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ON A DISPARITY BETWEEN WILLINGNESS TO PAY AND WILLINGNESS TO ACCEPT UNDER THE RANK–DEPENDENT UTILITY MODEL

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Abstract. Willingness to pay $\mathrm{WTP}(X)$ for a lottery X, represented by a finitely-supported probability distribution on $\mathbb{R}$, is the highest amount an individual is willing to pay for X. Willingness to accept $\mathrm{WTA}(X)$ is the smallest amount for which an individual would accept the sell of X. We deal with these notions under Rank-Dependent Utility, one of the behavioral models of decision making under risk. Applying some results concerning a comparison of quasideviation means, we characterize the properties of willingness to pay and willingness to accept related to the experimentally observed disparity between them.

1. Introduction

Assume that $\mathscr{X}$ is a family of lotteries, that is all finitely-supported probability distributions on $\mathbb{R}$. For every $n \in \mathbb{N}$, with $n \geqslant 2$, denote by $\langle x_1, \ldots, x_n; p_1, \ldots, p_n \rangle$ the lottery in which payoffs $x_1 < x_2 < \ldots < x_n$ occur with probabilities $p_1, \ldots, p_n \in (0,1)$, respectively.

The lottery pricing problem is an important issue in many models of decision making under risk. It consists on assigning to each lottery a real number, interpreted as a price for the lottery. In experimental settings, it has been observed that the price of a given lottery depends on whether a decision maker intends to buy or sell it. Therefore in many models a buying price and a selling price for a given lottery are defined in different ways. A buying price of a lottery X is the highest amount an individual is willing to pay for X. A selling price of a lottery X is the smallest amount for which an individual would accept the sell of X. There are several ways of defining the buying and selling prices. Some of them have been recently discussed in [1].

In a recent paper [6], a model based on the assumption that the decision maker derives preference over lotteries from changes in the initial wealth implied by accepting the given lottery, has been investigated. This approach has been inspired by earlier works [5] and [9]. In this setting, for every $X \in \mathscr{X}$, a buying price for X, called

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willingness to accept (WTA(X)), and a selling price for X, called willingness to pay (WTP(X)), are determined by

$$E[u(\mathrm{WTA}(X)-X)]=0$$

and

$$E[u(X-\mathrm{WTP}(X))]=0,$$

respectively, where $u:\mathbb{R}\to\mathbb{R}$ is a strictly increasing continuous function such that $u(0)=0$. Thus, for every $X\in\mathscr{X}$, both prices are defined implicitly.

It follows from several experiments that $\mathrm{WTA}(X)$ for a given $X\in\mathscr{X}$ usually is much higher than $\mathrm{WTP}(X)$ (see e.g. [4], [10], [11]). In a recent paper [3] a disparity between $\mathrm{WTA}(X)$ and $\mathrm{WTP}(X)$ has been characterized in terms of the properties of the generating function u.

In this paper we investigate willingness to accept and willingness to pay under Rank-Dependent Utility, one of the alternative behavioral models of decision making under risk. In order to define them let us recall that, if $X=\langle x_1,\dots,x_n;p_1,\dots,p_n\rangle\in\mathscr{X}$, then the Choquet integral with respect to a probability distortion function g, that is a non-decreasing function mapping $[0,1]$ into $[0,1]$ and satisfying the boundary conditions $g(0)=0$ and $g(1)=1$, is given by

$$E_g[X]=\sum_{k=1}^{n}\left[g\left(\sum_{i=k}^{n}p_i\right)-g\left(\sum_{i=k+1}^{n}p_i\right)\right]x_k, \tag{1}$$

with a convention $\sum_{i=n+1}^{n}p_i=0$. Then, for every $X\in\mathscr{X}$, the numbers $\mathrm{WTA}(X)$ and $\mathrm{WTP}(X)$ are defined by

$$E_g[u(\mathrm{WTA}(X)-X)]=0 \tag{2}$$

and

$$E_g[u(X-\mathrm{WTP}(X))]=0, \tag{3}$$

respectively, where $u:\mathbb{R}\to\mathbb{R}$ is a strictly increasing continuous function with $u(0)=0$. It is not difficult to note (cf. Remark 3) that, for every $X\in\mathscr{X}$, the numbers $\mathrm{WTA}(X)$ and $\mathrm{WTP}(X)$ are uniquely determined by (2) and (3), respectively.

