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SCHUR CONVEXITY OF DUAL FORM OF THE COMPLETE SYMMETRIC FUNCTION

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Abstract. In this paper, the Schur-convexity, the Schur-geometric-convexity and the Schur-harmonic-convexity of dual form of the complete symmetric function are investigated. As consequences, some new inequalities are established via majorization theory.

1. Introduction

Throughout this paper, $\mathbf{R}_+^n = \{x|x = (x_1,x_2,\ldots,x_n),\ x_i > 0,\ i = 1,2\ldots,n\}$ and

$$x = (x_1,x_2,\ldots,x_n) \in \mathbf{R}_+^n, \quad y = (y_1,y_2,\ldots,y_n) \in \mathbf{R}_+^n.$$

For simplicity, $\mathbf{R}_+$ stands for $\mathbf{R}_+^1$.

Whiteley [18] introduced the Whiteley's symmetric function $T_n^{[r,s]}(x)$ through the following equation

$$\sum_{r=0}^{\infty} T_n^{[r,s]}(x)t^r = \begin{cases} \prod_{i=1}^n (1+x_it)^s & s > 0, \\ \prod_{i=1}^n (1-x_it)^s & s < 0 \end{cases}$$

The complete symmetric function which is Whiteley's symmetric function as $s = -1$ reads as follows

$$c_r(x) = T_n^{[r,-1]}(x) = \sum_{i_1+i_2+\ldots+i_n=r} x_1^{i_1}\ldots x_n^{i_n}, \tag{1.1}$$

where $c_0(x) = 1,\ r \in \{1,2,\ldots,n\},\ \ i_1,i_2,\ldots,i_n$ are non-negative integers.

K. Z. Guan [9] discussed the Schur convexity of $c_r(x)$ and proved that $c_r(x)$ is Schur convex in $\mathbf{R}_+^n$. Subsequently, Y. M. Chu et al. [3] proved that $c_r(x)$ is Schur multiplicatively convex and harmonic convex in $\mathbf{R}_+^n$.

It is not difficult to show that the equivalent definition of the complete symmetric function is

$$c_r(x) = \sum_{1\leqslant i_1\leqslant i_2\leqslant\ldots\leqslant i_r\leqslant n} \prod_{j=1}^r x_{i_j}. \tag{1.2}$$

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In fact, for any $x=(x_1,x_2,\ldots,x_n)\in R_+^n$, let

$$A=\{x_1^{i_1}\cdots x_n^{i_n} : i_1+\ldots+i_n=r\},$$

$$B=\{x_{j_1}\cdots x_{j_r} : 1\leqslant j_1\leqslant\ldots\leqslant j_r\leqslant n\}.$$

It suffices to show that $A=B$ in order to prove that (1.1) is equivalent to (1.2). For any $x_1^{i_1}\cdots x_n^{i_n}\in A$, without loss of generality, we may assume $i_1\cdots i_n$ are not equal to zero. And then we can choose appropriate $j_1,\ldots,j_r$ such that $j_1=\ldots=j_{i_1}=1$, $j_{i_1+1}=\ldots=j_{i_1+i_2}=2,\ldots,j_{i_1+\ldots+i_{n-1}+1}=\ldots=j_{i_1+\ldots+i_n}=n$, this indicates that $x_1^{i_1}\cdots x_n^{i_n}=x_{j_1}\cdots x_{j_r}$ which implies $A\subseteq B$. On the other hand, $B\subseteq A$ is obviously true. Together with $A\subseteq B$ we have that $A=B$.

The main purpose of this paper is to discuss the Schur-convexity, Schur-geometric-convexity and Schur-harmonic-convexity of the dual form of the complete symmetric function, that is, we shall study the function given by

$$f(x,r)=\prod_{i_1+i_2+\ldots+i_n=r}\sum_{j=1}^{n} i_j x_j=\prod_{1\leqslant i_1\leqslant i_2\leqslant\ldots\leqslant i_r\leqslant n}\sum_{j=1}^{r} x_{i_j}, \tag{1.3}$$

where $i_1,i_2,\ldots,i_n$ are nonnegative integers. As applications, some new inequalities are established.

