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ON OPERATOR BOHR TYPE INEQUALITIES

LIMIN ZOU AND CHUANJIANG HE

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Abstract. The purpose of this paper is to discuss inequalities related to operator versions of the classical Bohr inequality. We obtain refinements of some inequalities due to Cheung and Pečarić [J. Math. Anal. Appl. 323 (2006) 403–412] and Zhang [J. Math. Anal. Appl. 333 (2007) 1264–1271]. Moreover, we present two inequalities for multiple operators, which are similar to ones proposed by Chansangiam et al. [J. Math. Anal. Appl. 356 (2009) 525–536].

1. Introduction

Let $B(H)$ be the algebra of all bounded linear operators on a complex separable Hilbert space H. For $A \in B(H)$, A^* denotes the adjoint operator of A. The absolute value of operator A is defined by $|A| = (A^*A)^{1/2}$. If $A,\ B \in B(H)$ are self-adjoint, then $A \geqslant B$ means that $A - B$ is a positive operator. Let M_n be the space of $n \times n$ complex matrices. For $X \in M_n$, X_{ij} denotes the sub-matrix of X resulting from the deletion of row i and column j. Throughout this paper, we assume that $p,\ q \in \mathbb{R}$ with $\frac{1}{p} + \frac{1}{q} = 1$, where $\mathbb{R}$ is the set of real numbers.

The classical Bohr inequality [2] for scalars asserts that for complex numbers z_1, z_2 and $p,\ q > 1$,

$$|z_1 - z_2|^2 \leqslant p\,|z_1|^2 + q\,|z_2|^2 .$$

A number of generalizations of Bohr inequality for operator in $B(H)$ have been established [1, 3–11] over the years. In 2003, Hirzallah [7, Theorem 1] obtained an operator version of Bohr inequality, which says that if $A, B \in B(H)$ and $1 < p \leqslant q$, then

$$|A - B|^2 + |(1-p)A - B|^2 \leqslant p\,|A|^2 + q\,|B|^2 . \tag{1.1}$$

Inequality (1.1) was extended to all possible cases of p, q by Cheung and Pečarić [4, Theorem 1, Corollary 1, and Theorem 2] as follows:
(i) If $p < 1$, then

$$\begin{aligned} p\,|A|^2 + q\,|B|^2 &\leqslant |A - B|^2 + |(1-p)A - B|^2 , \\ p\,|A|^2 + q\,|B|^2 &\leqslant |A - B|^2 + |A - (1-q)B|^2 . \end{aligned} \tag{1.2}$$

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(ii) If $1 < p \leqslant 2$, then

$$|A-B|^2 + |(1-p)A-B|^2 \leqslant p|A|^2 + q|B|^2 \leqslant |A-B|^2 + |A-(1-q)B|^2. \quad (1.3)$$

(iii) If $p > 2$, then

$$|A-B|^2 + |A-(1-q)B|^2 \leqslant p|A|^2 + q|B|^2 \leqslant |A-B|^2 + |(1-p)A-B|^2. \quad (1.4)$$

Recently, Zhang [10, Theorem 7] presented a generalization of operator Bohr inequality for multiple operators, which states that if k is a positive integer and $A_j \in B(H)$, $j = 1, \cdots, k$, then for any positive numbers $t_1, \cdots, t_k$ with $\sum_{j=1}^{k} t_j = 1$,

$$\left|\sum_{j=1}^{k} t_j A_j\right|^2 \leqslant \sum_{j=1}^{k} t_j |A_j|^2. \quad (1.5)$$

Inequality (1.5) was generalized by Chansangiam et al. [3, Theorem 26] to the following form:

Define $X = [x_{ij}] \in M_k$, where

$$x_{ij} = \begin{cases} \alpha_i^2 - \beta_i, & i = j \\ \alpha_i \alpha_j, & i \neq j \end{cases}, \quad \alpha_i,\ \beta_i \in \mathbb{R},\ \ 1 \leqslant i,\ j \leqslant k.$$

If $X \leqslant 0$, then

$$\left|\sum_{j=1}^{k} \alpha_j A_j\right|^2 \leqslant \sum_{j=1}^{k} \beta_j |A_j|^2. \quad (1.6)$$

If $X \geqslant 0$, then

$$\left|\sum_{j=1}^{k} \alpha_j A_j\right|^2 \geqslant \sum_{j=1}^{k} \beta_j |A_j|^2. \quad (1.7)$$

In this paper, we give refinements of inequalities (1.2)–(1.5). Moreover, we obtain two inequalities for multiple operators, which are similar to inequalities (1.6) and (1.7).

