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ON k–QUASI–M–HYPONORMAL OPERATORS

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Abstract. In this present article we introduce a new class of operators which we will be called the class of k-quasi-M-hyponormal operators that includes hyponormal an M-hyponormal operators. A part from other results, we show that following results hold for a k-quasi M-hyponormal operator T:
(i) T has the Bishop's property (β).
(ii) The spectral mapping theorem holds for the essential approximate point spectrum of T.
(iii) Every non-zero isolated point in the spectrum of T is a simple pole of the resolvent of T.

1. Introduction

Let $B(H)$ be the algebra of all bounded linear operators acting on infinite dimensional separable complex Hilbert space H. Let T be an operator in $B(H)$. As an easy extension of normal operators, hyponormal operators have been studied by many mathematicians. Though there are many unsolved interesting problems for hyponormal operators (e.g., the invariant subspace problem), one of recent trends in operator theory is studying natural extensions of hyponormal operators. So we introduce some of these non-hyponormal operators. The operator T is said to be a hyponormal operator $T^*T \geqslant TT^*$, M-hyponormal if there exists a real positive number M such that

$$M^2(T-\lambda)^*(T-\lambda) \geqslant (T-\lambda)(T-\lambda)^* \text{ for all } \lambda \in \mathbb{C},$$

quasi-M-hyponormal if there exits a real positive number M such that

$$T^*(M^2(T-\lambda)^*(T-\lambda))T \geqslant T^*(T-\lambda)(T-\lambda)^*T \text{ for all } \lambda \in \mathbb{C}.$$

It is known that the class of M-hyponormal operators contains the class of hyponormal operators. In order to generalize these classes we introduce a new a class of operators which we call the class of k-quasi-M-hyponormal operators defined as follows:

DEFINITION 1.1. An operator T is said to be a k-quasi-M-hyponormal operator if there exists a real positive number M such that

$$T^{*k}(M^2(T-\lambda)^*(T-\lambda))T^k \geqslant T^{*k}(T-\lambda)(T-\lambda)^*T^k \text{ for all } \lambda \in \mathbb{C},$$

where k is a natural number.

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It is clear that

$$\text{Hyponormal} \Rightarrow M\text{-Hyponormal} \Rightarrow k\text{-quasi-}M\text{-hyponormal.}$$

An operator $T \in B(H)$ is said to have the single-valued extension property (or SVEP) if for every open subset G of $\mathbb{C}$ and any analytic function $f : G \to H$ such that $(T-z)f(z) \equiv 0$ on G, we have $f(z) \equiv 0$ on G. For $T \in B(H)$ and $x \in H$, the set $\rho_T(x)$ is defined to consist of elements $z_0 \in \mathbb{C}$ such that there exists an analytic function $f(z)$ defined in a neighborhood of z_0, with values in H, which verifies $(T-z)f(z) = x$, and it is called the local resolvent set of T at x. We denote the complement of $\rho_T(x)$ by $\sigma_T(x)$, called the local spectrum of T at x, and define the local spectral subspace of T , $H_T(F) = \{x \in H : \sigma_T(x) \subset F\}$ for each subset F of $\mathbb{C}$. An operator $T \in B(H)$ is said to have the property (β) if for every open subset G of $\mathbb{C}$ and every sequence $f_n : G \to H$ of H-valued analytic functions such that $(T-z)f_n(z)$ converges uniformly to 0 in norm on compact subsets of G, $f_n(z)$ converges uniformly to 0 in norm on compact subsets of G. An operator $T \in B(H)$ is said to have Dunford's property (C) if $H_T(F)$ is closed for each closed subset F of $\mathbb{C}$. It is well known that

$$\text{Property } (\beta) \Rightarrow \text{ Dunford's property}(C) \Rightarrow \text{ SVEP.}$$

