambda),\lambda),\ e(\lambda) \in E(x(\lambda),\lambda), \\ f(\lambda) \in F(x(\lambda),\lambda),\ g(\lambda) \in G(x(\lambda),\lambda),\ s(\lambda) \in S(x(\lambda),\lambda) \\ \text{such that } w \in W(n(e(\lambda),\lambda),a(\lambda),\lambda) - N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda) \\ +M(g(\lambda) - m(s(\lambda),\lambda), f(\lambda),\lambda)\}. \end{aligned}$$

In this paper, our main aim is to study the behavior of the solution set $S(\lambda)$, and the conditions on the mappings A,B,C,D,E,F,G,S,N,W,H, η, m,n,i,j under which the function $S(\lambda)$ is continuous or Lipschitz continuous with respect to the parameter $\lambda \in \Gamma$.

LEMMA 2.1. ([32]) *Let $\eta : \mathscr{H} \times \mathscr{H} \to \mathscr{H}$ be a single-valued operator, $H : \mathscr{H} \to \mathscr{H}$ be a strictly η-monotone operator and $M : \mathscr{H} \to 2^{\mathscr{H}}$ be an (H,η)-monotone operator. Then, the operator $(H+\lambda M)^{-1}$ is single-valued.*

By Lemma 2.1, we can define the resolvent operator $R^{H,\eta}_{M,\lambda}$ as follows:

DEFINITION 2.2. ([32]) Let $\eta : \mathscr{H} \times \mathscr{H} \longrightarrow \mathscr{H}$ be a single-valued operator, $H : \mathscr{H} \longrightarrow \mathscr{H}$ be a strictly η-monotone operator and $M : \mathscr{H} \longrightarrow 2^{\mathscr{H}}$ be an (H,η)-monotone operator. Then the resolvent operator $R^{H,\eta}_{M,\lambda} : \mathscr{H} \longrightarrow \mathscr{H}$ is defined by

$$R^{H,\eta}_{M,\lambda}(x) = (H+\lambda M)^{-1}(x), \forall x \in \mathscr{H}.$$

LEMMA 2.2. ([32]) *Let $\eta : \mathscr{H} \times \mathscr{H} \longrightarrow \mathscr{H}$ be a single-valued Lipschitz continuous operator with constant $\tau > 0$, $H : \mathscr{H} \longrightarrow \mathscr{H}$ be a strongly η-monotone operator with constant $\gamma > 0$ and $M : \mathscr{H} \longrightarrow 2^{\mathscr{H}}$ be an (H,η)-monotone operator. Then, the resolvent operator $R^{H,\eta}_{M,\lambda} : \mathscr{H} \longrightarrow \mathscr{H}$ is Lipschitz continuous with constant $\frac{\tau}{\gamma}$, i.e.,*

$$\|R^{H,\eta}_{M,\lambda}(x) - R^{H,\eta}_{M,\lambda}(y)\| \leqslant \frac{\tau}{\gamma}\|x-y\|, \forall x,y \in \mathscr{H}.$$

LEMMA 2.3. *Let $\eta : \mathscr{H} \times \mathscr{H} \to \mathscr{H}$ be a single-valued operator, $H : \mathscr{H} \to \mathscr{H}$ be a strictly η-monotone operator. For each fixed $\lambda \in \Gamma$, $x(\lambda) \in \mathscr{H}$ is a solution of the (PCGSNMIQVIP) (2.1) if and only if there exist $a(\lambda) \in A(i(x(\lambda),\lambda),\lambda), b(\lambda) \in B(x(\lambda),\lambda)$, $c(\lambda) \in C(x(\lambda),\lambda)$, $d(\lambda) \in D(x(\lambda),\lambda)$, $e(\lambda) \in E(x(\lambda),\lambda)$, $f(\lambda) \in F(x(\lambda),\lambda)$, $g(\lambda) \in G(x(\lambda),\lambda)$, $s(\lambda) \in S(x(\lambda),\lambda)$, such that the following relation holds:*

$$\begin{aligned} g(\lambda) = m(s(\lambda),\lambda) + R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\lambda) - m(s(\lambda),\lambda)) \\ - \rho W(n(e(\lambda),\lambda),a(\lambda),\lambda) + \rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda) + \rho w], \end{aligned}$$

where $R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}(u) = (H + \rho M(\cdot,f(\lambda),\lambda))^{-1}(u), \forall u \in \mathscr{H}$ and $\rho > 0$ is a constant.

Proof. For each fixed $\lambda \in \Gamma$, let $x(\lambda) \in \mathscr{H}$ be a solution of the (PCGSN-MIQVIP) (2.1) with $a(\lambda) \in A(i(x(\lambda),\lambda),\lambda)$, $b(\lambda) \in B(x(\lambda),\lambda)$, $c(\lambda) \in C(x(\lambda),\lambda)$, $d(\lambda) \in D(x(\lambda),\lambda)$, $e(\lambda) \in E(x(\lambda),\lambda)$, $f(\lambda) \in F(x(\lambda),\lambda)$, $g(\lambda) \in G(x(\lambda),\lambda)$, and $s(\lambda) \in S(x(\lambda),\lambda)$ if and only if

$$\begin{aligned}
& w \in W(n(e(\lambda),\lambda),a(\lambda),\lambda) - N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda) + M(g(\lambda)\\
&\qquad - m(s(\lambda),\lambda),f(\lambda),\lambda)\\
\Longleftrightarrow\ & \rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda) - \rho W(n(e(\lambda),\lambda),a(\lambda),\lambda) + \rho w \in \rho M(g(\lambda)\\
&\qquad - m(s(\lambda),\lambda),f(\lambda),\lambda)\\
\Longleftrightarrow\ & H(g(\lambda) - m(s(\lambda),\lambda)) + \rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)\\
&\qquad - \rho W(n(e(\lambda),\lambda),a(\lambda),\lambda) + \rho w\\
&\qquad \in H(g(\lambda) - m(s(\lambda),\lambda)) + \rho M(g(\lambda) - m(s(\lambda),\lambda),f(\lambda),\lambda)\\
\Longleftrightarrow\ & H(g(\lambda) - m(s(\lambda),\lambda)) + \rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)\\
&\qquad - \rho W(n(e(\lambda),\lambda),a(\lambda),\lambda) + \rho w\\
&\qquad \in (H + \rho M(\cdot,f(\lambda),\lambda))[g(\lambda) - m(s(\lambda),\lambda)]\\
\Longleftrightarrow\ & g(\lambda) - m(s(\lambda),\lambda) = R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\lambda) - m(s(\lambda),\lambda))\\
&\qquad + \rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda) - \rho W(n(e(\lambda),\lambda),a(\lambda),\lambda) + \rho w]\\
\Longleftrightarrow\ & g(\lambda) = m(s(\lambda),\lambda) + R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\lambda) - m(s(\lambda),\lambda))\\
&\qquad - \rho W(n(e(\lambda),\lambda),a(\lambda),\lambda) + \rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda) + \rho w].
\end{aligned}$$

This completes the proof. □

REMARK 2.2. Lemma 2.1 improves and extends Theorem 3.1 in [31], Lemma 2.1 in [26], Theorem 3.1 in [25], Lemma 2.1 in [24], Lemma 2.1 in [23], Lemma 3.1 in [21], Lemma 2.1 in [22], Lemma 3.1 in [12], Lemma 2.1 in [27], Lemma 2.1 in [30].