2. Main results

In this section, we shall present some operator Bohr type inequalities. To achieve it, we need an operator equality, which is equivalent to some existing results.

LEMMA 2.1. *Let $A, B \in B(H)$, $p,\ q \neq 0, 1$, and $0 \leqslant \lambda \leqslant 1$. Then*

$$p|A|^2 + q|B|^2 = |A-B|^2 + \frac{\lambda}{p-1}|(p-1)A+B|^2 + \frac{1-\lambda}{q-1}|A+(q-1)B|^2. \quad (2.1)$$

Proof. Note that

$$\frac{\lambda}{p-1}\left|(p-1)A+B\right|^2=\lambda\,(p-1)\,|A|^2+\frac{\lambda}{p-1}\,|B|^2+\lambda\,(A^*B+B^*A)\,,$$

$$\frac{1-\lambda}{q-1}\,|A+(q-1)\,B|^2=\frac{1-\lambda}{q-1}\,|A|^2+(1-\lambda)\,(q-1)\,|B|^2+(1-\lambda)\,(A^*B+B^*A)\,,$$

and $\frac{q}{p}=q-1=\frac{1}{p-1}$. By simple calculations, equality (2.1) follows from above equalities. This completes the proof. □

REMARK 2.1. Abramovich et al. [1, Equality 2.2] proved that for any $\alpha\in\mathbb{R}$,

$$\alpha\,(1-\alpha)\,|A-B|^2+|\alpha A+(1-\alpha)\,B|^2=\alpha\,|A|^2+(1-\alpha)\,|B|^2\,. \tag{2.2}$$

The special case for $0\leqslant\alpha\leqslant 1$ has been obtained earlier by Zhang [10, Theorem 2]. Note that for $\alpha\neq 0,\ 1$, equality (2.2) is equivalent to

$$|A-B|^2+\frac{\alpha}{1-\alpha}\left|A+\frac{1-\alpha}{\alpha}B\right|^2=\frac{1}{1-\alpha}\,|A|^2+\frac{1}{\alpha}\,|B|^2\,.$$

Taking $\frac{1}{1-\alpha}=p,\ \frac{1}{\alpha}=q$ and using equality

$$\frac{1}{p-1}\left|(p-1)A+B\right|^2=\frac{1}{q-1}\,|A+(q-1)\,B|^2$$

in the last equality, we can easily conclude that (2.2) is equivalent to (2.1). Fujii and Zuo [6, Theorem 4.1] proved that for $t\neq 0$,

$$|A+B|^2+\frac{1}{t}\,|tA-B|^2=(1+t)\,|A|^2+\left(1+\frac{1}{t}\right)|B|^2\,.$$

Simple calculations show that above equality is also equivalent to (2.1).

Now, we refine inequalities (1.2)–(1.4) by utilizing equality (2.1).

THEOREM 2.1. *Let $A,B\in B(H)$, $p,\ q\neq 0,1$, and $0\leqslant\lambda\leqslant 1$.*
(i) If $p<1$, then

$$\begin{aligned}p\,|A|^2+q\,|B|^2&\leqslant |A-B|^2+\lambda q\,|(1-p)\,A-B|^2\,,\\ p\,|A|^2+q\,|B|^2&\leqslant |A-B|^2+(1-\lambda)\,p\,|A-(1-q)\,B|^2\,.\end{aligned} \tag{2.3}$$

(ii) If $1<p\leqslant 2$, then

$$\begin{aligned}|A-B|^2+|(1-p)\,A-B|^2&\leqslant |A-B|^2+((q-1)\,(1-\lambda)+\lambda)\,|(1-p)\,A-B|^2\\ &\leqslant p\,|A|^2+q\,|B|^2\\ &\leqslant |A-B|^2+((p-1)\,\lambda+1-\lambda)\,|A-(1-q)\,B|^2\\ &\leqslant |A-B|^2+|A-(1-q)\,B|^2\,.\end{aligned} \tag{2.4}$$

(iii) If $p>2$, then

$$
\begin{aligned}
|A-B|^2+|(1-q)A-B|^2 &\leqslant |A-B|^2+((p-1)\lambda+1-\lambda)\,|A-(1-q)B|^2\\
&\leqslant p\,|A|^2+q\,|B|^2\\
&\leqslant |A-B|^2+((q-1)(1-\lambda)+\lambda)\,|(1-p)A-B|^2\\
&\leqslant |A-B|^2+|A-(1-p)B|^2.
\end{aligned}
\tag{2.5}
$$