The aim of this paper is to characterize a disparity between willingness to accept and willingness to pay in the model defined by (2)–(3). An effective tool for dealing with this issue is a notion of a quasideviation mean. Let us note that in [2] quasideviation means were applied to characterize some properties of the principle of equivalent utility, one of the important utility-based methods of insurance contracts pricing.

2. Preliminaries

We begin this section with introducing some notation. If $X=\langle x_1,\dots,x_n;p_1,\dots,p_n\rangle\in\mathscr{X}$, then we set

$$X+c:=\langle x_1+c,\dots,x_n+c;p_1,\dots,p_n\rangle \quad\text{for}\quad c\in\mathbb{R},$$

$$\alpha X := \langle \alpha x_1, \dots, \alpha x_n; p_1, \dots, p_n \rangle \quad \text{for} \quad \alpha \in (0, \infty),$$
$$-X := \langle -x_n, \dots, -x_1; p_n, \dots, p_1 \rangle$$

and

$$u(X) := \langle u(x_1), \dots, u(x_n); p_1, \dots, p_n \rangle$$

for any strictly increasing function $u : \mathbb{R} \to \mathbb{R}$.

REMARK 1. Let g be a probability distortion function. It follows from (1) that

$$E_g[X + c] = E_g[X] + c \quad \text{for} \quad X \in \mathscr{X},\ c \in \mathbb{R}, \tag{4}$$

$$E_g[\alpha X] = \alpha E_g[X] \quad \text{for} \quad X \in \mathscr{X},\ \alpha \in (0, \infty) \tag{5}$$

and

$$E_g[u(X)] \leqslant E_g[v(X)] \quad \text{for} \quad X \in \mathscr{X},$$

whenever $u, v : \mathbb{R} \to \mathbb{R}$ are strictly increasing functions such that $u(x) \leqslant v(x)$ for $x \in \mathbb{R}$. Furthermore, if $\bar{g} : [0,1] \to [0,1]$ is the probability distortion function conjugated to g, i.e.

$$\bar{g}(p) = 1 - g(1 - p) \quad \text{for} \quad p \in [0,1], \tag{6}$$

then

$$E_g[-X] = -E_{\bar{g}}[X] \quad \text{for} \quad X \in \mathscr{X}. \tag{7}$$

REMARK 2. In view of (1), for any probability distortion function g and $X = \langle x_1, \dots, x_n; p_1, \dots, p_n \rangle \in \mathscr{X}$, we have

$$E_g[X] = x_1 + \sum_{k=1}^{n-1} g\left(\sum_{i=k+1}^{n} p_i \right) (x_{k+1} - x_k). \tag{8}$$

Thus, if g_1 and g_2 are probability distortion functions such that

$$g_1(p) \leqslant g_2(p) \quad \text{for} \quad p \in [0,1],$$

then

$$E_{g_1}[X] \leqslant E_{g_2}[X] \quad \text{for} \quad X \in \mathscr{X}.$$

REMARK 3. Assume that $u : \mathbb{R} \to \mathbb{R}$ is a strictly increasing continuous function with $u(0) = 0$ and g is a probability distortion function. Let $X = \langle x_1, \dots, x_n; p_1, \dots, p_n \rangle \in \mathscr{X}$. Then, the function

$$\mathbb{R} \ni t \to E_g[u(t - X)] = \sum_{k=1}^{n} \left[g\left(\sum_{i=k}^{n} p_i \right) - g\left(\sum_{i=k+1}^{n} p_i \right) \right] u(t - x_k)$$

is continuous and strictly increasing. Furthermore, it takes negative values for $t < \min\{x_i : i \in \{1, \dots n\}\}$ and positive values for $t > \max\{x_i : i \in \{1, \dots n\}\}$. Thus, $\mathrm{WTP}(X)$ is uniquely determined by (3). In a similar way one can show that $\mathrm{WTA}(X)$ is uniquely determined by (2).

