Mathematical
Inequalities
& Applications
Volume 16, Number 4 (2013), 1151–1157
doi:10.7153/mia-16-89

MULTIPLE–SET SPLIT FEASIBILITY PROBLEMS FOR A FINITE FAMILY OF DEMICONTRACTIVE MAPPINGS IN HILBERT SPACES

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(*Communicated by Yeol Je Cho*)

Abstract. In this paper, we introduce an iterative algorithm for solving the multiple-set split feasibility problems for a finite family demicontractive mappings in Hilbert spaces. The results presented in this paper improve and extend some recent corresponding results in [4], [7], [9], [10], [13], [14].

1. Introduction

Let H_1 and H_2 be two real Hilbert spaces and let C and Q be nonempty closed convex subset of H_1 and H_2, respectively. The split feasibility problem (SFP) is formulated as finding a point x^* with the property

$$x^* \in C \quad Ax^* \in Q, \tag{1.1}$$

where $A : H_1 \to H_2$ is a bounded linear operator.

The SFP in finite-dimensional Hilbert spaces was first introduced by Censor and Elfving [1] for modeling inverse problems which arise from phase retrievals and in medical image reconstruction [2]. Recently, it has been found that the SFP can also be used in various disciplines such as image restoration, computer tomograph, and radiation therapy treatment planning [3–8].

The SFP in an infinite-dimensional Hilbert space can be found in [9–11].

Assuming that the SFP is consistent (i.e.(1.1)) has a solution), it is not hard to see that

$$x^* = P_C(I + \gamma A^*(P_Q - I)Ax^*, \forall x \in C, \tag{1.2}$$

where P_C and P_Q are the (orthogonal) projection onto C and Q, respectively, $\gamma > 0$, and A^* denotes the adjoint of A. That is, x^* solves the SFP (1.1) if and only if x^* solves the fixed point equation (1.2) [see 9]. This implies that we can use fixed point algorithms to solve SFP.

Mathematics subject classification (2010): 47H09, 47J25.

Keywords and phrases: Multiple-set split feasibility problem, demicontractive mapping, demi-closed, fixed point.

This work was supported by the Scientific Research Foundation of Kunming University (Grant No. XJL12008).

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T is said to be demicontractive (see for example [12]) if there exists a constant $\beta \in [0,1)$ such that

$$\|Tx-q\|^2 \leqslant \|x-q\|^2 + \beta\|x-Tx\|^2, \quad \forall (x,q) \in H \times Fix(T), \tag{1.3}$$

which is equivalent to

$$\langle x-Tx, x-q\rangle \geqslant \frac{1-\beta}{2}\|x-Tx\|^2, \quad \forall (x,q) \in H \times Fix(T), \tag{1.4}$$

(1.3) is also equivalent to

$$\langle x-Tx, q-Tx\rangle \leqslant \frac{1+\beta}{2}\|x-Tx\|^2, \quad \forall (x,q) \in H \times Fix(T), \tag{1.5}$$

An operator satisfying (1.3) will be referred to as a β-demicontractive mapping. It is worth noting that the class of demicontractive maps contains important operators such as the quasi-nonexpansive maps and the strictly pseudocontractive maps with fixed points.

The split common fixed point problems for quasi-nonexpansive mappings and demicontractive mappings in the setting of Hilbert space was introduced and studied by Moudafi [13, 14]. In [14], Moudafi proposed the following iterative algorithm for solving split common fixed problem for demicontractive mappings: for arbitrarily chosen $x^* \in H_1$,

$$\begin{cases} u_n = x_n + \gamma A^*(T-I)Ax_n, \\ x_{n+1} = (1-\alpha_n)u_n + \alpha_n U u_n, \quad n \in N \end{cases}$$

they proved that $\{x_n\}$ converges weakly to a split common fixed point x^*, where $U : H_1 \to H_1$ and $T : H_2 \to H_2$ are two demicontractive mappings, $A : H_1 \to H_2$ is a bounded linear operator, λ is the spectral radius of the operator AA^*. Inspired and motivated by the recent work of Moudafi [13,14], in this paper, we study the following multiple-set split feasibility problem $(MSSFP)$ for a finite family of demicontractive mappings in Hilbert spaces.

$$x^* \in C \quad \text{such that} \quad Ax^* \in Q, \tag{1.6}$$

where $A : H_1 \to H_2$ is a bounded linear operator, $K_i : H_1 \to H_1$ and $T_i : H_2 \to H_2$, $i = 1,2,\cdots,N$ are demicontractive mappings, $C := \cap_{i=1}^N F(K_i) \neq \emptyset$ and Q:=$\cap_{i=1}^N F(T_i) \neq \emptyset$. In the sequel, we use Γ to denote the set of solutions of $MSSFP$ (1.6), i.e.,

$$\Gamma = \{x \in C, Ax \in Q\}. \tag{1.7}$$

The results presented in this paper improve and extend some recent corresponding results in [4], [7], [9], [10], [13], [14].