Proof. Let $p<1$, then $q<1$ and $pq<0$. It follows that

$$\frac{1}{p-1}\,|(p-1)A+B|^2 \leqslant 0$$

and

$$\frac{1}{q-1}\,|A+(q-1)B|^2 \leqslant 0.$$

By (2.1), we obtain

$$
\begin{aligned}
p\,|A|^2+q\,|B|^2 &\leqslant |A-B|^2+\frac{\lambda}{p-1}\,|(1-p)A-B|^2\\
&\leqslant |A-B|^2+\left(\frac{\lambda}{p-1}+\lambda\right)|(1-p)A-B|^2\\
&= |A-B|^2+\lambda q\,|(1-p)A-B|^2
\end{aligned}
$$

and

$$
\begin{aligned}
p\,|A|^2+q\,|B|^2 &\leqslant |A-B|^2+\frac{1-\lambda}{q-1}\,|A-(1-q)B|^2\\
&\leqslant |A-B|^2+\left(\frac{1-\lambda}{q-1}+1-\lambda\right)|A-(1-q)B|^2\\
&= |A-B|^2+(1-\lambda)\,p\,|A-(1-q)B|^2.
\end{aligned}
$$

Then, inequality (2.3) holds.

Since $1<p\leqslant 2$ implies $q\geqslant 2$, it follows that $\frac{1}{p-1}\geqslant 1$ and $\frac{1}{q-1}\leqslant 1$. Now, we prove the second and third inequalities of (2.4). By (2.1), we have

$$
\begin{aligned}
p\,|A|^2+q\,|B|^2 &\geqslant |A-B|^2+\lambda\,|(p-1)A+B|^2+\frac{1-\lambda}{q-1}\,|A+(q-1)B|^2\\
&= |A-B|^2+\lambda\,|(p-1)A+B|^2+\frac{1-\lambda}{p-1}\,|(p-1)A+B|^2\\
&= |A-B|^2+\left(\lambda+\frac{1-\lambda}{p-1}\right)|(p-1)A+B|^2\\
&= |A-B|^2+((q-1)(1-\lambda)+\lambda)\,|(1-p)A-B|^2.
\end{aligned}
$$

This is the second inequality of (2.4). Meanwhile, by (2.1), we also have

$$\begin{aligned} p|A|^2+q|B|^2 &\leqslant |A-B|^2+\frac{\lambda}{p-1}|(p-1)A+B|^2+(1-\lambda)|A+(q-1)B|^2 \\ &= |A-B|^2+\frac{\lambda}{q-1}|A+(q-1)B|^2+(1-\lambda)|A+(q-1)B|^2 \\ &= |A-B|^2+\left(\frac{\lambda}{q-1}+1-\lambda\right)|A+(q-1)B|^2 \\ &= |A-B|^2+((p-1)\lambda+1-\lambda)|A-(1-q)B|^2. \end{aligned}$$

This is the third inequality of (2.4). Next, we prove the first inequality of (2.4). A simple calculation shows that

$$\begin{aligned} |(1-p)A-B|^2 &- ((q-1)(1-\lambda)+\lambda)|(1-p)A-B|^2 \\ &= (1-\lambda)(2-q)|(1-p)A-B|^2 \\ &\leqslant 0. \end{aligned}$$

Consequently,

$$|A-B|^2+|(1-p)A-B|^2 \leqslant ((q-1)(1-\lambda)+\lambda)|(1-p)A-B|^2.$$

Finally, we prove the fourth inequality of (2.4). Similarly, we have

$$\begin{aligned} |A-(1-q)B|^2 &- ((p-1)\lambda+1-\lambda)|A-(1-q)B|^2 \\ &= \lambda(2-p)|A-(1-q)B|^2 \\ &\geqslant 0. \end{aligned}$$

So,

$$|A-B|^2+((p-1)\lambda+1-\lambda)|A-(1-q)B|^2 \leqslant |A-B|^2+|A-(1-q)B|^2$$

Similar to the case of $1<p\leqslant 2$, we can prove inequality (2.5), so we omit the details. This completes the proof. □

REMARK 2.2. Inequalities (2.3)–(2.5) are refinements of inequalities (1.2)–(1.4), respectively. Zou et al. [11, Lemma 2.1] proved that

$$|A-B|^2+\frac{2}{p}|(p-1)A+B|^2 \leqslant p|A|^2+q|B|^2 \leqslant |A-B|^2+\frac{2}{q}|A+(q-1)B|^2 \tag{2.6}$$

for $1<p\leqslant 2$ and

$$|A-B|^2+\frac{2}{q}|A+(q-1)B|^2 \leqslant p|A|^2+q|B|^2 \leqslant |A-B|^2+\frac{2}{p}|(p-1)A+B|^2 \tag{2.7}$$

for $p>2$. Taking $\lambda=\dfrac{q-2}{q-p}$ in (2.4) and (2.5), we obtain inequalities (2.6) and (2.7) respectively.