REMARK 4. Let $u:\mathbb{R}\to\mathbb{R}$ be a strictly increasing continuous function such that $u(0)=0$ and let g be a probability distortion function. If $X=\langle x_1,x_2;p,1-p\rangle\in\mathscr{X}^{(2)}$ then, applying (1), from (2) and (3) we deduce that

$$(1-g(p))u(\mathrm{WTA}(X)-x_2)+g(p)u(\mathrm{WTA}(X)-x_1)=0 \tag{9}$$

and

$$(1-g(1-p))u(x_1-\mathrm{WTP}(X))+g(1-p)u(x_2-\mathrm{WTP}(X))=0, \tag{10}$$

respectively.

As we have already mentioned, a notion of a quasideviation mean, introduced in [7], will play an important role in a characterization of the disparity between the willingness to accept and the willingness to pay. To recall the notion, assume that $I\subseteq\mathbb{R}$ is an open interval. A function $D:I^2\to\mathbb{R}$ is said to be a quasideviation, provided it satisfies the following conditions:

- for every $x,y\in I$, $D(x,y)$ has the same sign as $x-y$;
- for every $x\in I$, the function $I\ni t\to D(x,t)$ is continuous;
- for every $x,y\in I$ with $x<y$, the function

  $$(x,y)\ni t\to\frac{D(y,t)}{D(x,t)}$$

  is strictly increasing.

Let

$$\Delta_n:=\{\overline{\lambda}\in[0,\infty)^n:\overline{\lambda}=(\lambda_1,\dots,\lambda_n)\wedge\sum_{i=1}^n\lambda_i>0\}\ \text{ for } n\in\mathbb{N}.$$

According to [8, Theorem 1], if $D:I^2\to\mathbb{R}$ is a quasideviation, then for every $n\in\mathbb{N}$, $\overline{x}=(x_1,\dots,x_n)\in I^n$ and $\overline{\lambda}=(\lambda_1,\dots,\lambda_n)\in\Delta_n$, the equation

$$\sum_{i=1}^n\lambda_iD(x_i,t)=0 \tag{11}$$

has a unique solution t_0. Moreover

$$\min\{x_i:i\in\{1,..,n\}\}\leqslant t_0\leqslant\max\{x_i:i\in\{1,..,n\}\},$$

whence equation (11) defines a mean. Following [7], we denote it by $\tilde{\mathfrak{M}}_D(\overline{x};\overline{\lambda})$.

A series of properties of quasideviation means have been characterized in [8]. The following result, which is a particular case of [8, Theorem 7], will be useful in our further considerations.

THEOREM 1. *Assume that $I\subseteq\mathbb{R}$ is an open interval and $D_1,D_2:I^2\to\mathbb{R}$ are quasideviations. Then the following statements are equivalent:*

(i) *for every* $x_1,x_2 \in I$ *and* $\lambda \in [0,1]$

$$\tilde{\mathfrak{M}}_{D_1}((x_1,x_2);(\lambda,1-\lambda)) \leqslant \tilde{\mathfrak{M}}_{D_2}((x_1,x_2);(\lambda,1-\lambda)); \tag{12}$$

(ii) *there exists a function* $A : I \to (0,\infty)$ *such that*

$$D_1(x,y) \leqslant A(y)D_2(x,y) \quad \textit{for} \quad x,y \in I. \tag{13}$$

REMARK 5. Let $I \subseteq \mathbb{R}$ be an open interval and $D : I^2 \to \mathbb{R}$ be a quasideviation. Then, from the definition of the quasideviation mean we derive that

$$\tilde{\mathfrak{M}}_D((x_1,x_2);(\lambda,1-\lambda)) = \tilde{\mathfrak{M}}_D((x_2,x_1);(1-\lambda,\lambda)) \quad \text{for} \quad x_1,x_2 \in I,\ \lambda \in [0,1],$$

$$\tilde{\mathfrak{M}}_D((x,x);(\lambda,1-\lambda)) = x \quad \text{for} \quad x \in I,\ \lambda \in [0,1],$$

$$\tilde{\mathfrak{M}}_D((x_1,x_2);(1,0)) = x_1 \quad \text{for} \quad x_1,x_2 \in I,$$

and

$$\tilde{\mathfrak{M}}_D((x_1,x_2);(0,1)) = x_2 \quad \text{for} \quad x_1,x_2 \in I.$$

