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A NEW CHARACTERIZATION OF THE GENERALIZED WEIGHTED COMPOSITION OPERATOR FROM H^∞ INTO THE ZYGMUND SPACE

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Abstract. A new criterion for the boundedness, as well as for the compactness of the generalized weighted composition operators from H^∞ into the Zygmund space are given in this paper.

1. Introduction

Let $\mathbb{D}$ be the open unit disk in the complex plane $\mathbb{C}$ and $H(\mathbb{D})$ be the space of analytic functions on $\mathbb{D}$. Let $H^\infty = H^\infty(\mathbb{D})$ denote the space of bounded analytic functions f on $\mathbb{D}$ with norm $\|f\|_\infty = \sup_{z\in\mathbb{D}}|f(z)|$. An $f \in H(\mathbb{D})$ is said to belong to the Bloch space $\mathscr{B}$ if

$$\|f\|_{\mathscr{B}} = \sup_{z\in\mathbb{D}}|f'(z)|(1-|z|^2) < \infty.$$

In this paper, we shall consider the Zygmund space $\mathscr{Z}$ consisting of the functions $f \in H(\mathbb{D}) \cap C(\overline{\mathbb{D}})$ such that

$$\|f\| = \sup \frac{|f(e^{i(\theta+h)}) + f(e^{i(\theta-h)}) - 2f(e^{i\theta})|}{h} < \infty,$$

where the supremum is taken over all $\theta \in \mathbb{R}$ and $h > 0$. As a consequence of Theorem 5.3 of [3] and the Closed Graph Theorem, a function $f \in H(\mathbb{D})$ belongs to $\mathscr{Z}$ if and only if $\sup_{z\in\mathbb{D}}(1-|z|^2)|f''(z)| < \infty$. Furthermore, the quantity

$$\|f\|_{\mathscr{Z}} = |f(0)| + |f'(0)| + \sup_{z\in\mathbb{D}}(1-|z|^2)|f''(z)|,$$

yields a Banach space structure on $\mathscr{Z}$. For some results on the Zygmund space and operators on them, see, for example [10, 11, 14, 15].

The differentiation operator D is defined by $Df = f'$, $f \in H(\mathbb{D})$. For a nonnegative integer n, we define

$$(D^0f)(z) = f(z), \quad (D^nf)(z) = f^{(n)}(z), \quad n \geqslant 1, \; f \in H(\mathbb{D}).$$

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Let φ be an analytic self-map of $\mathbb{D}$, $u \in H(\mathbb{D})$ and let n be a nonnegative integer. Define the linear operator $D^n_{\varphi,u}$, called the generalized weighted composition operator, by

$$(D^n_{\varphi,u}f)(z) = u(z)\cdot(D^n f)(\varphi(z)), \quad f \in H(\mathbb{D}), \quad z \in \mathbb{D}.$$

When $n = 0$ and $u(z) = 1$, $D^n_{\varphi,u}$ is the composition operator C_φ, which is defined by $C_\varphi f = f\circ\varphi$ for $f \in H(\mathbb{D})$. If $n = 0$, then $D^n_{\varphi,u}$ is the weighted composition operator uC_φ, which is defined as follows

$$uC_\varphi f = u(f\circ\varphi), \quad f \in H(\mathbb{D}).$$

If $n = 1$, $u(z) = \varphi'(z)$, then $D^n_{\varphi,u} = DC_\varphi$. When $u(z) = 1$, $D^n_{\varphi,u} = C_\varphi D^n$. DC_φ and $C_\varphi D^n$ were studied, for example, in [4, 7, 12, 13, 15, 20, 19, 24, 27]. For the study of the generalized weighted composition operator on various function spaces, see, for example, [6, 22, 23, 30, 31, 32, 35, 36, 37]. A basic problem concerning concrete operators on various Banach function spaces is to relate their operator theoretic properties to the function theoretic properties of the involving symbols, which has attracted a lot of attention recently, see, for example, [1, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37].