2. Definitions and Lemmas

Schur-convexity of symmetric functions introduced by Schur [17] plays an important role in establishing new inequalities, see, for example, [5–7, 9–15, 17, 19–23, 25]. The Schur-geometric-convexity and Schur-harmonic-convexity involving some special functions have been investigated, see, for example, [1–5, 8, 16, 20, 24, 26, 27]. We recall some well known definitions and lemmas in order to carry out our work.

Let $x_{[1]}\geqslant x_{[2]}\geqslant\cdots\geqslant x_{[n]}$, $y_{[1]}\geqslant y_{[2]}\geqslant\cdots\geqslant y_{[n]}$ be their ordered components.

DEFINITION 1. ([15]) The n-tuple x is said to be majorized by y (in symbols $x\prec y$), if

$$\sum_{i=1}^{m} x_{[i]}\leqslant\sum_{i=1}^{m} y_{[i]},\quad m=1,2,\ldots,n-1$$

and

$$\sum_{i=1}^{n} x_{[i]}=\sum_{i=1}^{n} y_{[i]}.$$

DEFINITION 2. ([26]) The n-tuple x is said to be logarithmically majorized by y (in symbols $\log x\prec\log y$) if

$$\prod_{i=1}^{m} x_{[i]}\leqslant\prod_{i=1}^{m} y_{[i]},\quad m=1,2,\ldots,n-1$$

and

$$\prod_{i=1}^{n} x_{[i]} = \prod_{i=1}^{n} y_{[i]}.$$

DEFINITION 3. ([26]) A function $f : \mathbf{R}_+^n \to \mathbf{R}_+$ is called Schur-geometrically-convex if

$$\log x \prec \log y \Rightarrow f(x) \leqslant f(y).$$

LEMMA 1. ([26]) *Suppose that $f : \boldsymbol{R}_+^n \to \boldsymbol{R}_+$ is symmetric and differentiable, if*

$$(x_1 - x_2)\left(x_1 \frac{\partial f}{\partial x_1} - x_2 \frac{\partial f}{\partial x_2}\right) \geqslant 0 \tag{2.1}$$

for all $x_1 \neq x_2$, then f is Schur-geometrically-convex.

REMARK 1. In order to show that f is Schur-geometrically-convex, it suffices to show that inequality (2.1) holds for $x_1 > x_2$, because when $x_1 < x_2$,

$$(x_1 - x_2)\left(x_1 \frac{\partial f}{\partial x_1} - x_2 \frac{\partial f}{\partial x_2}\right) = (x_2 - x_1)\left(x_2 \frac{\partial f}{\partial x_2} - x_1 \frac{\partial f}{\partial x_1}\right) \geqslant 0.$$

DEFINITION 4. ([1]) A function $f : \mathbf{R}_+^n \to \mathbf{R}_+$ is called Schur-harmonically-convex if

$$x \prec y \Rightarrow f\left(\frac{1}{x}\right) \leqslant f\left(\frac{1}{y}\right),$$

where

$$\frac{1}{x} = \left(\frac{1}{x_1}, \frac{1}{x_2}, \ldots, \frac{1}{x_n}\right), \qquad \frac{1}{y} = \left(\frac{1}{y_1}, \frac{1}{y_2}, \ldots, \frac{1}{y_n}\right).$$

LEMMA 2. ([1]) *Suppose that $f : \boldsymbol{R}_+^n \to \boldsymbol{R}_+$ is symmetric and differentiable, if*

$$(x_1 - x_2)\left(x_1^2 \frac{\partial f}{\partial x_1} - x_2^2 \frac{\partial f}{\partial x_2}\right) \geqslant 0 \tag{2.2}$$

for all $x_1 \neq x_2$, then f is Schur-harmonically-convex.