Hence, inequality (12) is satisfied for every $x_1,x_2 \in I$ and $\lambda \in [0,1]$ if and only if it is satisfied for every $\lambda \in (0,1)$ and $x_1,x_2 \in I$ with $x_1 < x_2$. Therefore, the following result is a direct consequence of Theorem 1.

COROLLARY 1. *Assume that* $I \subseteq \mathbb{R}$ *is an open interval and* $D_1,D_2 : I^2 \to \mathbb{R}$ *are quasideviations. The inequality* (12) *is valid for every* $\lambda \in (0,1)$ *and* $x_1,x_2 \in I$ *with* $x_1 < x_2$ *if and only if there exists a function* $A : I \to (0,\infty)$ *such that* (13) *holds.*

3. Results

In the whole section we assume that $u : \mathbb{R} \to \mathbb{R}$ is a strictly increasing continuous function such that $u(0) = 0$ and g is a continuous probability distortion function.

The following result shows the relationship between willingness to accept, willingness to pay and quasideviation means.

LEMMA 1. *Let* $D_1,D_2 : \mathbb{R}^2 \to \mathbb{R}$ *be defined by*

$$D_1(x,y) = u(x-y) \quad \textit{for} \quad (x,y) \in \mathbb{R}^2 \tag{14}$$

and

$$D_2(x,y) = -u(y-x) \quad \textit{for} \quad (x,y) \in \mathbb{R}^2. \tag{15}$$

Then D_1 *and* D_2 *are quasideviations and, for every* $p \in (0,1)$ *and* $x_1,x_2 \in \mathbb{R}$ *with* $x_1 < x_2$*, we have*

$$\mathrm{WTA}(\langle x_1,x_2,p,1-p\rangle) = \tilde{\mathfrak{M}}_{D_2}((x_1,x_2);(g(p),1-g(p))) \tag{16}$$

and

$$\mathrm{WTP}(\langle x_1,x_2,p,1-p\rangle) = \tilde{\mathfrak{M}}_{D_1}((x_1,x_2);(1-g(1-p),g(1-p))). \tag{17}$$

Proof. Since u is continuous, strictly increasing and $u(0)=0$, it is not difficult to check that D_1 and D_2 are quasideviations. Furthermore, applying (9) and (10), we obtain (17) and (16), respectively. $\square$

REMARK 6. Let u be the identity on $\mathbb{R}$. Then, making use of (4) and (7), from (2) and (3) one can easily derive that

$$\mathrm{WTA}(X) = E_{\bar{g}}[X] \quad \text{for} \quad X \in \mathscr{X}$$

and

$$\mathrm{WTP}(X) = E_g[X] \quad \text{for} \quad X \in \mathscr{X},$$

respectively. Furthermore, according to Lemma 1, a function $D_0 : \mathbb{R}^2 \to \mathbb{R}$, given by

$$D_0(x,y) = x - y \quad \text{for} \quad (x,y) \in \mathbb{R}^2, \tag{18}$$

is a quasideviation and, for every $p \in (0,1)$ and $x_1, x_2 \in \mathbb{R}$ with $x_1 < x_2$, we get

$$E_g[\langle x_1, x_2, p, 1-p\rangle] = \tilde{\mathfrak{M}}_{D_0}((x_1,x_2);(1-g(1-p), g(1-p))) \tag{19}$$

and

$$E_{\bar{g}}[\langle x_1, x_2, p, 1-p\rangle] = \tilde{\mathfrak{M}}_{D_0}((x_1,x_2);(g(p), 1-g(p))). \tag{20}$$

In the next theorem we establish a characterization of the disparity between willingness to accept and willingness to pay in the case where u is odd.