2. Preliminaries

Throughout this paper, Let H_1 and H_2 be two real Hilbert spaces. We denote the strong convergence, weak convergence of a sequence $\{x_n\}$ to a point $x \in X$ by $x_n \to x$, $x_n \rightharpoonup x$, respectively, and $F(T)$ is the fixed point set of a mapping T.

Let E be a Banach space. A mapping $T : E \to E$ is said to be demi-closed at origin, if for any sequence $\{x_n\} \subset E$ with $x_n \rightharpoonup x^*$ and $\|(I-T)x_n\| \to 0$, then $x^* = Tx^*$.

A Banach space E is said to satisfy Opial's condition, if for any sequence $\{x_n\}$ in E, $x_n \rightharpoonup x^*$ implies that

$$\liminf_{n\to\infty} \|x_n - x^*\| < \liminf_{n\to\infty} \|x_n - y\|, \quad \forall y \in E \text{ with } y \neq x^*. \tag{2.1}$$

It is well known that every Hilbert space satisfies Opial's condition.

A mapping $T : C \to E$ is said to be semi-compact, if for any sequence $\{x_n\}$ in C such that $\|x_n - Tx_n\| \to 0, (n \to \infty)$, there exists subsequence $\{x_{n_j}\}$ of $\{x_n\}$ such that $\{x_{x_j}\}$ converges strongly to $x^* \in C$.

We state a key property of the relaxed operator $T_\alpha := (1-\alpha)I + \alpha T$.

Let T be a β-demicontractive self-mapping on H with $Fix(T) \neq \emptyset$ and set $T_\alpha := (1-\alpha)I + \alpha T$ for $\alpha \in (0,1]$. Then, T_α is quasi-nonexpansive if $\alpha \in [0, 1-\beta]$.

Indeed, for any arbitrary element $(x,q) \in H \times Fix(T)$, we have

$$\|T_\alpha x - q\|^2 = \|x-q\|^2 - 2\alpha\langle x-q, x-Tx\rangle + \alpha^2\|Tx-x\|^2, \tag{2.2}$$

which according to (1.3) yields

$$\|T_\alpha x - q\|^2 \leqslant \|x-q\|^2 - \alpha(1-\beta-\alpha)\|Tx-x\|^2. \tag{2.3}$$

Furthermore, it is obvious that $Fix(T) = Fix(T_\alpha)$ if $\alpha \neq 0$. As a consequence, the operator T_α is quasi-nonexpansive for $\alpha \in [0, 1-\beta]$ and $Fix(T)$ is a closed convex subset of H.

3. Main Results

THEOREM 1. *Let H_1 and H_2 be two Hilbert spaces and A be a bounded linear operator from H_1 to H_2, let $S_i : H_1 \to H_2$ be β_i-demicontractive and $T_i : H_2 \to H_2$ be μ_i-demicontractive satisfying $\cap_{i=1}^N F(S_i) = C$ and $\cap_{i=1}^N F(T_i) = Q$, $i = 1, 2, \cdots, N$. For any $x_0 \in H_1$, $\{x_n\}$ is defined by*

$$\begin{cases} y_n = x_n + \gamma A^*(T_n - I)Ax_n, \\ x_{n+1} = (1-\alpha_n)y_n + \alpha_n S_n y_n, \quad n \in N, \end{cases} \tag{3.1}$$

where λ is the spectral radius of the operator A^*A, $T_n = T_{n(mod\ N)}$, $S_n = S_{n(mod\ N)}$, $\forall n \geqslant 1$, γ and $\{\alpha_n\}$ satisfy the following conditions:

(1) $C := \cap_{i=1}^N F(S_i) \neq \emptyset$, $Q := \cap_{i=1}^N F(T_i) \neq \emptyset$;

(2) $\mu = \max_{1\leqslant i\leqslant N} \mu_i < 1$, $\gamma \in (0, \frac{1-\mu}{\lambda})$;

(3) $\beta = \max_{1\leqslant i\leqslant N} \beta_i < 1$, $\alpha_n \in (\delta, 1-\beta-\delta)$.