LEMMA 3. ([12]) *Let $x = (x_1, x_2, \ldots, x_n) \in \boldsymbol{R}_+^n$ and $\sum_{i=1}^{n} x_i = s$. If $c \geqslant s$, then*

$$\frac{c - x}{\frac{nc}{s} - 1} = \left(\frac{c - x_1}{\frac{nc}{s} - 1}, \frac{c - x_2}{\frac{nc}{s} - 1}, \ldots, \frac{c - x_n}{\frac{nc}{s} - 1}\right) \prec (x_1, x_2, \ldots, x_n) = x.$$

LEMMA 4. ([9]) *Let $x = (x_1, x_2, \ldots, x_n) \in \boldsymbol{R}_+^n$ and $\sum_{i=1}^{n} x_i = s$. If $c \geqslant 0$, then*

$$\frac{c + x}{\frac{nc}{s} + 1} = \left(\frac{c + x_1}{\frac{nc}{s} + 1}, \frac{c + x_2}{\frac{nc}{s} + 1}, \ldots, \frac{c + x_n}{\frac{nc}{s} + 1}\right) \prec (x_1, x_2, \ldots, x_n) = x.$$

3. Main results

In this section, we mainly investigate the Schur-geometric-convexity, the Schur-convexity and the Schur-harmonic-convexity with respect to $f(x,r)$.

THEOREM 1. *The complete symmetric function given by*

$$f(x,r)=\prod_{i_1+i_2+\ldots+i_n=r}\sum_{j=1}^{n} i_j x_j$$

is Schur-geometrically-convex in R_+^n.

Proof. The cases $r=1$ and $r=2$ can be easily proved.

Next, we consider the case $r\geqslant 3$ and $x_1>x_2$.

$$f(x,r)=\prod_{\substack{i_1+i_2+\ldots+i_n=r\\ i_1\neq 0,\ i_2=0}}\sum_{j=1}^{n} i_j x_j\cdot\prod_{\substack{i_1+i_2+\ldots+i_n=r\\ i_1=0,\ i_2\neq 0}}\sum_{j=1}^{n} i_j x_j\cdot\prod_{\substack{i_1+i_2+\ldots+i_n=r\\ i_1\neq 0,\ i_2\neq 0}}\sum_{j=1}^{n} i_j x_j\cdot\prod_{\substack{i_1+i_2+\ldots+i_n=r\\ i_1=0,\ i_2=0}}\sum_{j=1}^{n} i_j x_j.$$

It is not difficult to show that

$$\begin{aligned}x_1\frac{\partial f(x,r)}{\partial x_1}&=f(x,r)\Bigg(\sum_{\substack{i_1+i_2+\ldots+i_n=r\\ i_1\neq 0,\ i_2=0}}\frac{i_1x_1}{\sum_{j=1}^{n} i_j x_j}+\sum_{\substack{i_1+i_2+\ldots+i_n=r\\ i_1\neq 0,\ i_2\neq 0}}\frac{i_1x_1}{\sum_{j=1}^{n} i_j x_j}\Bigg)\\&=f(x,r)\Bigg(\sum_{\substack{k+k_3+\ldots+k_n=r\\ k\neq 0}}\frac{kx_1}{kx_1+\sum_{j=3}^{n} k_j x_j}+\sum_{\substack{k+m+i_3+\ldots+i_n=r\\ k\neq 0,\ m\neq 0}}\frac{kx_1}{kx_1+mx_2+\sum_{j=3}^{n} i_j x_j}\Bigg),\end{aligned}$$

and

$$\begin{aligned}x_2\frac{\partial f(x,r)}{\partial x_2}&=f(x,r)\Bigg(\sum_{\substack{i_1+i_2+\ldots+i_n=r\\ i_1=0,\ i_2\neq 0}}\frac{i_2x_2}{\sum_{j=1}^{n} i_j x_j}+\sum_{\substack{i_1+i_2+\ldots+i_n=r\\ i_1\neq 0,\ i_2\neq 0}}\frac{i_2x_2}{\sum_{j=1}^{n} i_j x_j}\Bigg)\\&=f(x,r)\Bigg(\sum_{\substack{k+k_3+\ldots+k_n=r\\ k\neq 0}}\frac{kx_2}{kx_2+\sum_{j=3}^{n} k_j x_j}+\sum_{\substack{k+m+i_3+\ldots+i_n=r\\ k\neq 0,\ m\neq 0}}\frac{kx_2}{kx_2+mx_1+\sum_{j=3}^{n} i_j x_j}\Bigg).\end{aligned}$$