Let $T \in B(H)$. $\ker T$ denotes the null space of T and let $\alpha(T) = \dim \ker T$. ran T denotes the range of T and let

$$\beta(T) = \dim N(T^*) = \dim \overline{\operatorname{ran} T}^{\perp},$$

where $\overline{\operatorname{ran} T}$ denotes the closure of $\operatorname{ran} T$. T is called semi-Fredholm if it has closed range and either $\alpha(T) < \infty$ or $\beta(T) < \infty$. T is called Fredholm if it is semi-Fredholm and both $\alpha(T) < \infty, \beta(T) < \infty$. T is called Weyl if it is Fredholm of index zero, i.e., $i(T) = \alpha(T) - \beta(T) = 0$. The Weyl spectrum of T is defined by

$$w(T) = \{\lambda \in \mathbb{C} \,|T - \lambda \text{ is not Weyl }\}.$$

$\pi_{00}(T)$ denotes the set of all eigenvalues of T such that λ is an isolated point of $\sigma(T)$ and $0 < \alpha(T-\lambda) < \infty$. We write $\sigma_e(T)$ for the essential spectrum of T. We say T to be isoloid if every isolated point in $\sigma(T)$ is an eigenvalue of T. The essential approximate point spectrum $\sigma_{ea}(T)$ of T is defined by $\sigma_{ea}(T) = \bigcap_K \{\sigma_a(T+K) :$ K is a compact operator$\}$, where $\sigma_a(T)$ denotes the approximate point spectrum of T.

2. Main Results

It is well known that if T is M-hyponormal or quasi-M-hyponormal and a closed subspace $\mathscr{M}$ of H is T-invariant, then $T|_{\mathscr{M}}$ is M-hyponormal [6]. By the same way we obtain a similar result for a k-quasi-M-hyponormal operator.

PROPOSITION 2.1. *Let $\mathscr{M}$ be a closed T-invariant subspace of H. Then the restriction $T|_{\mathscr{M}}$ of a k-quasi-M-hyponormal operator T to $\mathscr{M}$ is a k-quasi-M-hyponormal operator.*

Proof. Let

$$T = \begin{pmatrix} A & C \\ 0 & B \end{pmatrix} \quad \text{on } H = \mathscr{M} \oplus \mathscr{M}^{\perp}.$$

Since T is k-quasi-M-hyponormal, there exists a real positive number M such that

$$T^{*k}(M^2(T-\lambda)^*(T-\lambda))T^k \geqslant T^{*k}(T-\lambda)(T-\lambda)^*T^k \text{ for all } \lambda \in \mathbb{C},$$

where k is a natural number. Hence

$$M^2 \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}^{*k} \begin{pmatrix} A-\lambda & C \\ 0 & B-\lambda \end{pmatrix}^{*} \begin{pmatrix} A-\lambda & C \\ 0 & B-\lambda \end{pmatrix} \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}^{k}$$

$$- \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}^{*k} \begin{pmatrix} A-\lambda & C \\ 0 & B-\lambda \end{pmatrix} \begin{pmatrix} A-\lambda & C \\ 0 & B-\lambda \end{pmatrix}^{*} \begin{pmatrix} A & C \\ 0 & B \end{pmatrix}^{k} \geqslant 0.$$

Therefore

$$\begin{pmatrix} A^{*k}\{M^2(A-\lambda)^*(A-\lambda)-(A-\lambda)(A-\lambda)^*\}A^k & E \\ F & G \end{pmatrix} \geqslant 0,$$

for some operators E, F and G. Hence

$$A^{*k}\{M^2(A-\lambda)^*(A-\lambda)-(A-\lambda)(A-\lambda)^*\}A^k \geqslant 0.$$

This implies that $A = T\,|_{\mathscr{M}}$ is k-quasi-M-hyponormal. □

The following lemma is a structural result.

LEMMA 2.1. *Let $T \in B(H)$ be a k-quasi-M-hyponormal operator, the range of T^k be not dense and*

$$T = \begin{pmatrix} T_1 & T_2 \\ 0 & T_3 \end{pmatrix} \quad \text{on } H = \overline{\operatorname{ran} T^k} \oplus \ker T^{*k}.$$

Then T_1 is M-hyponormal, $T_3^k = 0$ and $\sigma(T) = \sigma(T_1) \cup \{0\}$.