It is well known that the composition operator is bounded on the Bloch space by the Schwarz-Pick lemma. Composition operators and weighted composition operators on Bloch-type spaces were studied, for example, in [16, 17, 26, 28, 33]. In [28], Wulan, Zheng and Zhu obtained a characterization for the compactness of the composition operators acting on the Bloch space as follows:

THEOREM A. *Let φ be an analytic self-map of $\mathbb{D}$. Then $C_\varphi : \mathscr{B} \to \mathscr{B}$ is compact if and only if*

$$\lim_{j\to\infty} \|\varphi^j\|_{\mathscr{B}} = 0.$$

Stević has studied generalized weighted composition operators $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ in [23] (see also [32]), and, among others, obtained the following result.

THEOREM B. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. Then the following statements holds.*

(a) $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is bounded if and only if

$$\sup_{z\in\mathbb{D}} \frac{(1-|z|^2)|u''(z)|}{(1-|\varphi(z)|^2)^n} < \infty, \qquad \sup_{z\in\mathbb{D}} \frac{(1-|z|^2)|u(z)||\varphi'(z)|^2}{(1-|\varphi(z)|^2)^{n+2}} < \infty$$

and

$$\sup_{z\in\mathbb{D}} \frac{(1-|z|^2)|2u'(z)\varphi'(z)+u(z)\varphi''(z)|}{(1-|\varphi(z)|^2)^{n+1}} < \infty$$

(b) Suppose that $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is bounded, then $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is compact if and only if

$$\lim_{|\varphi(z)|\to 1} \frac{(1-|z|^2)|u''(z)|}{(1-|\varphi(z)|^2)^n} = 0, \qquad \lim_{|\varphi(z)|\to 1} \frac{(1-|z|^2)|u(z)||\varphi'(z)|^2}{(1-|\varphi(z)|^2)^{n+2}} = 0$$

and

$$\lim_{|\varphi(z)|\to 1} \frac{(1-|z|^2)|2u'(z)\varphi'(z)+u(z)\varphi''(z)|}{(1-|\varphi(z)|^2)^{n+1}} = 0.$$

In [6], Li and Fu obtained a new characterization for the boundedness, as well as the compactness for $D^n_{\varphi,u} : \mathscr{B} \to \mathscr{Z}$. Among others, they proved the following result.

THEOREM C. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a positive integer. Then the following statements hold.*

(a) $D^n_{\varphi,u} : \mathscr{B} \to \mathscr{Z}$ is bounded if and only if $u \in \mathscr{Z}$,

$$\sup_{z\in\mathbb{D}}(1-|z|^2)|u(z)||\varphi'(z)|^2 < \infty,$$
$$\sup_{z\in\mathbb{D}}(1-|z|^2)|2u'(z)\varphi'(z)+u(z)\varphi''(z)| < \infty,$$

and

$$\max\{\sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}k_{\varphi(w),1}\|_{\mathscr{Z}}, \sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}k_{\varphi(w),2}\|_{\mathscr{Z}}, \sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}k_{\varphi(w),3}\|_{\mathscr{Z}}\} < \infty,$$

where

$$k_{a,j}(z) = \left(\frac{1-|a|^2}{1-\overline{a}z}\right)^j, \quad j = 1,2,3.$$

(b) If $D^n_{\varphi,u} : \mathscr{B} \to \mathscr{Z}$ is bounded, then $D^n_{\varphi,u} : \mathscr{B} \to \mathscr{Z}$ is compact if and only if

$$\lim_{|\varphi(w)|\to 1}\|D^n_{\varphi,u}k_{\varphi(w),j}\|_{\mathscr{Z}} = 0, \quad j = 1,2,3.$$

Motivated by these observations, in this work we show that $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is bounded (respectively, compact) if and only if the sequence $(\|D^n_{\varphi,u}I^j\|_{\mathscr{Z}})_{j=n}^\infty$ is bounded (respectively, convergent to 0 as $j \to \infty$), where $I^j(z) = z^j$.

Throughout the paper, C denotes a positive constant which may differ from one occurrence to the other. The notation $A \asymp B$ means that there exists a positive constant C such that $B/C \leqslant A \leqslant CB$.

2. Main results and proofs

In this section we formulate our main results and give their proofs. For this purpose, we need the next criterion following from standard arguments similar to those outlined in Proposition 3.11 of [2].