Thus,

$$x_1\frac{\partial f(x,r)}{\partial x_1}-x_2\frac{\partial f(x,r)}{\partial x_2}=f(x,r)(\Lambda_1+\Lambda_2),$$

where

$$\Lambda_1=\sum_{\substack{k+k_3+\ldots+k_n=r\\ k\neq 0}}\left(\frac{kx_1}{kx_1+\sum_{j=3}^{n} k_j x_j}-\frac{kx_2}{kx_2+\sum_{j=3}^{n} k_j x_j}\right),$$

and

$$\Lambda_2 = \sum_{\substack{k+m+i_3+\ldots+i_n=r \\ k\neq 0, m\neq 0}} \left(\frac{kx_1}{kx_1+mx_2+\sum_{j=3}^{n} i_j x_j} - \frac{kx_2}{kx_2+mx_1+\sum_{j=3}^{n} i_j x_j} \right).$$

Notice that $h(t) = \frac{t}{c+t} = 1 - \frac{c}{c+t}$ $(c > 0)$ is increasing in $(0, +\infty)$, this yields $\Lambda_1 \geqslant 0$. Meanwhile, let $a = \sum_{j=3}^{n} i_j x_j$, then

$$\begin{aligned} \frac{kx_1}{kx_1+mx_2+\sum_{j=3}^{n} i_j x_j} - \frac{kx_2}{kx_2+mx_1+\sum_{j=3}^{n} i_j x_j} &= \frac{kx_1}{kx_1+mx_2+a} - \frac{kx_2}{kx_2+mx_1+a} \\ &= \frac{mk(x_1^2-x_2^2)+ak(x_1-x_2)}{(kx_1+mx_2+a)(kx_2+mx_1+a)} \\ &\geqslant 0. \end{aligned}$$

Thus, $\Lambda_2 \geqslant 0$.

Consequently,

$$x_1 \frac{\partial f(x,r)}{\partial x_1} - x_2 \frac{\partial f(x,r)}{\partial x_2} \geqslant 0$$

which completes the proof. □

THEOREM 2. *The complete symmetric function $f(x,r)$ is increasing with respect to each $x_i (i = 1,2,\ldots,n)$ and Schur-concave for each $r \in \{1,2,...,n\}$ in R_+^n.*

Proof. It is not difficult to show that $\ln f(x,r)$ is increasing with respect to each x_i $(i = 1,2,\ldots,n)$ and concave in R_+^n, and notice that $\ln f(x,r)$ is symmetric, this leads to $\ln f(x,r)$ is Schur-concave by the proposition C.2 in [15, p. 67]. And then $f(x,r)$ is also increasing with respect to each x_i $(i = 1,2,\ldots,n)$ and Schur-concave for each $r \in \{1,2,...,n\}$ in R_+^n. □

THEOREM 3. *The complete symmetric function given by*

$$f(x,r) = \prod_{1\leqslant i_1\leqslant i_2\leqslant\ldots\leqslant i_r\leqslant n} \sum_{j=1}^{r} x_{i_j}$$

is Schur-harmonically-convex in R_+^n.

Proof. In view of

$$x_1 \frac{\partial f(x,r)}{\partial x_1} - x_2 \frac{\partial f(x,r)}{\partial x_2} \geqslant 0$$

and $\frac{\partial f(x,r)}{\partial x_1} > 0$, $\frac{\partial f(x,r)}{\partial x_2} > 0$, and $x_1 > x_2$, one has $x_1^2 \frac{\partial f(x,r)}{\partial x_1} - x_2^2 \frac{\partial f(x,r)}{\partial x_2} \geqslant 0$, which completes the proof. □

4. Some applications

In this section, some new inequalities involving the dual form of the complete symmetric function shall be established by utilizing the previous results and majorilization theory.