LEMMA 2.4. ([28]) *Let (X,d) be a complete metric space and $T_1,T_2 : X \longrightarrow C(X)$ be two set-valued contractive mappings with same contractive constant $\theta \in (0,1)$, i.e.,*

$$\tilde{D}(T_i(x),T_i(y)) \leqslant \theta d(x,y), \forall x,y \in X, i = 1,2,$$

Then

$$\tilde{D}(F(T_1),F(T_2)) \leqslant \frac{1}{1-\theta}\sup_{x\in X}\tilde{D}(T_1(x),T_2(x)),$$

where $F(T_1)$ and $F(T_2)$ are fixed-point sets of T_1 and T_2, respectively.

DEFINITION 2.3. ([25, 26]) A set-valued mapping $E : \mathscr{H}\times\Gamma \longrightarrow C(\mathscr{H})$ is said to be

(i) strongly monotone in the first argument if there exists a constant $\delta > 0$ such that

$$\langle s_1 - s_2, x - y\rangle \geqslant \delta\|x-y\|^2, \forall (x,y,\lambda) \in \mathscr{H}\times\mathscr{H}\times\Gamma, s_1 \in E(x,\lambda), s_2 \in E(y,\lambda).$$

(ii) Lipschitz continuous in the first argument if there exists a constant $\sigma > 0$ such that

$$\tilde{D}(E(x,\lambda),E(y,\lambda)) \leqslant \sigma\|x-y\|, \forall(x,y,\lambda) \in \mathscr{H} \times \mathscr{H} \times \Gamma.$$

(iii) continuous in the first argument if for any $x \in X$ and given $\varepsilon > 0$, there exists a $\delta > 0$, such that for any $y \in X$ and $\| x-y \| < \delta$, we have $\tilde{D}(E(x,\lambda),E(y,\lambda)) < \varepsilon, \forall\lambda \in \Gamma$.

DEFINITION 2.4. ([25, 26]) Let $C,D : \mathscr{H} \times \Gamma \longrightarrow C(\mathscr{H})$ be two set-valued mappings and $N : \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \Gamma \longrightarrow \mathscr{H}$ be single-valued mapping.

(i) N is said to be Lipschitz continuous in the first argument if there exists a constant $L_{(N,1)} > 0$ such that

$$\|N(x,c,d,\lambda) - N(y,c,d,\lambda)\| \leqslant L_{(N,1)}\|x-y\|,$$
$$\forall(x,y,c,d,\lambda) \in \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \Gamma.$$

(ii) N is said to be relaxed Lipschitz continuous in the second argument with respect to C if there exists a constant $s > 0$ such that

$$\langle N(b,u,d,\lambda) - N(b,v,d,\lambda), x-y\rangle \leqslant -s\|x-y\|^2,$$
$$\forall(b,d,x,y,\lambda) \in \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \Gamma, u \in C(x,\lambda), v \in C(y,\lambda).$$

(iii) N is said to be generalized pseudo-contractive in the third argument with respect to D if there exists a constant $t > 0$ such that

$$\langle N(b,c,u,\lambda) - N(b,c,v,\lambda), x-y\rangle \leqslant t\|x-y\|^2,$$
$$\forall(b,c,x,y,\lambda) \in \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \Gamma, u \in D(x,\lambda), v \in D(y,\lambda).$$

In a similar way, we can define the Lipschitz continuity of N in the second and third argument.

ASSUMPTION 2.1. There is a constant $\mu > 0$ such that

$$\|R^{H,\eta}_{M(\cdot,x,\lambda),\rho}(w) - R^{H,\eta}_{M(\cdot,y,\lambda),\rho}(w)\| \leqslant \mu\|x-y\|, \forall(x,y,w,\lambda) \in \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \Gamma.$$

3. Main Results

THEOREM 3.1. *Let $\eta : \mathscr{H} \times \mathscr{H} \to \mathscr{H}$ be a Lipschitz continuous operator with constant σ. Let $H : \mathscr{H} \to \mathscr{H}$ be strongly η-monotone, Lipschitz continuous with constants γ, L_H, respectively. Let $A,B,C,D,E,F,G,S : \mathscr{H} \times \Gamma \longrightarrow C(\mathscr{H})$ be set-valued mappings such that A,B,C,D,E,F,G and S are Lipschitz continuous in the first argument with constants L_A, L_B, L_C, L_D, L_E, L_F, L_G and L_S, respectively, and G be strongly monotone in the first argument with constant $\delta > 0$. Let $N : \mathscr{H} \times \mathscr{H} \times \mathscr{H} \times \Gamma \longrightarrow \mathscr{H}$ be relaxed Lipschitz continuous in the second argument with respect to C and generalized pseudo-contractive in the third argument with respect* *to D with constants s and t, respectively.* $N(\cdot,\cdot,\cdot,\cdot)$ *be Lipschitz continuous in the first, second and third arguments with constants* $L_{(N,1)}$, $L_{(N,2)}$ *and* $L_{(N,3)}$, *respectively. Let* $W:\mathscr{H}\times\mathscr{H}\times\Gamma\longrightarrow\mathscr{H}$ *be Lipschitz continuous in the first and second arguments with constants* $L_{(W,1)}$ *and* $L_{(W,2)}$, *respectively. Let* $m,n,i,j:\mathscr{H}\times\Gamma\longrightarrow\mathscr{H}$ *be Lipschitz continuous in the first argument with constants* L_m, L_n, L_i *and* L_j, *respectively. Let* $M:\mathscr{H}\times\mathscr{H}\times\Gamma\longrightarrow 2^{\mathscr{H}}$ *be a set-valued mapping such that for each given* $(z,\lambda)\in\mathscr{H}\times\Gamma$, $M(\cdot,z,\lambda):\mathscr{H}\longrightarrow 2^{\mathscr{H}}$ *is an* (H,η)*-monotone mapping with* $(G(\mathscr{H},\lambda)-m(\mathscr{H},\lambda))\cap domM(\cdot,z,\lambda)\neq\emptyset$. *Suppose Assumption 2.1 holds and there exists a constant* $\rho>0$ *such that*