REMARK 2.3. Cheung and Pečarić [4, Theorems 3, 4] obtained some Bohr type inequalities for operators in $B(H)$ by using inequalities (1.2)–(1.4). By utilizing inequalities (2.3)–(2.5), we can refine some results obtained by Cheung and Pečarić [4, Theorems 3, 4].

REMARK 2.4. Chansangiam et al. [3] established some generalizations of Bohr inequality for operators in $B(H)$, such as inequalities (19)–(34) in [3]. The crucial tool for these results is Theorem 10 in [3], i.e., inequalities (1.2)–(1.4) of this paper. We can obtain some refinements of these inequalities by using (2.3)–(2.5).

The following result is a refinement of inequality (1.5).

THEOREM 2.2. *Let k be a positive integer and let $A_j \in B(H)$, $j = 1, \cdots, k$. Then for any positive numbers $t_1, \cdots, t_k$ with $\sum_{j=1}^{k} t_j = 1$, $t_j \neq 1$, it holds that*

$$\left|\sum_{j=1}^{k} t_j A_j\right|^2 + \frac{1}{2}\min\{t_i,\ 1-t_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{t_j}{1-t_i} A_j\right|^2 \leqslant \sum_{j=1}^{k} t_j \left|A_j\right|^2 \tag{2.8}$$

for $1 \leqslant i \leqslant k$.

Proof. For $0 \leqslant \alpha \leqslant 1$, by (2.2), we have

$$|\alpha A + (1-\alpha)B|^2 + \frac{1}{2}\min\{\alpha,\ 1-\alpha\}|A-B|^2 \leqslant \alpha|A|^2 + (1-\alpha)|B|^2. \tag{2.9}$$

Note that

$$|t_1A_1 + \cdots + t_kA_k|^2 = \left|t_iA_i + (1-t_i)\sum_{j=1,\, j\neq i}^{k} \frac{t_j}{1-t_i} A_j\right|^2,\ 1 \leqslant i \leqslant k.$$

It follows from (2.9) and (1.5) that

$$\begin{aligned}
|t_1A_1 + \cdots + t_kA_k|^2 \quad & + \frac{1}{2}\min\{t_i,\ 1-t_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{t_j}{1-t_i} A_j\right|^2 \\
&= \left|t_iA_i + (1-t_i)\sum_{j=1,\, j\neq i}^{k} \frac{t_j}{1-t_i} A_j\right|^2 \\
&\quad + \frac{1}{2}\min\{t_i, 1-t_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{t_j}{1-t_i} A_j\right|^2 \\
&\leqslant t_i|A_i|^2 + (1-t_i)\left|\sum_{j=1,\, j\neq i}^{k} \frac{t_j}{1-t_i} A_j\right|^2 \\
&\leqslant t_1|A_1|^2 + \cdots + t_k|A_k|^2
\end{aligned}$$

for $1 \leqslant i \leqslant k$. Thus,

$$\left|\sum_{j=1}^{k} t_j A_j\right|^2 + \frac{1}{2}\min\{t_i,\, 1-t_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{t_j}{1-t_i}A_j\right|^2 \leqslant \sum_{j=1}^{k} t_j\left|A_j\right|^2.$$

This completes the proof. □

Finally, we present two inequalities for multiple operators, which are similar to inequalities (1.6) and (1.7).

Let k be a positive integer. For $\beta_j \in \mathbb{R}$, $1 \leqslant j \leqslant k$ and any positive numbers $\alpha_1, \cdots, \alpha_k$ with $\sum\limits_{j=1}^{k} \alpha_j = 1$, $\alpha_j \neq 1$, we define $Y = [y_{js}] \in M_k$, where

$$y_{js} = \begin{cases} \alpha_j^2 - (1-\alpha_i)\beta_j, & j = s \\ \alpha_j\alpha_s, & j \neq s \end{cases}, \quad 1 \leqslant j, s \leqslant k,\ i \neq j. \tag{2.10}$$

THEOREM 2.3. *Let $A_j \in B(H)$, $j = 1, \cdots, k$. If $Y = [y_{js}]$ is as (2.10) and $Y \leqslant 0$, then*

$$\left|\sum_{j=1}^{k} \alpha_j A_j\right|^2 + \frac{1}{2}\min\{\alpha_i,\, 1-\alpha_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i}A_j\right|^2 \leqslant \alpha_i\left|A_i\right|^2 + \sum_{j=1, j\neq i}^{k} \beta_j\left|A_j\right|^2$$

for $1 \leqslant i \leqslant k$.