Proof. Let

$$T = \begin{pmatrix} T_1 & T_2 \\ 0 & T_3 \end{pmatrix} \quad \text{on } H = \overline{\operatorname{ran} T^k} \oplus \ker T^{*k}$$

and let P be the orthogonal projection onto $\overline{\operatorname{ran} T^k}$. Since T is k-quasi-M-hyponormal, we have

$$P(M^2(T-\lambda)^*(T-\lambda)-(T-\lambda)(T-\lambda)^*)P \geqslant 0 \text{ for all } \lambda \in \mathbb{C}.$$

Therefore

$$P(M^2(T-\lambda)^*(T-\lambda))P - P(T-\lambda)P(T-\lambda)^*)P \geqslant 0 \text{ for all } \lambda \in \mathbb{C}.$$

Hence

$$M^2(T_1-\lambda)^*(T_1-\lambda)-(T_1-\lambda)(T_1-\lambda)^* \geqslant 0 \text{ for all } \lambda \in \mathbb{C}.$$

This shows that T_1 is M-hyponormal on $\overline{\text{ran } T^k}$. Further, we have

$$\langle T_3^k x_2, y_2\rangle = \langle T^k(I-P)x,(I-P)y\rangle = \langle (I-P)x, T^{*k}(I-P)y\rangle = 0,$$

for any $x=\begin{pmatrix} x_1 \\ x_2 \end{pmatrix}, \quad y=\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} \in H$. Thus $T_3^{*k}=0$. Since $\sigma(T_3)=\{0\}$, we have

$$\sigma(T)=\sigma(T_1)\cup\{0\}. \quad \square$$

As a consequence we obtain the following corollary.

COROLLARY 2.1. *Let $T \in B(H)$ be k-quasi-M-hyponormal operator. If $T|_{\overline{\text{ran } T^k}}$ is invertible, then T is similar to a direct sum of a M-hyponormal and a nilpotent operator.*

Proof. Since by assumption $0 \notin \sigma(T_1)$, we have $\sigma(T_1)\cap\sigma(T_3)=\varnothing$. Then there exists an operator S such that $T_1S-ST_3=T_2$ [13]. Since $\begin{pmatrix} I & S \\ 0 & I \end{pmatrix}^{-1} = \begin{pmatrix} I & -S \\ 0 & I \end{pmatrix}$, hence

$$T=\begin{pmatrix} T_1 & T_2 \\ 0 & T_3 \end{pmatrix} = \begin{pmatrix} I & S \\ 0 & I \end{pmatrix}^{-1} \begin{pmatrix} T_1 & 0 \\ 0 & T_3 \end{pmatrix}\begin{pmatrix} I & S \\ 0 & I \end{pmatrix}. \quad \square$$

REMARK 2.1. A bounded operator T is said to belong to the class $H(p)$ if there exists a natural $p := p(\lambda)$ such that:

$$H_0(\lambda I - T) = \ker(\lambda I - T)^p$$

for all $\lambda \in \mathbb{C}$, where $H_0(T)$ is the quasi-nilpotent part of $T \in B(H)$ defined by

$$H_0(T)=\{x\in H: \lim_n ||T^n x||^{\frac{1}{n}}=0\}.$$

Property $H(p)$ is satisfied by every generalized scalar operator, and in particular for M-hyponormal operators on Hilbert space, see [11]. M-hyponormal operators are restrictions of generalised scalar operators and hence have property β [8, 12].