$$
\begin{aligned}
&a=\sqrt{1-2\delta+L_G^2}+L_mL_S+\frac{\sigma}{\gamma}(1+L_H(L_G+L_mL_S))+\mu L_F<1,\\
&p=L_{(N,2)}L_C+L_{(N,3)}L_D>L_{(N,1)}L_jL_B+L_{(W,1)}L_nL_E+L_{(W,2)}L_AL_i=q,\\
&s>t+\frac{\gamma q}{\sigma}(1-a)+\sqrt{(p^2-q^2)\left[1-\frac{\gamma^2}{\sigma^2}(1-a)^2\right]},
\end{aligned}
\tag{3.1}
$$

$$
\left|\rho-\frac{s-t-\frac{\gamma q}{\sigma}(1-a)}{p^2-q^2}\right|<\frac{\sqrt{[s-t-\frac{\gamma q}{\sigma}(1-a)]^2-(p^2-q^2)[1-\frac{\gamma^2}{\sigma^2}(1-a)^2]}}{p^2-q^2}.
$$

Then, for each $\lambda\in\Gamma$, *we have the following:*

(1) the solution set $S(\lambda)$ *of the (PCGSNMIQVIP) (2.1) is nonempty;*

(2) $S(\lambda)$ *is a closed set in* $\mathscr{H}$.

Proof. (1) Define a set-valued mapping $K:\mathscr{H}\times\Gamma\longrightarrow 2^{\mathscr{H}}$ by

$$
\begin{aligned}
K(x,\lambda)=&\bigcup_{a\in A(i(x,\lambda),\lambda),b\in B(x,\lambda),c\in C(x,\lambda),d\in D(x,\lambda),e\in E(x,\lambda),f\in F(x,\lambda),g\in G(x,\lambda),s\in S(x,\lambda)}\Big[x-g(\lambda)\\
&+m(s(\lambda),\lambda)+R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}\Big(H(g(\lambda)-m(s(\lambda),\lambda))\\
&-\rho W(n(e(\lambda),\lambda),a(\lambda),\lambda)+\rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)+\rho w\Big)\Big],\\
&\forall(x,\lambda)\in\mathscr{H}\times\Gamma.
\end{aligned}
$$

For any $(x,\lambda)\in\mathscr{H}\times\Gamma$, since $A(i(x,\lambda),\lambda)$, $B(x,\lambda)$, $C(x,\lambda)$, $D(x,\lambda)$, $E(x,\lambda)$, $F(x,\lambda)$, $G(x,\lambda)$, $S(x,\lambda)\in C(\mathscr{H})$, $m(\cdot,\lambda)$ and $R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}$ are Lipschitz continuous, we have $K(x,\lambda)\in C(\mathscr{H})$. Now for each fixed $\lambda\in\Gamma$, we prove that $K(x,\lambda)$ is a set-valued contractive mapping. For any $(x,\lambda),(y,\lambda)\in\mathscr{H}\times\Gamma$ and any $u\in K(x,\lambda)$, there exist $a_1\in A(i(x,\lambda),\lambda),b_1\in B(x,\lambda),c_1\in C(x,\lambda),d_1\in D(x,\lambda),e_1\in E(x,\lambda),f_1\in F(x,\lambda),g_1\in G(x,\lambda),s_1\in S(x,\lambda)$ such that

$$
\begin{aligned}
u=&x-g_1+m(s_1,\lambda)+R^{H,\eta}_{M(\cdot,f_1,\lambda),\rho}[H(g_1-m(s_1,\lambda))-\rho W(n(e_1,\lambda),a_1,\lambda)\\
&+\rho N(j(b_1,\lambda),c_1,d_1,\lambda)+\rho w].
\end{aligned}
$$

Since $A(i(y,\lambda),\lambda)$, $B(y,\lambda)$, $C(y,\lambda)$, $D(y,\lambda)$, $E(y,\lambda)$, $F(y,\lambda)$, $G(y,\lambda)$, $S(y,\lambda)\in C(\mathscr{H})$, so there exist $a_2\in A(i(y,\lambda),\lambda),b_2\in B(y,\lambda),c_2\in C(y,\lambda),d_2\in D(y,\lambda),e_2\in$

$E(y,\lambda), f_2 \in F(y,\lambda), g_2 \in G(y,\lambda), s_2 \in S(y,\lambda)$ such that

$$\begin{aligned}
\|a_1 - a_2\| &\leqslant \tilde{D}(A(i(x,\lambda),\lambda), A(i(y,\lambda),\lambda)),\\
\|b_1 - b_2\| &\leqslant \tilde{D}(B(x,\lambda), B(y,\lambda)),\\
\|c_1 - c_2\| &\leqslant \tilde{D}(C(x,\lambda), C(y,\lambda)),\\
\|d_1 - d_2\| &\leqslant \tilde{D}(D(x,\lambda), D(y,\lambda)),\\
\|e_1 - e_2\| &\leqslant \tilde{D}(E(x,\lambda), E(y,\lambda)),\\
\|f_1 - f_2\| &\leqslant \tilde{D}(F(x,\lambda), F(y,\lambda)),\\
\|g_1 - g_2\| &\leqslant \tilde{D}(G(x,\lambda), G(y,\lambda)),\\
\|s_1 - s_2\| &\leqslant \tilde{D}(S(x,\lambda), S(y,\lambda)).
\end{aligned} \tag{3.2}$$