COROLLARY 1. *Let* $x = (x_1, x_2, \cdots, x_n) \in R_+^n$, $\sum_{i=1}^{n} x_i = s$.

(1) If $c \geqslant s$, *then* $f(\frac{1}{x}, r) \geqslant f\left(\frac{\frac{nc}{s}-1}{c-x}, r\right)$.

(2) If $c \geqslant 0$, *then* $f(\frac{1}{x}, r) \geqslant f\left(\frac{\frac{nc}{s}+1}{c+x}, r\right)$.

Proof. By employing the Schur-harmonic-convexity of $f(x,r)$, parts (1) and (2) follow from Lemmas 3 and 4, respectively. □

COROLLARY 2. *If* $x = (x_1, x_2, \cdots, x_n) \in R_+^n$, *then*

$$\prod_{1 \leqslant i_1 \leqslant i_2 \leqslant \ldots \leqslant i_r \leqslant n} \sum_{j=1}^{r} x_{i_j} \geqslant \prod_{1 \leqslant i_1 \leqslant i_2 \leqslant \ldots \leqslant i_r \leqslant n} (r\sqrt[n]{x_1 x_2 \cdots x_n}). \tag{4.1}$$

Proof. Corollary 2 follows from the Schur-geometric-convexity of $f(x,r)$ and the fact $\log x \succ \log \sqrt[n]{x}$, where $\sqrt[n]{x} = (\sqrt[n]{x_1 x_2 \cdots x_n}, \sqrt[n]{x_1 x_2 \cdots x_n}, \ldots, \sqrt[n]{x_1 x_2 \cdots x_n})$. □

COROLLARY 3. *Let* $A = (a_{ij})_{n \times n} (n \geqslant 3)$ *be a positive definite Hermitian matrix, and* I *denote* $n \times n$ *unit matrix. Then,*

$$\prod_{1 \leqslant i_1 \leqslant i_2 \leqslant \ldots \leqslant i_r \leqslant n} \sum_{j=1}^{r} \frac{\lambda_{i_j}}{tr(A)} \geqslant \prod_{1 \leqslant i_1 \leqslant i_2 \leqslant \ldots \leqslant i_r \leqslant n} \left(r\sqrt[n]{det(A)}/tr(A)\right) \tag{4.2}$$

and

$$\prod_{1 \leqslant i_1 \leqslant i_2 \leqslant \ldots \leqslant i_r \leqslant n} \sum_{j=1}^{r} \frac{\lambda_{i_j}}{1+\lambda_{i_j}} \geqslant \prod_{1 \leqslant i_1 \leqslant i_2 \leqslant \ldots \leqslant i_r \leqslant n} \left(r\sqrt[n]{det(A)/det(I+A)}\right), \tag{4.3}$$

where λ_i $(1 \leqslant i \leqslant n)$ *is eigenvalue of matrix* A, $tr(A) = \sum_{i=1}^{n} \lambda_i$ *and* $det(A) = \prod_{i=1}^{n} \lambda_i$.

Proof. As mentioned in [10], we know that

$$\log\left(\frac{\sqrt[n]{\det(A)}}{\operatorname{tr}(A)}, \ldots, \frac{\sqrt[n]{\det(A)}}{\operatorname{tr}(A)}\right) \prec \log\left(\frac{\lambda_1}{\operatorname{tr}(A)}, \ldots, \frac{\lambda_n}{\operatorname{tr}(A)}\right)$$

and

$$\log(\sqrt[n]{\det(A)/\det(I+A)}, \ldots, \sqrt[n]{\det(A)/\det(I+A)}) \prec \log\left(\frac{\lambda_1}{1+\lambda_1}, \ldots, \frac{\lambda_n}{1+\lambda_n}\right).$$

By using Theorem 1, we assert that both (4.2) and (4.3) hold. □

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