Proof. It is known that $-Y \geqslant 0$ is equivalent to $-Z = \dfrac{-Y}{(1-\alpha_i)^2} \geqslant 0$, which implies $Z_{ii} \leqslant 0$. It follows from (1.6) and (2.9) that

$$\begin{aligned}\alpha_i\left|A_i\right|^2 &+ (1-\alpha_i)\sum_{j=1, j\neq i}^{k} \frac{\beta_j}{1-\alpha_i}\left|A_j\right|^2 \geqslant \alpha_i\left|A_i\right|^2 + (1-\alpha_i)\left|\sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i}A_j\right|^2 \\ &\geqslant \left|\alpha_i A_i + (1-\alpha_i)\sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i}A_j\right|^2 \\ &\quad + \frac{1}{2}\min\{\alpha_i,\, 1-\alpha_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i}A_j\right|^2 \\ &= \left|\alpha_1 A_1 + \cdots + \alpha_k A_k\right|^2 + \frac{1}{2}\min\{\alpha_i,\, 1-\alpha_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i}A_j\right|^2\end{aligned}$$

for $1 \leqslant i \leqslant k$. This completes the proof. □

THEOREM 2.4. *Let $A_j \in B(H)$, $j = 1, \cdots, k$. If $Y = [y_{js}]$ is as (2.10) and $Y \geqslant 0$, then*

$$\left|\sum_{j=1}^{k} \alpha_j A_j\right|^2 + \min\{\alpha_i,\, 1-\alpha_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i}A_j\right|^2 \geqslant \alpha_i\left|A_i\right|^2 + \sum_{j=1, j\neq i}^{k} \beta_j\left|A_j\right|^2$$

for $1 \leqslant i \leqslant k$.

Proof. By (2.2), we know that for $0 \leqslant \alpha \leqslant 1$,

$$\alpha |A|^2 + (1-\alpha)|B|^2 \leqslant |\alpha A + (1-\alpha)B|^2 + \min\{\alpha,\ 1-\alpha\}|A-B|^2. \tag{2.11}$$

It is known that $Y \geqslant 0$ is equivalent to $Z = \frac{Y}{(1-\alpha_i)^2} \geqslant 0$, which implies $Z_{ii} \geqslant 0$. It follows from (1.7) and (2.11) that

$$\begin{aligned}
\alpha_i |A_i|^2 &+ (1-\alpha_i)\sum_{j=1,j\neq i}^{k} \frac{\beta_j}{1-\alpha_i}|A_j|^2 \leqslant \alpha_i |A_i|^2 + (1-\alpha_i)\left|\sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i} A_j\right|^2 \\
&\leqslant \left|\alpha_i A_i + (1-\alpha_i)\sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i} A_j\right|^2 \\
&\quad + \min\{\alpha_i,\ 1-\alpha_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i} A_j\right|^2 \\
&= |\alpha_1 A_1 + \cdots + \alpha_k A_k|^2 + \min\{\alpha_i,\ 1-\alpha_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i} A_j\right|^2
\end{aligned}$$

for $1 \leqslant i \leqslant k$. This completes the proof. □

REMARK 2.5. By Theorem 2.4 and inequality (2.8), we have

$$\alpha_i |A_i|^2 + \sum_{j=1,j\neq i}^{k} \beta_j |A_j|^2 \leqslant \sum_{j=1}^{k} \alpha_j |A_j|^2 + \frac{1}{2}\min\{\alpha_i,\ 1-\alpha_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i} A_j\right|^2.$$

Consequently,

$$\sum_{j=1,j\neq i}^{k} \beta_j |A_j|^2 \leqslant \sum_{j=1,j\neq i}^{k} \alpha_j |A_j|^2 + \frac{1}{2}\min\{\alpha_i,\ 1-\alpha_i\}\left|A_i - \sum_{j=1,\, j\neq i}^{k} \frac{\alpha_j}{1-\alpha_i} A_j\right|^2.$$

This is also an inequality for multiple operators.

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Limin Zou
College of Mathematics and Statistics, Chongqing University
Chongqing, 401331 P. R. China
and
School of Mathematics and Statistics
Chongqing Three Gorges University
Chongqing, 404100 P. R. China
e-mail: limin-zou@163.com

Chuanjiang He
College of Mathematics and Statistics, Chongqing University
Chongqing, 401331 P. R. China
e-mail: chuanjianghe@sina.com