Let

$$\begin{aligned}
v = y - g_2 + m(s_2,\lambda) + R^{H,\eta}_{M(\cdot,f_2,\lambda),\rho}[&H(g_2 - m(s_2,\lambda)) - \rho W(n(e_2,\lambda), a_2, \lambda)\\
&+ \rho N(j(b_2,\lambda), c_2, d_2, \lambda) + \rho w],
\end{aligned}$$

then we have $v \in K(y,\lambda)$. It follows that

$$\begin{aligned}
\|u - v\| = \|&x - g_1 + m(s_1,\lambda) + R^{H,\eta}_{M(\cdot,f_1,\lambda),\rho}[H(g_1 - m(s_1,\lambda)) - \rho W(n(e_1,\lambda), a_1, \lambda)\\
&+ \rho N(j(b_1,\lambda), c_1, d_1, \lambda) + \rho w]\\
&- [y - g_2 + m(s_2,\lambda) + R^{H,\eta}_{M(\cdot,f_2,\lambda),\rho}[H(g_2 - m(s_2,\lambda)) - \rho W(n(e_2,\lambda), a_2, \lambda)\\
&+ \rho N(j(b_2,\lambda), c_2, d_2, \lambda) + \rho w]]\|\\
\leqslant \|&x - g_1 + m(s_1,\lambda) - (y - g_2 + m(s_2,\lambda))\|\\
&+ \|R^{H,\eta}_{M(\cdot,f_1,\lambda),\rho}[H(g_1 - m(s_1,\lambda)) - \rho W(n(e_1,\lambda), a_1, \lambda)\\
&+ \rho N(j(b_1,\lambda), c_1, d_1, \lambda) + \rho w] - R^{H,\eta}_{M(\cdot,f_2,\lambda),\rho}[H(g_2 - m(s_2,\lambda))\\
&- \rho W(n(e_2,\lambda), a_2, \lambda) + \rho N(j(b_2,\lambda), c_2, d_2, \lambda) + \rho w]\|.
\end{aligned} \tag{3.3}$$

From the definition of $R^{H,\eta}_{M(\cdot,f,\lambda),\rho}$ and Assumption 2.1, we have

$$\begin{aligned}
&\|R^{H,\eta}_{M(\cdot,f_1,\lambda),\rho}[H(g_1 - m(s_1,\lambda)) - \rho W(n(e_1,\lambda), a_1, \lambda) + \rho N(j(b_1,\lambda), c_1, d_1, \lambda) + \rho w]\\
&\quad - R^{H,\eta}_{M(\cdot,f_2,\lambda),\rho}[H(g_2 - m(s_2,\lambda)) - \rho W(n(e_2,\lambda), a_2, \lambda) + \rho N(j(b_2,\lambda), c_2, d_2, \lambda) + \rho w]\|\\
&\leqslant \|R^{H,\eta}_{M(\cdot,f_1,\lambda),\rho}[H(g_1 - m(s_1,\lambda)) - \rho W(n(e_1,\lambda), a_1, \lambda) + \rho N(j(b_1,\lambda), c_1, d_1, \lambda) + \rho w]\\
&\quad - R^{H,\eta}_{M(\cdot,f_1,\lambda),\rho}[H(g_2 - m(s_2,\lambda)) - \rho W(n(e_2,\lambda), a_2, \lambda) + \rho N(j(b_2,\lambda), c_2, d_2, \lambda) + \rho w]\|\\
&\quad + \|R^{H,\eta}_{M(\cdot,f_1,\lambda),\rho}[H(g_2 - m(s_2,\lambda)) - \rho W(n(e_2,\lambda), a_2, \lambda) + \rho N(j(b_2,\lambda), c_2, d_2, \lambda) + \rho w]\\
&\quad - R^{H,\eta}_{M(\cdot,f_2,\lambda),\rho}[H(g_2 - m(s_2,\lambda)) - \rho W(n(e_2,\lambda), a_2, \lambda) + \rho N(j(b_2,\lambda), c_2, d_2, \lambda) + \rho w]\|\\
&\leqslant \frac{\sigma}{\gamma}\|H(g_1 - m(s_1,\lambda)) - \rho W(n(e_1,\lambda), a_1, \lambda) + \rho N(j(b_1,\lambda), c_1, d_1, \lambda) + \rho w\\
&\quad - [H(g_2 - m(s_2,\lambda)) - \rho W(n(e_2,\lambda), a_2, \lambda) + \rho N(j(b_2,\lambda), c_2, d_2, \lambda) + \rho w]\|\\
&\quad + \mu\|f_1 - f_2\|
\end{aligned}$$

$$
\begin{aligned}
&\leqslant \frac{\sigma}{\gamma}\|x-y-[H(g_1-m(s_1,\lambda))-H(g_2-m(s_2,\lambda))]\| \\
&\quad+\frac{\sigma}{\gamma}\|x-y+\rho[N(j(b_1,\lambda),c_1,d_1,\lambda)-N(j(b_2,\lambda),c_2,d_2,\lambda)]\| \\
&\quad+\frac{\sigma\rho}{\gamma}\|W(n(e_1,\lambda),a_1,\lambda)-W(n(e_2,\lambda),a_2,\lambda)\|+\mu\|f_1-f_2\| \\
&\leqslant \frac{\sigma}{\gamma}(\|x-y\|+\|H(g_1-m(s_1,\lambda))-H(g_2-m(s_2,\lambda))\|) \\
&\quad+\frac{\sigma}{\gamma}\|x-y+\rho[N(j(b_1,\lambda),c_1,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_2,\lambda)]\| \\
&\quad+\frac{\sigma\rho}{\gamma}\|N(j(b_1,\lambda),c_2,d_2,\lambda)-N(j(b_2,\lambda),c_2,d_2,\lambda)\| \\
&\quad+\frac{\sigma\rho}{\gamma}\|W(n(e_1,\lambda),a_1,\lambda)-W(n(e_2,\lambda),a_2,\lambda)\|+\mu\|f_1-f_2\|. \qquad (3.4)
\end{aligned}
$$

From (3.3) and (3.4), we have

$$
\begin{aligned}
\|u-v\| &\leqslant \|x-y-(g_1-g_2)\|+\|m(s_1,\lambda)-m(s_2,\lambda)\| \\
&\quad+\frac{\sigma}{\gamma}(\|x-y\|+\|H(g_1-m(s_1,\lambda))-H(g_2-m(s_2,\lambda))\|) \\
&\quad+\frac{\sigma}{\gamma}\|x-y+\rho[N(j(b_1,\lambda),c_1,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_2,\lambda)]\| \\
&\quad+\frac{\sigma\rho}{\gamma}\|N(j(b_1,\lambda),c_2,d_2,\lambda)-N(j(b_2,\lambda),c_2,d_2,\lambda)\| \\
&\quad+\frac{\sigma\rho}{\gamma}\|W(n(e_1,\lambda),a_1,\lambda)-W(n(e_2,\lambda),a_2,\lambda)\|+\mu\|f_1-f_2\|. \qquad (3.5)
\end{aligned}
$$

Since G is strongly monotone and Lipschitz continuous in the first argument, we have

$$
\begin{aligned}
\|x-y-(g_1-g_2)\|^2 &= \|x-y\|^2-2\langle g_1-g_2,x-y\rangle+\|g_1-g_2\|^2 \\
&\leqslant \|x-y\|^2-2\delta\|x-y\|^2+[\tilde{D}(G(x,\lambda),G(y,\lambda))]^2 \\
&\leqslant \|x-y\|^2-2\delta\|x-y\|^2+L_G^2\|x-y\|^2 \\
&= (1-2\delta+L_G^2)\|x-y\|^2. \qquad (3.6)
\end{aligned}
$$