If $S_i - I$ and $T_i - I$ $(i = 1, 2, \cdots, N)$ are demi-closed at zero and $\Gamma \neq \emptyset$, then the sequences $\{x_n\}$ and $\{y_n\}$ both converge weakly to a split common fixed point $x^* \in \Gamma$.

Proof. Taking $q \in \Gamma$, this means that $q \in \cap_{i=1}^{N} F(S_i) = C$ and $q \in \cap_{i=1}^{N} F(T_i) = Q$. Using (2.3), we have

$$\begin{aligned}\|x_{n+1}-q\|^2 &\leqslant \|y_n-q\|^2 - \alpha_n(1-\beta_n-\alpha_n)\|S_n y_n - y_n\|^2\\ &\leqslant \|y_n-q\|^2 - \alpha_n(1-\beta-\alpha_n)\|S_n y_n - y_n\|^2,\end{aligned} \tag{3.2}$$

where

$$\begin{aligned}\|y_n-q\|^2 &= \|x_n+\gamma A^*(T_n-I)Ax_n-q\|^2\\ &= \|x_n-q\|^2+\gamma^2\|A^*(T_n-I)Ax_n\|^2+2\gamma\langle x_n-q, A^*(T_n-I)Ax_n\rangle\\ &= \|x_n-q\|^2+\gamma^2\langle (T_n-I)Ax_n, AA^*(T_n-I)Ax_n\rangle\\ &\quad +2\gamma\langle x_n-q, A^*(T_n-I)Ax_n\rangle.\end{aligned} \tag{3.3}$$

From the definition of λ, we have

$$\begin{aligned}\gamma^2\langle (T_n-I)Ax_n, AA^*(T_n-I)Ax_n\rangle &\leqslant \lambda\gamma^2\langle (T_n-I)Ax_n, (T_n-I)Ax_n\rangle\\ &\leqslant \lambda\gamma^2\|(T_n-I)Ax_n\|^2.\end{aligned} \tag{3.4}$$

It follows from (1.5) that

$$\begin{aligned}2\gamma\langle x_n-q, A^*(T_n-I)Ax_n\rangle &= 2\gamma\langle A(x_n-q), (T_n-I)Ax_n\rangle\\ &= 2\gamma\langle A(x_n-q+(T_n-I)Ax_n-(T_n-I)Ax_n, (T_n-I)Ax_n\rangle\\ &= 2\gamma(\langle T_nAx_n-Aq, (T_n-I)Ax_n\rangle - \|(T_n-I)Ax_n\|^2)\\ &\leqslant 2\gamma[\tfrac{1+\mu}{2}\|(T_n-I)Ax_n\|^2-\|(T_n-I)Ax_n\|^2]\\ &= -\gamma(1-\mu)\|(T_n-I)Ax_n\|^2.\end{aligned} \tag{3.5}$$

So,

$$\begin{aligned}\|x_{n+1}-q\|^2 &\leqslant \|x_n-q\|^2+\lambda\gamma^2\|(T_n-I)Ax_n\|^2-\gamma(1-\mu)\|(T_n-I)Ax_n\|^2\\ &\quad -\alpha_n(1-\beta-\alpha_n)\|S_ny_n-y_n\|^2\\ &\leqslant \|x_n-q\|^2-\gamma(1-\mu-\lambda\gamma)\|(T_n-I)Ax_n\|^2\\ &\quad -\alpha_n(1-\beta-\alpha_n)\|S_ny_n-y_n\|^2.\end{aligned} \tag{3.6}$$

By the assumption on γ, β, μ and α_n, we obtain

$$\|x_{n+1}-q\| \leqslant \|x_n-q\|, \quad n \geqslant 1. \tag{3.7}$$

This implies $\{x_n\}$ is bounded. From (3.7), we know that the sequence $\{\|x_n-q\|\}$ is monotonically decreasing, therefore, $\lim_{n\to\infty}\|x_n-q\|$ exists for each $q \in \Gamma$. It follows from (3.6) that

$$\begin{aligned}&\gamma(1-\mu-\lambda\gamma)\|(T_n-I)Ax_n\|^2+\alpha_n(1-\beta-\alpha_n)\|S_ny_n-y_n\|^2\\ &\leqslant \|x_n-q\|^2-\|x_{n+1}-q\|^2 \to 0, \text{ as } n\to\infty.\end{aligned}$$

