Operators and Matrices
Volume 6, Number 2 (2012), 263–270
doi:10.7153/oam-06-18

# COMPLETELY CO–BOUNDED SCHUR MULTIPLIERS

GILLES PISIER

*Abstract.* A linear map $u\colon\ E \to F$ between operator spaces is called completely co-bounded if it is completely bounded as a map from $E$ to the opposite of $F$. We give several simple results about completely co-bounded Schur multipliers on $B(\ell_2)$ and the Schatten class $S_p$. We also consider Herz-Schur multipliers on groups.

In this short note, we wish to draw attention to the notion of "completely co-bounded" mapping between two operator spaces. Recall that an operator space can be defined as a Banach space $E$ given together with an isometric embedding $E \subset B(H)$ into the space $B(H)$ of all bounded operators on a Hilbert space $H$. The theory of operator spaces started around 1987 with Ruan's thesis and has been considerably developed after that (notably by Effros-Ruan and Blecher-Paulsen, see [2, 8]), with applications mainly to Operator Algebra Theory. In this theory, the morphisms between operator spaces are the completely bounded maps (c.b. in short), defined as follows. First note that if $E \subset B(H)$ is any subspace, then the space $M_n(E)$ of $n \times n$ matrices with entries in $E$ inherits the norm induced by $M_n(B(H))$. The latter space is of course itself equipped with the norm of single operators acting naturally on $H \oplus \cdots \oplus H$ ($n$ times). Then, a linear map $u\colon\ E \to F$ is called completely bounded (c.b. in short) if

$$\|u\|_{\text{cb}} \overset{\text{def}}{=} \sup_{n\geqslant 1} \|u_n\colon\ M_n(E) \to M_n(F)\| < \infty \tag{1}$$

where, for each $n \geqslant 1$, $u_n$ is defined by $u_n([a_{ij}]) = [u(a_{ij})]$. One denotes by $CB(E,F)$ the space of all such maps.

Given an operator space $E$, the opposite $E^{op}$ is the same Banach space as $E$, but equipped with the operator space structure (o.s.s. in short) associated to any embedding $E \subset B(H)$ such that for any $a = [a_{ij}] \in M_n(E)$, we have $\|a\|_{M_n(B(H))} = \|[a_{ji}]\|_{M_n(E)}$. Thus $\|a\|_{M_n(E^{op})} = \|[a_{ji}]\|_{M_n(E)}$. It is easy to check that the (isometric linear) mapping $T \mapsto {}^tT \in B(H^*)$ realizes such an embedding (warning: here ${}^tT\colon\ H^* \to H^*$ designates the adjoint of $T$ in the Banach space sense).

We call a map $u\colon\ E \to F$ between operator spaces completely co-bounded if the same map is c.b. from $E$ to $F^{op}$. This definition is inspired by existing work on completely co-positive maps (cf. e.g. [4, 5] and references there). I started to think

*Mathematics subject classification* (2010): 47L07,47L25.
*Keywords and phrases*: Completely bounded map; operator space; transposition; multiplier.
Partially supported by NSF grant 0503688.

© Element, Zagreb
Paper OaM-06-18

about this notion after hearing Marciniak's lecture on co-positive multipliers at the 2004 Quantum probability conference in Będlewo.

While this definition seems at first glance a pointless variation, easy to reduce to the usual case, we hope in what follows to convince the reader that it has a natural place in operator space theory and that it suggests many interesting questions. As a first motivation for this notion, we should mention that the non-commutative Grothendieck theorem, that came out of work by the author and Haagerup, can be rephrased as saying that, if $A, B$ are $C^*$-algebras, any bounded linear mapping $u\colon\ A \to B^*$ is the sum of a c.b. mapping and a co-c.b. one, see [9, p. 189] for details and more references.

DEFINITION 1. A linear map $u\colon\ E \to F$ between operator spaces will be called completely co-bounded if it is completely bounded as a mapping from $E$ into $F^{op}$ the opposite operator space. We then denote

$$\|u\|_{cob} = \|u\colon\ E \to F^{op}\|_{cb}.$$

REMARK 2. Obviously, $\|u\colon\ E \to F^{op}\|_{cb} = \|u\colon\ E^{op} \to F\|_{cb}$ and $(F^{op})^* = F^{*op}$ completely isometrically. Therefore, $u\colon\ E \to F$ is completely co-bounded iff the same is true for $u^*$ and $\|u\|_{cob} = \|u^*\|_{cob}$, since this is valid for c.b. maps (cf. e.g. [2, 8]).