THEOREM 2.1. *Let $T \in B(H)$ be a k-quasi-M-hyponormal operator. Then T has Bishop's property (β). Hence T has the single valued extension property.*

Proof. If the range of T^k is dense, then T is M-hyponormal. Hence, T has Bishop's property (β) by Remark 2.1. So, we assume that the range of T^k is not dense. By Lemma 2.1, we have

$$T=\begin{pmatrix} T_1 & T_2 \\ 0 & T_3 \end{pmatrix} \quad \text{on } H=\overline{\text{ran } T^k}\oplus \ker T^{*k}.$$

Let D be an open subset of $\mathbb{C}$ and $f_n(z)$ be analytic functions on D to H. Assume $(T-z)f_n(z) \to 0$ uniformly on every compact subset of D. Put $f_n(z) = f_{n1}(z) \oplus f_{n2}(z)$ on $H = \overline{\operatorname{ran} T^k} \oplus \ker T^{*k}$. Then we can write

$$\begin{pmatrix} T_1 - z & T_2 \\ 0 & T_3 - z \end{pmatrix} \begin{pmatrix} f_{n1}(z) \\ f_{n2}(z) \end{pmatrix} = \begin{pmatrix} (T_1 - z)f_{n1}(z) + T_2 f_{n2}(z) \\ (T_3 - z)f_{n2}(z) \end{pmatrix} \to 0.$$

Since T_3 is nilpotent, T_3 has Bishop's property (β). Hence $f_{n2}(z) \to 0$ uniformly on every compact subset of D. Then $(T_1 - z)f_{n1}(z) \to 0$. Since T_1 is M-hyponormal, T_1 has Bishop's property (β) by Remark 2.1. Hence $f_{n1}(z) \to 0$ uniformly on every compact subset of D. Thus T has Bishop's property (β). □

As a simple consequence of the preceding result, we obtain

COROLLARY 2.2. *Let T be a k-quasi-M-hyponormal operator. Then the following assertions hold:*

(i) $\sigma_{ea}(f(T)) = f(\sigma_{ea}(T))$, *for every analytic function f on some open neighborhood of $\sigma(T)$.*

(ii) *T obeys a-Browder's theorem, that is $\sigma_{ea}(T) = \sigma_{ab}(T)$ (where $\sigma_{ab}(T) = \bigcap_K \{\sigma_a(T+K) : TK = KT$ and K is a compact operator$\}$).*

(iii) *a-Browder's theorem holds for $f(T)$ for every analytic function f on some open neighborhood of $\sigma(T)$.*

Proof. Note that above theorem implies that T has SVEP. By [2], (i) follows. Assertion (ii) is a consequence of [10, Corollary 2.3]. Since $\sigma_{ea}(f(T)) = f(ea(T))$, the rest of the argument follows as in [10, Corollary 2.3]. □

THEOREM 2.2. *An operator quasi-similar to a k-quasi-M-hyponormal operator has SVEP.*

Proof. Let T be k-quasi-M-hyponormal. Suppose S is an operator quasi-similar to T. Then there exist an injective operator A with dense range such that $AS = TA$. Let U be an open set and $f : U \mapsto H$ be an analytic function for which $(S - zI)f(z) = 0$ on U. Then $0 = A(S - zI)f(z) = (T - zI)Af(z)$ for all z in U. Since T has SVEP, we find $Af(z) = 0$. Since A is injective, it is immediate that $f(z) = 0$ for all z in U. This finishes the proof. □

It is well known that hyponormal, M-hyponormal and quasi-M-hyponormal operators are isoloids. In the following theorem we prove more.

THEOREM 2.3. *A k-quasi-M-hyponormal operator is isoloid.*

Proof. Let T be k-quasi-M-hyponormal with representation given in Lemma 2.1. Let z be an isolated point in $\sigma(T)$. Since $\sigma(T) = \sigma(T_1) \cup \{0\}$, z is an isolated point in $\sigma(T_1)$ or $z = 0$. If z isolated point in $\sigma(T_1)$, then $z \in \sigma_p(T_1)$ because M-hyponormal operators are isoloids [15]. Assume that $z = 0$ and $z \notin \sigma(T_1)$. Then for $x \in \ ker(T_3)$, $-T_1^{-1}T_2 x \oplus x \in \ker T$. This completes the proof. □

THEOREM 2.4. *Let $T \in B(H)$ be k-quasi-M-hyponormal operator. Write*

$$T = \begin{pmatrix} T_1 & T_2 \\ 0 & T_3 \end{pmatrix}$$

on $H = \overline{\operatorname{ran}(T^k)} \oplus \ker(T^{k*})$. Then the following statements hold.