THEOREM 2. *Assume that u is is odd. Then the following statements are pairwise equivalent:*

(i)
$$\mathit{WTP}(X) \leqslant \mathit{WTA}(X) \quad \textit{for} \quad X \in \mathscr{X}^{(2)};$$

(ii)
$$\mathit{WTP}(X) \leqslant \mathit{WTA}(X) \quad \textit{for} \quad X \in \mathscr{X};$$

(iii)
$$g(p) \leqslant \bar{g}(p) \quad \textit{for} \quad p \in [0,1]. \tag{21}$$

Proof. Since u is odd, in view of (14)–(15), we get $D_1 = D_2$. Thus, if (i) is valid, then taking $x_1, x_2 \in \mathbb{R}$ such that $x_1 < x_2$ and applying Lemma 1, for every $p \in (0,1)$, we obtain

$$\mathfrak{m} := \tilde{\mathfrak{M}}_{D_1}((x_1,x_2);(1-g(1-p), g(1-p))) \leqslant \tilde{\mathfrak{M}}_{D_1}((x_1,x_2);(g(p),1-g(p))) =: \mathfrak{M}.$$

Therefore, as u is strictly increasing, for every $p \in (0,1)$, we have

$$\begin{aligned}
&(1-g(1-p))u(x_1-\mathfrak{M}) + g(1-p)u(x_2-\mathfrak{M}) \\
&\leqslant (1-g(1-p))u(x_1-\mathfrak{m}) + g(1-p)u(x_2-\mathfrak{m}) \\
&= 0 = g(p)u(x_1-\mathfrak{M}) + (1-g(p))u(x_2-\mathfrak{M}).
\end{aligned}$$

Hence

$$(1-g(1-p)-g(p))(u(x_2-\mathfrak{M})-u(x_1-\mathfrak{M}))\geqslant 0 \quad \text{for} \quad p\in(0,1).$$

Since $x_1<x_2$, this implies that

$$1-g(1-p)-g(p)\geqslant 0 \quad \text{for} \quad p\in(0,1).$$

Thus, as $g(0)=0$ and $g(1)=1$, taking into account (6), we obtain (21). This proves the implication $(i)\Longrightarrow(iii)$.

If (21) holds then, as u is odd, taking into account (2)–(3) and (7), in view of Remark 2, we get

$$E_g[u(WTP(X)-X)]=E_g[-u(X-WTP(X))]=-E_{\overline{g}}[u(X-WTP(X))]$$

$$\leqslant -E_g[u(X-WTP(X))]=0=E_g[u(WTA(X)-X)] \quad \text{for} \quad X\in\mathscr{X}.$$

Hence, applying Remark 3, we obtain (ii) and so the implication $(iii)\Longrightarrow(ii)$ is proved. Clearly, we have also $(ii)\Longrightarrow(i)$. $\square$

Now, we are going to characterize a disparity between willingness to accept and willingness to pay under the assumption that the probability distortion function g is self-conjugated, this is it satisfies $\overline{g}=g$.

REMARK 7. Note that any self-conjugated probability distortion function is of the form

$$g(p)=\begin{cases} g_0(p) & \text{for } p\in[0,1/2],\\ 1-g_0(1-p) & \text{for } p\in(1/2,1],\end{cases}$$

where $g_0:[0,1/2]\to[0,1/2]$ is a continuous non-decreasing function such that $g_0(0)=0$ and $g_0(1/2)=1/2$. The natural examples of self-conjugated probability weighting functions are the Goldstein-Einhorn functions of the form

$$g(p)=\frac{p^r}{p^r+(1-p)^r} \quad \text{for} \quad p\in[0,1]$$

where $r\in(0,\infty)$.