LEMMA 1. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. The operator $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is compact if and only if $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is bounded and for any bounded sequence $(f_k)_{k\in\mathbb{N}}$ in H^∞ which converges to zero uniformly on compact subsets of $\mathbb{D}$, we have $\|D^n_{\varphi,u}f_k\|_{\mathscr{Z}} \to 0$ as $k \to \infty$.*

Next, we state and prove our main results in this paper.

THEOREM 1. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a nonnegative integer. Then $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is bounded if and only if*

$$\sup_{j \geqslant n} \|D^n_{\varphi,u} I^j\|_{\mathscr{Z}} < \infty, \quad \text{where } I^j(z) = z^j. \tag{1}$$

Proof. When $n = 0$, this is just the corresponding result in [1]. Next we only consider the case $n \geqslant 1$.

Suppose that $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is bounded. For any $j \in \mathbb{N}$, the function I^j is bounded in H^∞ and $\|I^j\|_\infty = 1$. Then, notice that for $j < n$, $(D^n_{\varphi,u} I^j)'(z) = 0$, the result follows by the boundedness of the operator $D^n_{\varphi,u}$ from H^∞ to $\mathscr{Z}$.

Conversely, assume that (1) holds and let $Q := \sup_{j \geqslant n} \|D^n_{\varphi,u} I^j\|_{\mathscr{Z}}$. Applying the operator $D^n_{\varphi,u}$ to I^j with $j = n, n+1, n+2$, we obtain

$$\begin{aligned}
(D^n_{\varphi,u} I^n)''(z) &= u''(z) n!, \\
(D^n_{\varphi,u} I^{n+1})''(z) &= (n+1)! [u''(z)\varphi(z) + 2u'(z)\varphi'(z) + u(z)\varphi''(z)]
\end{aligned}$$

and

$$\begin{aligned}
(D^n_{\varphi,u} I^{n+2})''(z) = \frac{(n+2)!}{2} & [u''(z)\varphi^2(z) + 4u'(z)\varphi(z)\varphi'(z) \\
& + 2u(z)\varphi'^2(z) + 2u(z)\varphi(z)\varphi''(z)],
\end{aligned}$$

while for $j < n$, $(D^n_{\varphi,u} I^j)'(z) = 0$. Thus, using the boundedness of the function φ, we have

$$\sup_{z \in \mathbb{D}} (1 - |z|^2)|u''(z)| \leqslant \frac{1}{n!} \|D^n_{\varphi,u} I^n\|_{\mathscr{Z}} \leqslant \frac{Q}{n!} < \infty, \tag{2}$$

$$\begin{aligned}
& \sup_{z \in \mathbb{D}} (1 - |z|^2)|2u'(z)\varphi'(z) + u(z)\varphi''(z)| \\
\leqslant & \frac{1}{(n+1)!} \|D^n_{\varphi,u} I^{n+1}\|_{\mathscr{Z}} + \sup_{z \in \mathbb{D}} (1 - |z|^2)|u''(z)| \leqslant \frac{(n+2)Q}{(n+1)!} < \infty
\end{aligned} \tag{3}$$

and

$$\begin{aligned}
\sup_{z \in \mathbb{D}} (1 - |z|^2)|u(z)||\varphi'(z)|^2 & \leqslant \frac{1}{(n+2)!} \|D^n_{\varphi,u} I^{n+2}\|_{\mathscr{Z}} + \frac{1}{2} \sup_{z \in \mathbb{D}} (1 - |z|^2)|u''(z)| \\
& \quad + \sup_{z \in \mathbb{D}} (1 - |z|^2)|2u'(z)\varphi'(z) + u(z)\varphi''(z)| \\
& \leqslant \frac{Q}{(n+2)!} + \frac{Q}{2n!} + \frac{(n+2)Q}{(n+1)!} < \infty.
\end{aligned} \tag{4}$$

For $a \in \mathbb{D}$, set

$$f_a(z) = \frac{1-|a|^2}{(1-\overline{a}z)}, \quad g_a(z) = \left(\frac{1-|a|^2}{1-\overline{a}z}\right)^2, \quad h_a(z) = \left(\frac{1-|a|^2}{1-\overline{a}z}\right)^3.$$