(1) *$\sigma_{T_3}(x_2) \subset \sigma_T(x_1 \oplus x_2)$ and $\sigma_{T1}(x_1) = \sigma_T(x_1 \oplus 0)$ where $x_1 \oplus x_2 \in H$.*

(2) *$R_{T_1}(F) \oplus 0 \subset H_T(F)$ where $R_{T_1}(F) := \{y \in \overline{\operatorname{ran}(T^k)} : \sigma_{T_1}(y) \subset F\}$ for any set $F \subset \mathbb{C}$.*

Proof. Let $T \in B(H)$ be k-quasi-M-hyponormal. Write

$$T = \begin{pmatrix} T_1 & T_2 \\ 0 & T_3 \end{pmatrix}$$

on $H = \overline{\operatorname{ran}(T^k)} \oplus \ker(T^{k*})$, where $T_3^k = 0$ and T_1 is M-hyponormal.

(1) Let $x_1 \oplus x_2 \in H = \overline{\operatorname{ran}(T^k)} \oplus \ker(T^{k*})$. If $\lambda_0 \in \rho_T(x_1 \oplus x_2)$, then there is an H-valued analytic function f defined on a neighborhood U of λ_0 such that $(T-\lambda)f(\lambda) = x_1 \oplus x_2$ for all $\lambda \in U$. We can write $f = f_1 \oplus f_2$ where $f_1 \in O(U, \overline{\operatorname{ran}(T^k)})$ and $f_2 \in O(U, \ker(T^{k*}))$, where $O(U,H)$ denotes the Fréchet space of H-valued analytic functions on U with respect to the uniform topology. Then we get

$$\begin{pmatrix} T_1-\lambda & T_2 \\ 0 & T_3-\lambda \end{pmatrix} \begin{pmatrix} f_1(\lambda) \\ f_2(\lambda) \end{pmatrix} \equiv \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}.$$

Thus $(T_3-\lambda)f_2(\lambda) \equiv x_2$. Hence $\lambda_0 \in \rho_{T_3}(x_2)$. On the other hand, if $\lambda_0 \in \rho_T(x_1 \oplus 0)$, then there is an H-valued analytic function g defined on a neighborhood U of λ_0 such that $(T-\lambda)g(\lambda) = x_1 \oplus 0$ for all $\lambda \in U$. If we set $g = g_1 \oplus g_2$ where $g_1 \in O(U, \operatorname{ran}(T^k))$ and $g_2 \in O(U, \ker(T^{k*}))$, then we get

$$\begin{pmatrix} T_1-\lambda & T_2 \\ 0 & T_3-\lambda \end{pmatrix} \begin{pmatrix} g_1(\lambda) \\ g_2(\lambda) \end{pmatrix} \equiv \begin{pmatrix} x_1 \\ 0 \end{pmatrix}.$$

Thus $(T_1-\lambda)g_1(\lambda) + T_2 g_2(\lambda) \equiv x_1$ and $(T_3-\lambda)g_2(\lambda) \equiv 0$. Since T_3 is nilpotent of order k, it has the single-valued extension property, which implies that $g_2(\lambda) \equiv 0$. Thus $(T_1-\lambda)g_1(\lambda) \equiv x_1$, and so $\lambda_0 \in \rho_{T_1}(x_1)$. Conversely, let $\lambda_0 \in \rho_{T_1}(x_1)$. Then there exists a function $g_1 \in O(U, \overline{\operatorname{ran}(T^k)})$ for some neighborhood U of λ_0 such that $(T_1-\lambda)g_1(\lambda) \equiv x_1$. Then $(T-\lambda)g_1(\lambda) \oplus 0 \equiv x_1 \oplus 0$. Hence $\lambda_0 \in \rho_T(x_1 \oplus 0)$.