Since $\gamma > 0$, $1-\mu-\lambda\gamma > 0$ and $1-\beta-\alpha_n > 0$, we can obtain

$$\lim_{n\to+\infty}\|(T_n-I)Ax_n\| = 0. \tag{3.8}$$

$$\lim_{n\to+\infty}\|(S_n-I)y_n\|=0. \tag{3.9}$$

For each fixed $j\in\{1,2,\cdots,N\}$, then $\{\|Ax_{iN+j}-T_jAx_{iN+j}\|\}$ is a subsequence of $\{\|(T_n-I)Ax_n\|\}$, so, we have

$$\lim_{i\to\infty}\|Ax_{iN+j}-T_jAx_{iN+j}\|=0. \tag{3.10}$$

Similarly,

$$\lim_{i\to\infty}\|y_{iN+j}-S_jy_{iN+j}\|=0. \tag{3.11}$$

It follows from (3.3), (3.4) and (3.5) that

$$\|y_n-q\|^2\leqslant\|x_n-q\|^2-\gamma(1-\mu-\lambda\gamma)\|(T_n-I)Ax_n\|^2.$$

Thus, since $\{x_n\}$ is bounded, we know that $\{y_n\}$ is bounded. Therefore, there exists a subsequence $\{y_{n_i}\}$ of $\{y_n\}$ such that $y_{n_i}\rightharpoonup x^*\in H_1$. Moreover, for any positive integer $j=1,2,\cdots,N$, there exists a subsequence $\{n_i(l)\}$ of $\{n_i\}$ with $n_i(l)(\bmod N)=j$ such that $y_{n_i(l)}\rightharpoonup x^*$. It follows from (3.11) that

$$\lim_{n_i(l)\to\infty}\|y_{n_i(j)}-S_jy_{n_i(j)}\|=0. \tag{3.12}$$

Since S_j is demi-closed at origin for each $j=1,2,\cdots,N$, it follows that $x^*\in C=\bigcap_{j=1}^N F(S_j)$. It follows from (3.1) and (3.10) that

$$x_{n_i}=y_{n_i}-\gamma A^*(T_{n_i}-I)Ax_{n_i}\rightharpoonup x^*. \tag{3.13}$$

Since A is a bounded linear operator, we know that $Ax_{n_i}\rightharpoonup Ax^*$. For any positive integer $k=1,2,\cdots,N$, there exists a subsequence $\{n_i(k)\}$ of $\{n_i\}$ with $n_i(k)(\bmod N)=k$ such that $Ax_{n_i(k)}\rightharpoonup Ax^*$. Using (3.10), we have

$$\lim_{n_i(k)\to\infty}\|Ax_{n_i(k)}-S_kAx_{n_i(k)}\|=0. \tag{3.14}$$

Since T_k is demi-closed at origin for each $k=1,2,\cdots,N$, it follows that $Ax^*\in Q=\bigcap_{k=1}^N F(T_k)$. Therefore, $x^*\in\Gamma$, i.e., x^* is a solution of *MSSFP* (1.6).

Finally, we show that $x_n\rightharpoonup x^*$ and $y_n\rightharpoonup x^*$.

Assume that there exists another subsequence $\{y_{n_j}\}$ of $\{y_n\}$ such that $\{y_{n_j}\}$ converges weakly to a point $y^*\in H$ with $y^*\neq x^*$. Using the same argument above, we know that $y^*\in\Gamma$. In addition, from (3.3) and (3.8), we know that $\lim_{n\to\infty}\|y_n-q\|$ exists for each $q\in\Gamma$. Thus, since each Hilbert space possesses Opial property, we have

$$\begin{aligned}\liminf_{i\to\infty}\|y_{n_i}-x^*\|<\liminf_{i\to\infty}\|y_{n_i}-y^*\|&=\lim_{n\to\infty}\|y_n-y^*\|\\&=\liminf_{j\to\infty}\|y_{n_j}-y^*\|<\liminf_{j\to\infty}\|y_{n_j}-x^*\|\\&=\lim_{n\to\infty}\|y_n-x^*\|=\liminf_{i\to\infty}\|y_{n_i}-x^*\|,\end{aligned}$$

which is a contradiction. This implies that $\{y_n\}$ converges weakly to the point $x^* \in \Gamma$. On the other hand, from (3.1) and (3.8), we can obtain

$$x_n = y_n - \gamma A^*(T_n - I)Ax_n \rightharpoonup x^*, \quad (n \to \infty).$$

Therefore, the sequences $\{x_n\}$ and $\{y_n\}$ both converge weakly to a split common fixed point $x^* \in \Gamma$. The proof is completed. □