By the Lipschitz continuity of m and S in the first argument and the Lipschitz continuity of H, we obtain

$$
\|m(s_1,\lambda)-m(s_2,\lambda)\| \leqslant L_m\|s_1-s_2\| \leqslant L_m\tilde{D}(S(x,\lambda),S(y,\lambda)) \leqslant L_mL_S\|x-y\|. \quad (3.7)
$$

And

$$
\begin{aligned}
\|H(g_1-m(s_1,\lambda))-H(g_2-m(s_2,\lambda))\| &\leqslant L_H\|g_1-m(s_1,\lambda)-[g_2-m(s_2,\lambda)]\| \\
&\leqslant L_H(\|g_1-g_2\|+\|m(s_1,\lambda)-m(s_2,\lambda)\|) \leqslant L_H(\|g_1-g_2\|+L_m\|s_1-s_2\|) \\
&\leqslant L_H(\tilde{D}(G(x,\lambda),G(y,\lambda))+L_m\tilde{D}(S(x,\lambda),S(y,\lambda))) \leqslant L_H(L_G+L_mL_S). \qquad (3.8)
\end{aligned}
$$

Since C and D are Lipschitz continuous in the first argument, $N(\cdot,\cdot,\cdot,\cdot)$ is Lipschitz continuous and relaxed Lipschitz continuous with respect to C in the second argument, and $N(\cdot,\cdot,\cdot,\cdot)$ is Lipschitz continuous and generalized pseudo-contractive with respect to D in the third argument, we have

$$\begin{aligned}
&\|x-y+\rho[N(j(b_1,\lambda),c_1,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_2,\lambda)]\|^2\\
&\quad=\|x-y\|^2+2\rho\langle N(j(b_1,\lambda),c_1,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_2,\lambda),x-y\rangle\\
&\qquad+\rho^2\|N(j(b_1,\lambda),c_1,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_2,\lambda)\|^2\\
&\quad\leqslant\|x-y\|^2+2\rho\langle N(j(b_1,\lambda),c_1,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_1,\lambda),x-y\rangle\\
&\qquad+2\rho\langle N(j(b_1,\lambda),c_2,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_2,\lambda),x-y\rangle\\
&\qquad+\rho^2(\|N(j(b_1,\lambda),c_1,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_1,\lambda)\|\\
&\qquad+\|N(j(b_1,\lambda),c_2,d_1,\lambda)-N(j(b_1,\lambda),c_2,d_2,\lambda)\|)^2\\
&\quad\leqslant\|x-y\|^2-2\rho s\|x-y\|^2+2\rho t\|x-y\|^2+\rho^2(L_{(N,2)}L_C+L_{(N,3)}L_D)^2\|x-y\|^2\\
&\quad=[1-2\rho(s-t)+\rho^2(L_{(N,2)}L_C+L_{(N,3)}L_D)^2]\|x-y\|^2.
\end{aligned}\tag{3.9}$$

Since B and j are Lipschitz continuous in the first argument, $N(\cdot,\cdot,\cdot,\cdot)$ is Lipschitz continuous in the first argument, we have

$$\begin{aligned}
&\|N(j(b_1,\lambda),c_2,d_2,\lambda)-N(j(b_2,\lambda),c_2,d_2,\lambda)\|\\
&\qquad\leqslant L_{(N,1)}\|j(b_1,\lambda)-j(b_2,\lambda)\|\leqslant L_{(N,1)}L_j\|b_1-b_2\|\\
&\qquad\leqslant L_{(N,1)}L_j\tilde{D}(B(x,\lambda),B(y,\lambda))\leqslant L_{(N,1)}L_jL_B\|x-y\|.
\end{aligned}\tag{3.10}$$

Since A, E, n, and i are Lipschitz continuous in the first argument, $W(\cdot,\cdot,\cdot)$ is Lipschitz continuous in the first and second arguments, we have

$$\begin{aligned}
&\|W(n(e_1,\lambda),a_1,\lambda)-W(n(e_2,\lambda),a_2,\lambda)\|\\
&\qquad\leqslant\|W(n(e_1,\lambda),a_1,\lambda)-W(n(e_2,\lambda),a_1,\lambda)\|\\
&\qquad+\|W(n(e_2,\lambda),a_1,\lambda)-W(n(e_2,\lambda),a_2,\lambda)\|\\
&\qquad\leqslant L_{(W,1)}\|n(e_1,\lambda)-n(e_2,\lambda)\|+L_{(W,2)}\|a_1-a_2\|\\
&\qquad\leqslant L_{(W,1)}L_n\|e_1-e_2\|+L_{(W,2)}\tilde{D}(A(i(x,\lambda),\lambda),A(i(y,\lambda),\lambda))\\
&\qquad\leqslant L_{(W,1)}L_n\tilde{D}(E(x,\lambda),E(y,\lambda))+L_{(W,2)}L_A\|i(x,\lambda)-i(y,\lambda)\|\\
&\qquad\leqslant L_{(W,1)}L_nL_E\|x-y\|+L_{(W,2)}L_AL_i\|x-y\|\\
&\qquad=(L_{(W,1)}L_nL_E+L_{(W,2)}L_AL_i)\|x-y\|.
\end{aligned}\tag{3.11}$$

Since F is Lipschitz continuous in the first argument, we have

$$\|f_1-f_2\|\leqslant L_F\|x-y\|.\tag{3.12}$$

Combining (3.5)–(3.12), we obtain

$$\begin{aligned}
\|u-v\|\leqslant{}&[\sqrt{1-2\delta+L_G^2}+L_mL_S+\frac{\sigma}{\gamma}(1+L_H(L_G+L_mL_S))\\
&+\frac{\sigma}{\gamma}\sqrt{1-2\rho(s-t)+\rho^2(L_{(N,2)}L_C+L_{(N,3)}L_D)^2}+\frac{\sigma\rho}{\gamma}(L_{(N,1)}L_jL_B
\end{aligned}$$

$$
\begin{aligned}
&+ L_{(W,1)}L_nL_E + L_{(W,2)}L_AL_i) + \mu L_F]\|x-y\| \\
&= (a+t(\rho))\|x-y\| = \theta\|x-y\|,
\end{aligned}
$$

where

$$
\begin{aligned}
a &= \sqrt{1-2\delta+L_G^2} + L_mL_S + \frac{\sigma}{\gamma}(1+L_H(L_G+L_mL_S)) + \mu L_F, \\
t(\rho) &= \frac{\sigma}{\gamma}[\sqrt{1-2\rho(s-t)+\rho^2(L_{(N,2)}L_C+L_{(N,3)}L_D)^2} \\
&\quad + \rho(L_{(N,1)}L_jL_B + L_{(W,1)}L_nL_E + L_{(W,2)}L_AL_i)],
\end{aligned}
$$

and $\theta = a + t(\rho)$.