From the definition of f_a, g_a and h_a, it is easy to see that f_a, g_a and h_a have bounded norms in H^∞. Since

$$f_a(z) = (1-|a|^2)\sum_{j=0}^{\infty} \overline{a}^j z^j, \qquad g_a(z) = (1-|a|^2)^2 \sum_{j=1}^{\infty} j\overline{a}^{j-1}z^{j-1}$$

and

$$h_a(z) = \frac{1}{2}(1-|a|^2)^3 \sum_{j=2}^{\infty} j(j-1)\overline{a}^{j-2}z^{j-2},$$

using linearity we get

$$\|D^n_{\varphi,u}f_a\|_{\mathscr{Z}} \leqslant (1-|a|^2)\sum_{j=0}^{\infty}|a|^j\|D^n_{\varphi,u}I^j\|_{\mathscr{Z}} \leqslant 2Q,$$

$$\|D^n_{\varphi,u}g_a\|_{\mathscr{Z}} \leqslant (1-|a|^2)^2\sum_{j=1}^{\infty}j|a|^{j-1}\|D^n_{\varphi,u}I^{j-1}\|_{\mathscr{Z}} \leqslant 4Q$$

and

$$\|D^n_{\varphi,u}h_a\|_{\mathscr{Z}} \leqslant \frac{1}{2}(1-|a|^2)^3\sum_{j=2}^{\infty}j(j-1)|a|^{j-2}\|D^n_{\varphi,u}I^{j-2}\|_{\mathscr{Z}} \leqslant 4Q.$$

Therefore, by the arbitrariness of $a \in \mathbb{D}$, we get

$$\max\Big\{\sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}f_{\varphi(w)}\|_{\mathscr{Z}},\ \sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}g_{\varphi(w)}\|_{\mathscr{Z}},\ \sup_{w\in\mathbb{D}}\|D^n_{\varphi,u}h_{\varphi(w)}\|_{\mathscr{Z}}\Big\} < \infty. \tag{5}$$

From (2), (3), (4), (5) and Theorem C, we see that $D^n_{\varphi,u} : \mathscr{B} \to \mathscr{Z}$ is bounded and hence $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is bounded, as desired. □

THEOREM 2. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a non-negative integer. Suppose that $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is bounded, then $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is compact if and only if*

$$\lim_{j\to\infty}\|D^n_{\varphi,u}I^j\|_{\mathscr{Z}} = 0, \ \textit{where}\ I^j(z) = z^j. \tag{6}$$

Proof. When $n = 0$, this is just the corresponding result in [1]. Next we only consider the case $n \geqslant 1$.

Assume that $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is compact. Since the sequence $\{I^j\}$ is bounded in H^∞ and converges to 0 uniformly on compact subsets, by Lemma 1 it follows that $\|D^n_{\varphi,u}I^j\|_{\mathscr{Z}} \to 0$ as $j \to \infty$.

Conversely, suppose that (6) holds. Fix $\varepsilon > 0$ and choose $N \in \mathbb{N}$ such that $\|D^n_{\varphi,u} I^j\|_{\mathscr{Z}} < \varepsilon/4$ for all $j \geqslant N$. Let $z_k \in \mathbb{D}$ be such that $|\varphi(z_k)| \to 1$ as $k \to \infty$. Arguing as in Theorem 1, we have