THEOREM 2. *Let H_1 and H_2 be two Hilbert spaces and A be a bounded linear operator from H_1 to H_2, let $S_i : H_1 \to H_2$ be β_i-demicontractive and $T_i : H_2 \to H_2$ be μ_i-demicontractive satisfying $\cap_{i=1}^N F(S_i) = C$ and $\cap_{i=1}^N F(T_i) = Q$, $i = 1,2,\cdots,N$. For any $x_0 \in H_1$, $\{x_n\}$ is defined by*

$$\begin{cases} y_n = x_n + \gamma A^*(T_n - I)Ax_n, \\ x_{n+1} = (1-\alpha_n)y_n + \alpha_n S_n y_n, \quad n \in N, \end{cases} \tag{3.15}$$

where λ is the spectral radius of the operator A^*A, $T_n = T_{n(mod\ N)}, S_n = S_{n(mod\ N)}, \forall n \geqslant 1$, γ and $\{\alpha_n\}$ satisfy the following conditions:

(1) $C := \cap_{i=1}^N F(S_i) \neq \emptyset$, $Q := \cap_{i=1}^N F(T_i) \neq \emptyset$;

(2) $\mu = \max_{1 \leqslant i \leqslant N} \mu_i < 1$, $\gamma \in (0, \frac{1-\mu}{\lambda})$;

(3) $\beta = \max_{1 \leqslant i \leqslant N} \beta_i < 1$, $\alpha_n \in (\delta, 1-\beta-\delta)$.

If one of the mappings $\{S_i : i = 1,2,\cdots,N\}$ is semi-compact and $\Gamma \neq \emptyset$, then the sequences $\{x_n\}$ and $\{y_n\}$ both converge strongly to a split common fixed point $x^* \in \Gamma$.

Proof. Without loss of generality, we may assume that S_1 is semi-compact, it follows from (3.12) that

$$\lim_{n_i(1) \to +\infty} \|y_{n_i(1)} - S_j y_{n_i(1)}\| = 0, \tag{3.16}$$

therefore, there exists a subsequence $\{y_{n_i(1)}\}$ of $\{y_n\}$ converges strongly to a point $p \in H_1$ (some point in H_1). Since $y_{n_1} \rightharpoonup x^* \in \Gamma$. We know that $p = x^*$. This implies that $\{y_{n_1}\}$ converges strongly to $x^* \in \Gamma$. From Theorem 3.1, we know that $\lim_{n\to\infty} \|y_n - p\|$ exists for each $p \in \Gamma$. Therefore $\{y_n\}$ converges strongly to x^*.

From (3.15) that, $x_n = y_n - \gamma A^*(T_n - I)Ax_n$, by (3.8), we obtain that $\{x_{n_1}\}$ converges strongly to $x^* \in \Gamma$. On the other hand, since $\lim_{n\to\infty} \|x_n - q\|$ exists, we know that $\{x_n\}$ converges strongly to $x^* \in \Gamma$. The proof is completed. □

The following corollary can be directly obtained from Theorem 3.2 since the type of demicontractive mappings contains the type of quasi-nonexpansive mappings.

COROLLARY 1. *Let H_1 and H_2 be two Hilbert spaces and A be a bounded linear operator from H_1 to H_2, let $S_i : H_1 \to H_2$ and $T_i : H_2 \to H_2$ be quasi-nonexpansive mappings satisfying $\cap_{i=1}^N F(S_i) = C$ and $\cap_{i=1}^N F(T_i) = Q$, $i = 1,2,\cdots,N$. For any $x_0 \in H_1$, $\{x_n\}$ is defined by*

$$\begin{cases} y_n = x_n + \gamma A^*(T_n - I)Ax_n, \\ x_{n+1} = (1-\alpha_n)y_n + \alpha_n S_n y_n, \quad n \in N, \end{cases} \tag{3.15}$$

where λ *is the spectral radius of the operator* A^*A, $T_n = T_{n(mod\ N)}$, $S_n = S_{n(mod\ N)}$, $\forall n \geqslant 1$, γ *and* $\{\alpha_n\}$ *satisfy the following conditions:*

(1) $C := \cap_{i=1}^N F(S_i) \neq \emptyset$, $Q := \cap_{i=1}^N F(T_i) \neq \emptyset$;

(2) $0 < \gamma\lambda < 1$, $\alpha_n \in (\delta, 1-\delta)$ *for some* $\delta \in (0,1)$.

If one of the mappings $\{S_i : i = 1,2,\cdots,N\}$ *is semi-compact and* $\Gamma \neq \emptyset$, *then the sequences* $\{x_n\}$ *and* $\{y_n\}$ *both converge strongly to a split common fixed point* $x^* \in \Gamma$.

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(Received November 6, 2012)

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