It follows from condition (3.1) that $\theta < 1$. Then, we have

$$d(u, K(y,\lambda)) = \inf_{v\in K(y,\lambda)} \|u-v\| \leqslant \theta\|x-y\|.$$

Since $u \in K(x,\lambda)$ is arbitrary, we have

$$\sup_{u\in K(x,\lambda)} d(u, K(y,\lambda)) \leqslant \theta\|x-y\|.$$

By using same argument, we can prove

$$\sup_{v\in K(y,\lambda)} d(K(x,\lambda), v) \leqslant \theta\|x-y\|.$$

By the definition of the Hausdorff metric $\tilde{D}$ on $C(\mathscr{H})$, we obtain that for all $(x,y,\lambda) \in \mathscr{H}\times\mathscr{H}\times\Gamma$, $\tilde{D}(K(x,\lambda),K(y,\lambda)) \leqslant \theta\|x-y\|$. That is $K(x,\lambda)$ is a set-valued contractive mapping which is uniform with respect to $\lambda \in \Gamma$. By a fixed-point theorem of Nadler [28], for each $\lambda\in\Gamma$, $K(x,\lambda)$ has a fixed point $x(\lambda)\in\mathscr{H}$, i.e., $x(\lambda)\in K(x,\lambda)$. By the definition of K and Lemma 2.3, $x(\lambda)\in S(\lambda)$ is a solution of the problem (2.1) and so $S(\lambda)\neq\emptyset$ for each $\lambda\in\Gamma$.

(2) For each $\lambda\in\Gamma$, let $\{x_n\}\subset S(\lambda)$ and $x_n \longrightarrow x_0$ as $n\longrightarrow\infty$. Then we have $x_n \in K(x_n,\lambda)$ for all $n = 1,2,\cdots$. By the proof of Conclusion (1), we have

$$\tilde{D}(K(x_n,\lambda),K(x_0,\lambda)) \leqslant \theta\|x_n-x_0\|.$$

It follows that

$$
\begin{aligned}
d(x_0,K(x_0,\lambda)) &\leqslant \|x_n-x_0\| + d(x_n,K(x_0,\lambda)) \\
&\leqslant \|x_n-x_0\| + \sup_{u\in K(x_n,\lambda)} d(u,K(x_0,\lambda)) \\
&\leqslant \|x_n-x_0\| + \tilde{D}(K(x_n,\lambda),K(x_0,\lambda)) \leqslant (1+\theta)\|x_n-x_0\| \longrightarrow 0.
\end{aligned}
$$

as $n\longrightarrow\infty$.

Hence, we have $x_0\in K(x_0,\lambda)$ and $x_0\in S(\lambda)$. Therefore, $S(\lambda)$ is a closed set in $\mathscr{H}$. □

REMARK 3.1. It is easily to know that $\delta \leqslant L_G$.

THEOREM 3.2. *Under the hypotheses of Theorem 3.1, further assume*

(i) For any $x \in \mathscr{H}$, $A(x,\lambda)$, $B(x,\lambda)$, $C(x,\lambda)$, $D(x,\lambda)$, $E(x,\lambda)$, $F(x,\lambda)$, $G(x,\lambda)$, $S(x,\lambda)$, $m(x,\lambda)$, $n(x,\lambda)$, $i(x,\lambda)$, *and* $j(x,\lambda)$ *are Lipschitz continuous(or continuous) in the second argument with constants* $\ell_A, \ell_B, \ell_C, \ell_D, \ell_E, \ell_F, \ell_G, \ell_S, \ell_m, \ell_n, \ell_i$, *and* ℓ_j, *respectively;*

(ii) For any $u,v,w,p,q,f,z \in \mathscr{H}$, $\lambda \longmapsto W(u,v,\lambda)$, $\lambda \longmapsto N(w,p,,q,\lambda)$, $\lambda \longmapsto R^{H,\eta}_{M(\cdot,f,\lambda),\rho}(z)$ *are Lipschitz continuous(or continuous) with Lipschitz constants* ℓ_W, ℓ_N, ℓ_R, *respectively.*

Then the solution set $S(\lambda)$ *of the (PCGSNMIQVIP) (2.1) is a Lipschitz continuous (or continuous) mapping from* Γ *to* $\mathscr{H}$.

Proof. For any $\lambda, \overline{\lambda} \in \Gamma$, it follows from Theorem 3.1 that $S(\lambda)$ and $S(\overline{\lambda})$ are both nonempty closed subset. By the proof of Theorem 3.1, $K(x,\lambda), K(x,\overline{\lambda})$ are both set-valued contractive mappings with same contractive constant $\theta \in (0,1)$. By Lemma 2.4, we have

$$\tilde{D}(S(\lambda), S(\overline{\lambda})) \leqslant \frac{1}{1-\theta} \sup_{x\in\mathscr{H}} \tilde{D}(K(x,\lambda), K(x,\overline{\lambda})). \tag{3.13}$$

Taking any $u \in K(x,\lambda)$, there exist $a(\lambda) \in A(i(x,\lambda),\lambda), b(\lambda) \in B(x,\lambda), c(\lambda) \in C(x,\lambda), d(\lambda) \in D(x,\lambda), e(\lambda) \in E(x,\lambda), f(\lambda) \in F(x,\lambda), g(\lambda) \in G(x,\lambda), s(\lambda) \in S(x,\lambda)$ such that

$$\begin{aligned} u = x - g(\lambda) + m(s(\lambda),\lambda) + R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\lambda) - m(s(\lambda),\lambda)) \\ - \rho W(n(e(\lambda),\lambda), a(\lambda),\lambda) + \rho N(j(b(\lambda),\lambda), c(\lambda), d(\lambda),\lambda) + \rho w]. \end{aligned} \tag{3.14}$$