THEOREM 3. *Assume that g is self-conjugated. Then the following statements are pairwise equivalent:*

(i)

$$WTP(X)\leqslant WTA(X) \quad \textit{for} \quad X\in\mathscr{X}^{(2)};$$

(ii)

$$WTP(X)\leqslant WTA(X) \quad \textit{for} \quad X\in\mathscr{X};$$

(iii)

$$u(x)+u(-x)\leqslant 0 \quad \textit{for} \quad x\in\mathbb{R}. \tag{22}$$

Proof. Let $D_1, D_2 : \mathbb{R}^2 \to \mathbb{R}$ be given by (14) and (15), respectively. Fix $\lambda \in (0,1)$ and $x_1, x_2 \in \mathbb{R}$ with $x_1 < x_2$. Since g is continuous with $g(0) = 0$ and $g(1) = 1$, there exists $p_\lambda \in (0,1)$ such that $g(p_\lambda) = \lambda$. Moreover, as g is self-conjugated, we have $g(1 - p_\lambda) = 1 - \lambda$. Thus, if (i) holds, then applying (16)–(17), we obtain

$$\tilde{\mathfrak{M}}_{D_1}((x_1,x_2);(\lambda,1-\lambda)) = WTP(\langle x_1,x_2,p_\lambda,1-p_\lambda\rangle)$$

$$\leqslant WTA(\langle x_1,x_2,p_\lambda,1-p_\lambda\rangle) = \tilde{\mathfrak{M}}_{D_2}((x_1,x_2);(\lambda,1-\lambda)).$$

Therefore, according to Corollary 1, there exists a function $A : \mathbb{R} \to (0,\infty)$ such that

$$D_1(x,y) \leqslant A(y) D_2(x,y) \quad \text{for} \quad (x,y) \in \mathbb{R}^2.$$

Setting in this inequality $y = 0$, in view of (14)–(15), we get

$$u(x) \leqslant -A(0)u(-x) \quad \text{for} \quad x \in \mathbb{R}. \tag{23}$$

Replacing in (23) x by $-x$ and adding obtained in this way inequality side by side to (23), we obtain

$$(u(x) + u(-x))(1 + A(0)) \leqslant 0 \quad \text{for} \quad x \in \mathbb{R}.$$

Since $A(0) > 0$, this gives (22). Thus, the implication $(i) \Longrightarrow (iii)$ is proved.

If (22) is satisfied then, as g is self-conjugated, applying Remark 1, in view of (2)–(3), we get

$$E_g[u(\mathrm{WTP}(X) - X)] \leqslant E_g[-u(-(\mathrm{WTP}(X) - X))]$$

$$= -E_g[u(X - \mathrm{WTP}(X)] = 0 = E_g[u(\mathrm{WTA}(X) - X)] \quad \text{for} \quad X \in \mathscr{X}.$$

Hence, taking into account Remark 3, we obtain (ii). In this way we have proved that $(iii) \Longrightarrow (ii)$. The implication $(ii) \Longrightarrow (i)$ is obvious. $\square$

The following result is a generalization of [3, Theorem 3.7].

THEOREM 4. *Assume that (21) holds. Then the following statements are pairwise equivalent:*

(i)

$$WTP(X) \leqslant E_g[X] \quad \textit{for} \quad X \in \mathscr{X}^{(2)}; \tag{24}$$

(ii)

$$E_{\bar{g}}[X] \leqslant WTA(X) \quad \textit{for} \quad X \in \mathscr{X}^{(2)}; \tag{25}$$

(iii)

$$WTP(X) \leqslant E_g[X] \leqslant E_{\bar{g}}[X] \leqslant WTA(X) \quad \textit{for} \quad X \in \mathscr{X}; \tag{26}$$

(iv) *there exists* $c \in (0,\infty)$ *such that*

$$u(x) \leqslant cx \quad \textit{for} \quad x \in \mathbb{R}. \tag{27}$$

Proof. First we show that $(i) \Longrightarrow (iv)$. To this end assume that (24) is valid and fix $\lambda \in (0,1)$ and $x_1, x_2 \in \mathbb{R}$ such that $x_1 < x_2$. Then, taking $p_\lambda \in (0,1)$ with $g(1-p_\lambda) = 1-\lambda$ and applying Lemma 1, in view of (19) and (24), we get

$$\tilde{\mathfrak{M}}_{D_1}((x_1,x_2);(\lambda,1-\lambda)) = WTP(\langle x_1,x_2,p_\lambda,1-p_\lambda\rangle)$$
$$\leqslant E_g[\langle x_1,x_2,p,1-p\rangle] = \tilde{\mathfrak{M}}_{D_0}((x_1,x_2);(\lambda,1-\lambda))$$

where $D_0 : \mathbb{R}^2 \to \mathbb{R}$ is given by (18). Thus, according to Corollary 1, there exists a function $A : \mathbb{R} \to (0,\infty)$ such that

$$D_1(x,y) \leqslant A(y)D_0(x,y) \quad \text{for} \quad (x,y) \in \mathbb{R}^2.$$

Setting in this inequality $y = 0$, in view of (14) and (18), we get (27) with $c := A(0) > 0$.