(2) If $x_1 \in R_{T_1}(F)$, then $\sigma_{T_1}(x_1) \subset F$. Since $\sigma_{T_1}(x_1) = \sigma_T(x_1 \oplus 0)$ by (1), $\sigma_T(x_1 \oplus 0) \subset F$. Thus $x_1 \oplus 0 \in H_T(F)$, and hence $R_{T_1}(F) \oplus 0 \subset H_T(F)$. $\square$

For $T \in B(H)$, the smallest nonnegative integer p such that $\ker(T^p) = \ker(T^{p+1})$ is called the ascent of T and denoted by $p(T)$. If no such integer exists, we set $p(T) = \infty$. The smallest nonnegative integer q such that $\operatorname{ran}(T^q) = \operatorname{ran}(T^{q+1})$ is called the descent of T and denoted by $q(T)$. If no such integer exists, we set $q(T) = \infty$.

THEOREM 2.5. *Let $T \in B(H)$ be k-quasi-M-hyponormal. If μ is a non-zero isolated point of $\sigma(T)$, then μ is a simple pole of the resolvent of T, that is, T is polaroid*

Proof. Assume that $\operatorname{ran}(T^k)$ is dense. Then T is M-hyponormal and [15] implies that μ is simple pole of the resolvent of T. So we may assume that T^k does not have dense range. Then by Lemma 2.1 the operator T can be decomposed as follows:

$$T = \begin{pmatrix} A & B \\ 0 & C \end{pmatrix} \text{ on } H = \overline{\operatorname{ran}(T^k)} \oplus \ker(T^{*k}),$$

where A is M-hyponormal and $C^k = 0$. Now if μ is a non-zero isolated point of $\sigma(T)$, then $\mu \in \operatorname{iso}\sigma(A)$ because $\sigma(T) = \sigma(A) \cup \{0\}$. Therefore μ is a simple pole of the resolvent of A because an M-hyponormal operator is isoloid and the M-hyponormal operator A can be written as follows:

$$A = \begin{pmatrix} A_1 & 0 \\ 0 & A_2 \end{pmatrix} \text{ on } \overline{\operatorname{ran}(T^k)} = \ker(A-\mu) \oplus \overline{\operatorname{ran}(A-\mu)},$$

where $\sigma(A_1) = \{\mu\}$. Therefore

$$T = \begin{pmatrix} \mu & 0 & B_1 \\ 0 & A_2 & B_2 \\ 0 & 0 & C \end{pmatrix} = \begin{pmatrix} \mu & D \\ 0 & F \end{pmatrix} \text{ on } H = \ker(A-\mu) \oplus \overline{\operatorname{ran}(A-\mu)} \oplus \ker(T^{*k}),$$

where

$$F = \begin{pmatrix} A_2 & B_2 \\ 0 & C \end{pmatrix}.$$

We claim that F is an invertible operator on $\operatorname{ran}(A-\mu) \oplus N(T^{*k})$. First we verify that $A_2 - \mu I$ is invertible. If not, then μ will be an isolated point in $\sigma(A_2)$. Since A_2 is M-hyponormal and M-hyponormal is isoloid, hence μ is an eigenvalue of A_2 and so $A_2 x = \mu x$ for some non-zero vector x in $\operatorname{ran}(A-\mu I)$. On the other hand, $Ax = A_2 x$ implying x is in $N(A-\mu I)$. Hence x must be a zero vector. This contradiction shows that $A_2 - \mu I$ is invertible. Since $C - \mu I$ is also invertible, it follows that $F - \mu I$ is invertible [5, Problem 71] and $(F-z)^{-1}$ is analytic on a neighborhood of μ. Hence μ is a simple pole of the resolvent

$$(z-T)^{-1} = \begin{pmatrix} \frac{1}{z-\mu} & \frac{1}{z-\mu} D(z-F)^{-1} \\ 0 & (z-F)^{-1} \end{pmatrix}$$

of T. □

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