It is easy to see that there exist $a(\overline{\lambda}) \in A(i(x,\overline{\lambda}),\overline{\lambda})$, $b(\overline{\lambda}) \in B(x,\overline{\lambda})$, $c(\overline{\lambda}) \in C(x,\overline{\lambda})$, $d(\overline{\lambda}) \in D(x,\overline{\lambda})$, $e(\overline{\lambda}) \in E(x,\overline{\lambda})$, $f(\overline{\lambda}) \in F(x,\overline{\lambda})$, $g(\overline{\lambda}) \in G(x,\overline{\lambda})$, $s(\overline{\lambda}) \in S(x,\overline{\lambda})$ such that

$$\begin{aligned} \|a(\lambda) - a(\overline{\lambda})\| &\leqslant \tilde{D}(A(i(x,\lambda),\lambda), A(i(x,\overline{\lambda}),\overline{\lambda})), \\ \|b(\lambda) - b(\overline{\lambda})\| &\leqslant \tilde{D}(B(x,\lambda), B(x,\overline{\lambda})), \\ \|c(\lambda) - c(\overline{\lambda})\| &\leqslant \tilde{D}(C(x,\lambda), C(x,\overline{\lambda})), \\ \|d(\lambda) - d(\overline{\lambda})\| &\leqslant \tilde{D}(D(x,\lambda), D(x,\overline{\lambda})), \\ \|e(\lambda) - e(\overline{\lambda})\| &\leqslant \tilde{D}(E(x,\lambda), E(x,\overline{\lambda})), \\ \|f(\lambda) - f(\overline{\lambda})\| &\leqslant \tilde{D}(F(x,\lambda), F(x,\overline{\lambda})), \\ \|g(\lambda) - g(\overline{\lambda})\| &\leqslant \tilde{D}(G(x,\lambda), G(x,\overline{\lambda})), \\ \|s(\lambda) - s(\overline{\lambda})\| &\leqslant \tilde{D}(S(x,\lambda), S(x,\overline{\lambda})). \end{aligned} \tag{3.15}$$

Let

$$\begin{aligned} v = x - g(\overline{\lambda}) + m(s(\overline{\lambda}),\overline{\lambda}) + R^{H,\eta}_{M(\cdot,f(\overline{\lambda}),\overline{\lambda}),\rho}[H(g(\overline{\lambda}) - m(s(\overline{\lambda}),\overline{\lambda})) \\ - \rho W(n(e(\overline{\lambda}),\overline{\lambda}), a(\overline{\lambda}),\overline{\lambda}) + \rho N(j(b(\overline{\lambda}),\overline{\lambda}), c(\overline{\lambda}), d(\overline{\lambda}),\overline{\lambda}) + \rho w], \end{aligned} \tag{3.16}$$

then $v \in K(x,\overline{\lambda})$.

It follows that

$$\begin{aligned}
\|u-v\| &= \|x-g(\lambda)+m(s(\lambda),\lambda)+R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\lambda)-m(s(\lambda),\lambda))\\
&\quad -\rho W(n(e(\lambda),\lambda),a(\lambda),\lambda)+\rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)+\rho w]\\
&\quad -(x-g(\overline{\lambda})+m(s(\overline{\lambda}),\overline{\lambda})+R^{H,\eta}_{M(\cdot,f(\overline{\lambda}),\overline{\lambda}),\rho}[H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))\\
&\quad -\rho W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})+\rho N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})+\rho w])\|\\
&\leqslant \|g(\lambda)-g(\overline{\lambda})\|+\|m(s(\lambda),\lambda)-m(s(\overline{\lambda}),\overline{\lambda})\|\\
&\quad +\|R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\lambda)-m(s(\lambda),\lambda))-\rho W(n(e(\lambda),\lambda),a(\lambda),\lambda)\\
&\quad +\rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)+\rho w]-R^{H,\eta}_{M(\cdot,f(\overline{\lambda}),\overline{\lambda}),\rho}[H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))\\
&\quad -\rho W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})+\rho N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})+\rho w]\|\\
&\leqslant \|g(\lambda)-g(\overline{\lambda})\|+\|m(s(\lambda),\lambda)-m(s(\overline{\lambda}),\overline{\lambda})\|\\
&\quad +\|R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\lambda)-m(s(\lambda),\lambda))-\rho W(n(e(\lambda),\lambda),a(\lambda),\lambda)\\
&\quad +\rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)+\rho w]\\
&\quad -R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))-\rho W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})\\
&\quad +\rho N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})+\rho w]\|\\
&\quad +\|R^{H,\eta}_{M(\cdot,f(\lambda),\lambda),\rho}[H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))-\rho W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})\\
&\quad +\rho N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})+\rho w]\\
&\quad -R^{H,\eta}_{M(\cdot,f(\overline{\lambda}),\lambda),\rho}[H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))-\rho W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})\\
&\quad +\rho N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})+\rho w]\|\\
&\quad +\|R^{H,\eta}_{M(\cdot,f(\overline{\lambda}),\lambda),\rho}[H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))\\
&\quad -\rho W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})+\rho N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})+\rho w]\\
&\quad -R^{H,\eta}_{M(\cdot,f(\overline{\lambda}),\overline{\lambda}),\rho}[H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))-\rho W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})\\
&\quad +\rho N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})+\rho w]\|\\
&\leqslant \|g(\lambda)-g(\overline{\lambda})\|+\|m(s(\lambda),\lambda)-m(s(\overline{\lambda}),\overline{\lambda})\|\\
&\quad +\frac{\sigma}{\gamma}\|H(g(\lambda)-m(s(\lambda),\lambda))-\rho W(n(e(\lambda),\lambda),a(\lambda),\lambda)\\
&\quad +\rho N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)+\rho w-[H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))\\
&\quad -\rho W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})+\rho N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})+\rho w]\|\\
&\quad +\mu\|f(\lambda)-f(\overline{\lambda})\|+\ell_R\|\lambda-\overline{\lambda}\|\\
&\leqslant \|g(\lambda)-g(\overline{\lambda})\|+\|m(s(\lambda),\lambda)-m(s(\overline{\lambda}),\overline{\lambda})\|\\
&\quad +\frac{\sigma}{\gamma}\|H(g(\lambda)-m(s(\lambda),\lambda))-H(g(\overline{\lambda})-m(s(\overline{\lambda}),\overline{\lambda}))\|\\
&\quad +\frac{\sigma\rho}{\gamma}\|W(n(e(\lambda),\lambda),a(\lambda),\lambda)-W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})\|
\end{aligned}$$

$$
\begin{aligned}
&+\frac{\sigma\rho}{\gamma}\|N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)-N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})\|\\
&+\mu\|f(\lambda)-f(\overline{\lambda})\|+\ell_R\|\lambda-\overline{\lambda}\|\\
\leqslant\ &(1+\frac{\sigma L_H}{\gamma})(\|g(\lambda)-g(\overline{\lambda})\|+\|m(s(\lambda),\lambda)-m(s(\overline{\lambda}),\overline{\lambda})\|)\\
&+\frac{\sigma\rho}{\gamma}\|W(n(e(\lambda),\lambda),a(\lambda),\lambda)-W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})\|\\
&+\frac{\sigma\rho}{\gamma}\|N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)-N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})\|\\
&+\mu\|f(\lambda)-f(\overline{\lambda})\|+\ell_R\|\lambda-\overline{\lambda}\|.
\end{aligned}\tag{3.17}
$$