The implication $(ii) \Longrightarrow (iv)$ can be proved in a similar way. Namely, assume that (25) holds and fix $\lambda \in (0,1)$ and $x_1, x_2 \in \mathbb{R}$ with $x_1 < x_2$. Then, taking $p_\lambda \in (0,1)$ such that $g(p_\lambda) = \lambda$ and applying Lemma 1, in view of (20) and (25), we get

$$\tilde{\mathfrak{M}}_{D_0}((x_1,x_2);(\lambda,1-\lambda)) = E_{\overline{g}}[\langle x_1,x_2,p,1-p\rangle]$$
$$\leqslant WTA(\langle x_1,x_2,p_\lambda,1-p_\lambda\rangle) = \tilde{\mathfrak{M}}_{D_2}((x_1,x_2);(\lambda,1-\lambda)).$$

Therefore, applying Corollary 1, we conclude that

$$D_0(x,y) \leqslant A(y)D_2(x,y) \quad \text{for} \quad (x,y) \in \mathbb{R}^2$$

with some function $A : \mathbb{R} \to (0,\infty)$. Setting in this inequality $y = 0$ and replacing x by $-x$, in view of (15) and (18), we get (27) with $c := 1/A(0) > 0$.

Now, assume that (27) holds with some $c \in (0,\infty)$. Then, taking into account Remark 1, in view of (2) and (3), for every $X \in \mathscr{X}$, we get

$$0 = E_g[u(\mathrm{WTA}(X)-X)] \leqslant E_g[c(\mathrm{WTA}(X)-X)] = c(\mathrm{WTA}(X)-E_{\overline{g}}[X])$$

and

$$0 = E_g[u(X-\mathrm{WTP}(X))] \leqslant E_g[c(X-\mathrm{WTP}(X))] = c(E_g[X]-\mathrm{WTP}(X)),$$

respectively. Furthermore, applying Remark 2, from (21) we deduce that

$$E_g[X] \leqslant E_{\overline{g}}[X] \quad \text{for} \quad X \in \mathscr{X}.$$

Thus, (26) is valid and so, the implication $(iv) \Longrightarrow (iii)$ is proved.

The implications $(iii) \Rightarrow (i)$ and $(iii) \Rightarrow (ii)$ are obvious. □

COROLLARY 2. *Assume that g is self-conjugated. Then the following statements are pairwise equivalent:*

(i)

$$WTP(X) \leqslant E_g[X] \quad \textit{for} \quad X \in \mathscr{X}^{(2)};$$

(*ii*)

$$E_g[X] \leqslant WTA(X) \ \ for \ \ X \in \mathscr{X}^{(2)};$$

(*iii*)

$$WTP(X) \leqslant E_g[X] \leqslant WTA(X) \ \ for \ \ X \in \mathscr{X};$$

(*iv*) (27) *holds with some* $c \in (0,\infty)$.

Taking into account (8), from Theorem 4 we derive the following result.

COROLLARY 3. *Assume that* g *satisfies* (21) *and there exists* $c \in (0,\infty)$ *such that* (27) *is valid. Then, for every* $X = \langle x_1,\dots,x_n;p_1,\dots,p_n\rangle \in \mathscr{X}$, *we have*

$$WTA(X) - WTP(X) \geqslant E_{\bar{g}}[X] - E_g[X] = \sum_{k=1}^{n-1}\left[\bar{g}\left(\sum_{i=k+1}^{n} p_i\right) - g\left(\sum_{i=k+1}^{n} p_i\right)\right](x_{k+1}-x_k).$$

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