$$\begin{aligned}
\|D^n_{\varphi,u} f_{\varphi(z_k)}\|_{\mathscr{Z}} &\leqslant (1-|\varphi(z_k)|^2) \sum_{j=0}^{\infty} |\varphi(z_k)|^j \|D^n_{\varphi,u} I^j\|_{\mathscr{Z}} \\
&= (1-|\varphi(z_k)|^2) \sum_{j=0}^{N-1} |\varphi(z_k)|^j \|D^n_{\varphi,u} I^j\|_{\mathscr{Z}} \\
&\quad + (1-|\varphi(z_k)|^2) \sum_{j=N}^{\infty} |\varphi(z_k)|^j \|D^n_{\varphi,u} I^j\|_{\mathscr{Z}} \\
&\leqslant 2Q(1-|\varphi(z_k)|^N) + \varepsilon. \\
\|D^n_{\varphi,u} g_{\varphi(z_k)}\|_{\mathscr{Z}} &\leqslant (1-|\varphi(z_k)|^2)^2 \sum_{j=1}^{\infty} j|\varphi(z_k)|^{j-1} \|D^n_{\varphi,u} I^{j-1}\|_{\mathscr{Z}} \\
&= (1-|\varphi(z_k)|^2)^2 \sum_{j=1}^{N} j|\varphi(z_k)|^{j} \|D^n_{\varphi,u} I^{j-1}\|_{\mathscr{Z}} \\
&\quad + (1-|\varphi(z_k)|^2)^2 \sum_{j=N+1}^{\infty} j|\varphi(z_k)|^{j} \|D^n_{\varphi,u} I^{j-1}\|_{\mathscr{Z}} \\
&\leqslant \frac{N(N+1)Q}{2}(1-|\varphi(z_k)|^2)^2 + \varepsilon. \\
\|D^n_{\varphi,u} h_{\varphi(z_k)}\|_{\mathscr{Z}} &\leqslant \frac{1}{2}(1-|\varphi(z_k)|^2)^3 \sum_{j=2}^{\infty} j(j-1)|\varphi(z_k)|^{j-2} \|D^n_{\varphi,u} I^{j-2}\|_{\mathscr{Z}} \\
&= \frac{1}{2}(1-|\varphi(z_k)|^2)^3 \sum_{j=2}^{N+1} j(j-1)|\varphi(z_k)|^{j-2} \|D^n_{\varphi,u} I^{j-2}\|_{\mathscr{Z}} \\
&\quad + \frac{1}{2}(1-|\varphi(z_k)|^2)^3 \sum_{j=N+2}^{\infty} j(j-1)|\varphi(z_k)|^{j-2} \|D^n_{\varphi,u} I^{j-2}\|_{\mathscr{Z}} \\
&\leqslant \frac{N(N+1)(N+2)Q}{6}(1-|\varphi(z_k)|^2)^3 + \varepsilon.
\end{aligned}$$

Since $|\varphi(z_k)| \to 1$ as $k \to \infty$, by the arbitrariness of ε, we get

$$\lim_{k\to\infty} \|D^n_{\varphi,u} f_{\varphi(z_k)}\|_{\mathscr{Z}} = 0, \lim_{k\to\infty} \|D^n_{\varphi,u} g_{\varphi(z_k)}\|_{\mathscr{Z}} = 0, \lim_{k\to\infty} \|D^n_{\varphi,u} h_{\varphi(z_k)}\|_{\mathscr{Z}} = 0,$$

i.e., we obtain

$$\lim_{|\varphi(a)|\to 1} \|D^n_{\varphi,u} f_{\varphi(a)}\|_{\mathscr{Z}} = 0, \lim_{|\varphi(a)|\to 1} \|D^n_{\varphi,u} g_{\varphi(a)}\|_{\mathscr{Z}} = 0, \lim_{|\varphi(a)|\to 1} \|D^n_{\varphi,u} h_{\varphi(a)}\|_{\mathscr{Z}} = 0. \quad (7)$$

From (7) and Theorem C, we see that $D^n_{\varphi,u} : \mathscr{B} \to \mathscr{Z}$ is compact and therefore $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ is compact. The proof is complete. □

From main results in [6, 23, 32] and Theorems 1 and 2 in this paper, we can immediately get the following two corollaries.

COROLLARY 1. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a positive integer. Then the following statements are equivalent.*

(a) $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ *is bounded;*

(b) $D^n_{\varphi,u} : \mathscr{B} \to \mathscr{Z}$ *is bounded;*

(c) $\sup_{j \geqslant n} \|D^n_{\varphi,u} I^j\|_{\mathscr{Z}} < \infty$, *where* $I^j(z) = z^j$.

COROLLARY 2. *Let $u \in H(\mathbb{D})$, φ be an analytic self-map of $\mathbb{D}$ and n be a positive integer. Suppose that $D^n_{\varphi,u} : H^\infty(or\mathscr{B}) \to \mathscr{Z}$ is bounded, then the following statements are equivalent.*

(a) $D^n_{\varphi,u} : H^\infty \to \mathscr{Z}$ *is compact;*

(b) $D^n_{\varphi,u} : \mathscr{B} \to \mathscr{Z}$ *is compact;*

(c) $\lim_{j\to\infty} \|D^n_{\varphi,u} I^j\|_{\mathscr{Z}} = 0$, *where* $I^j(z) = z^j$.

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