By the Lipschitz continuity of G in the second argument, we have

$$
\|g(\lambda)-g(\overline{\lambda})\|\leqslant\tilde{D}(G(x,\lambda),G(x,\overline{\lambda}))\leqslant\ell_G\|\lambda-\overline{\lambda}\|.\tag{3.18}
$$

By the Lipschitz continuity of m in the first and second arguments and the Lipschitz continuity of S in the second argument, we have

$$
\begin{aligned}
&\|m(s(\lambda),\lambda)-m(s(\overline{\lambda}),\overline{\lambda})\|\\
&\quad\leqslant\|m(s(\lambda),\lambda)-m(s(\overline{\lambda}),\lambda)\|+\|m(s(\overline{\lambda}),\lambda)-m(s(\overline{\lambda}),\overline{\lambda})\|\\
&\quad\leqslant L_m\|s(\lambda)-s(\overline{\lambda})\|+\ell_m\|\lambda-\overline{\lambda}\|\leqslant L_m\tilde{D}(S(x,\lambda),S(x,\overline{\lambda}))+\ell_m\|\lambda-\overline{\lambda}\|\\
&\quad\leqslant L_m\ell_S\|\lambda-\overline{\lambda}\|+\ell_m\|\lambda-\overline{\lambda}\|\leqslant(L_m\ell_S+\ell_m)\|\lambda-\overline{\lambda}\|.
\end{aligned}\tag{3.19}
$$

By the Lipschitz continuity of W in the first, second, and third arguments, the Lipschitz continuity of n and A in the first and second arguments, and the Lipschitz continuity of E and i in the second argument, we have

$$
\begin{aligned}
&\|W(n(e(\lambda),\lambda),a(\lambda),\lambda)-W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})\|\\
&\quad\leqslant\|W(n(e(\lambda),\lambda),a(\lambda),\lambda)-W(n(e(\overline{\lambda}),\overline{\lambda}),a(\lambda),\lambda)\|\\
&\qquad+\|W(n(e(\overline{\lambda}),\overline{\lambda}),a(\lambda),\lambda)-W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\lambda)\|\\
&\qquad+\|W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\lambda)-W(n(e(\overline{\lambda}),\overline{\lambda}),a(\overline{\lambda}),\overline{\lambda})\|\\
&\quad\leqslant L_{(W,1)}\|n(e(\lambda),\lambda)-n(e(\overline{\lambda}),\overline{\lambda})\|+L_{(W,2)}\|a(\lambda)-a(\overline{\lambda})\|+\ell_W\|\lambda-\overline{\lambda}\|\\
&\quad\leqslant L_{(W,1)}(L_n\ell_E+\ell_n)\|\lambda-\overline{\lambda}\|+L_{(W,2)}(L_A\ell_i+\ell_A)\|\lambda-\overline{\lambda}\|+\ell_W\|\lambda-\overline{\lambda}\|\\
&\quad\leqslant[L_{(W,1)}(L_n\ell_E+\ell_n)+L_{(W,2)}(L_A\ell_i+\ell_A)+\ell_W]\|\lambda-\overline{\lambda}\|.
\end{aligned}\tag{3.20}
$$

By the Lipschitz continuity of N in the first, second, third and fourth arguments, the Lipschitz continuity of j in the first and second arguments, and the Lipschitz continuity of B,C,D in the second argument, we have

$$
\begin{aligned}
&\|N(j(b(\lambda),\lambda),c(\lambda),d(\lambda),\lambda)-N(j(b(\overline{\lambda}),\overline{\lambda}),c(\overline{\lambda}),d(\overline{\lambda}),\overline{\lambda})\|\\
&\qquad\leqslant[L_{(N,1)}(L_j\ell_B+\ell_j)+L_{(N,2)}\ell_C+L_{(N,3)}\ell_D+\ell_N]\|\lambda-\overline{\lambda}\|.
\end{aligned}\tag{3.21}
$$

By the Lipschitz continuity of F in the second argument, we have

$$\|f(\lambda)-f(\overline{\lambda})\| \leqslant \tilde{D}(F(x,\lambda),F(x,\overline{\lambda})) \leqslant \ell_F\|\lambda-\overline{\lambda}\|. \tag{3.22}$$

In view of $(3.17)-(3.22)$, we obtain that

$$\|u-v\| \leqslant \Lambda\|\lambda-\overline{\lambda}\|,$$

where

$$\Lambda = \left(1+\frac{\sigma L_H}{\gamma}\right)(\ell_G+L_m\ell_S+\ell_m)+\frac{\sigma\rho}{\gamma}[L_{(W,1)}(L_n\ell_E+\ell_n)+L_{(W,2)}(L_A\ell_i+\ell_A)+\ell_W + L_{(N,1)}(L_j\ell_B+\ell_j)+L_{(N,2)}\ell_C+L_{(N,3)}\ell_D+\ell_N]+\mu\ell_F+\ell_R.$$

Then, we obtain

$$\sup_{u\in K(x,\lambda)} d(u,K(x,\overline{\lambda})) \leqslant \Lambda\|\lambda-\overline{\lambda}\|.$$

By using a similar argument as above, we have

$$\sup_{v\in K(x,\overline{\lambda})} d(K(x,\lambda),v) \leqslant \Lambda\|\lambda-\overline{\lambda}\|.$$

It follows that

$$\tilde{D}(K(x,\lambda),K(x,\overline{\lambda})) \leqslant \Lambda\|\lambda-\overline{\lambda}\|.$$

By Lemma 2.4,

$$\tilde{D}(S(\lambda),S(\overline{\lambda})) \leqslant \frac{\Lambda}{1-\theta}\|\lambda-\overline{\lambda}\|.$$

This proves that the solution set $S(\lambda)$ of (PCGSNMIQVIP) (2.1) is Lipschitz continuous in $\lambda\in\Gamma$. If, each mapping in Condition (i) and (ii) is assumed to be continuous in $\lambda\in\Gamma$, then by similar argument as above, we can show that $S(\lambda)$ is also continuous in $\lambda\in\Gamma$. This completes the proof. □

REMARK 3.2. Theorems 3.1 and 3.2 extend, improve and unify the corresponding results in [16–31].

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