REMARK 3. Clearly if $B$ is a $C^*$-algebra with $F \subset B$ and if $\alpha\colon B \to B$ is an anti-automorphism (for instance transposition on $B(\ell_2)$), then $u\colon\ E \to F$ is completely co-bounded iff $\alpha u$ is c.b. and $\|u\|_{cob} = \|\alpha u\|_{cb}$. It is well known that the transposition on $M_n$ has c.b. norm equal to $n$ (cf. e.g. [8, p. 418-419]). Therefore, the identity map on $B(H)$ is *not* completely co-bounded unless $H$ is finite dimensional. More generally, the identity map on a von Neumann algebra $B$ is completely co-bounded iff $B$ is of type $I_n$ for some finite $n$, i.e. iff $B$ is a direct sum of finitely many algebras of the form $M_n \otimes A_n$, with $A_n$ commutative.

At first glance, the reader may have serious doubts for the need of the preceding notion ! But hopefully the next result will provide some justification.

THEOREM 4. *A Schur multiplier $M_\varphi\colon\ [x_{ij}] \to [\varphi_{ij} x_{ij}]$ is completely co-bounded on $B(\ell_2)$ iff the matrix $[|\varphi_{ij}|]$ defines a bounded operator on $\ell_2$ and we have*

$$\|M_\varphi\|_{cob} = \|TM_\varphi\|_{cb} = \|[|\varphi_{ij}|]\|_{B(\ell_2)} \tag{2}$$

*where $T\colon\ B(\ell_2) \to B(\ell_2)$ denotes the transposition. Moreover, if $\|M_\varphi\|_{cob} \leqslant 1$, then $M_\varphi$ admits a factorization*

$$B(\ell_2) \xrightarrow{J} \ell_\infty(\mathbb{N} \times \mathbb{N}) \xrightarrow{\mathscr{M}_\varphi} B(\ell_2)$$

*where $J$ is the natural inclusion map and where $\|\mathscr{M}_\varphi\|_{cb} \leqslant 1$.*

*Proof.* Assume that $[|\varphi_{ij}|]$ is in $B(\ell_2)$. Then the mapping

$$\mathscr{M}_\varphi\colon\ \ell_\infty(\mathbb{N}\times\mathbb{N}) \longrightarrow B(\ell_2)$$
$$[x_{ij}] \longrightarrow [x_{ij}\varphi_{ij}]$$

is obviously bounded with $\|\mathscr{M}_\varphi\| = \|[|\varphi_{ij}|]\|_{B(\ell_2)}$. Actually, more generally, if $x_{ij} \in B(H)$ with $\|x_{ij}\| \leqslant 1$, then the matrix $[\varphi_{ij}x_{ij}]$ defines a bounded operator on $\ell_2(H)$ with norm easily seen to be majorized by $\|[|\varphi_{ij}|]\|_{B(\ell_2)}$. Indeed, for any pair $(a_i),(b_j)$ in the unit ball of $\ell_2^n(H)$, we have

$$|\sum_i \langle a_i, \sum_j \varphi_{ij}x_{ij}b_j\rangle| \leqslant \sum_{i,j} \|a_i\|\ |\varphi_{ij}|\ \|x_{ij}\|\|\|b_j\| \leqslant \|[|\varphi_{ij}|]\|_{B(\ell_2)},$$

and hence $\|[\varphi_{ij}x_{ij}]\|_{M_n(B(H))}\| \leqslant \|[|\varphi_{ij}|]\|_{B(\ell_2)}$. This shows that $\|\mathscr{M}_\varphi\| \leqslant \|\mathscr{M}_\varphi\|_{cb} \leqslant \|[|\varphi_{ij}|]\|_{B(\ell_2)}$.

Let $J$ be as above. Clearly we have

$$\|J\|_{cb} = 1$$

and hence

$$M_\varphi = \mathscr{M}_\varphi J$$

with $\|\mathscr{M}_\varphi\|_{cb} = \|[|\varphi_{ij}|]\|_{B(\ell_2)}$. But since $\ell_\infty(\mathbb{N}\times\mathbb{N})^{op}$ and $\ell_\infty(\mathbb{N}\times\mathbb{N})$ are identical we can factorize $M_\varphi$ as follows

$$M_\varphi\colon\ B(\ell_2) \xrightarrow{J} \ell_\infty(\mathbb{N}\times\mathbb{N}) = \ell_\infty(\mathbb{N}\times\mathbb{N})^{op} \xrightarrow{\mathscr{M}_\varphi} B(\ell_2)^{op}$$

it follows that

$$\|M_\varphi\colon\ B(\ell_2) \to B(\ell_2)^{op}\|_{cb} \leqslant \|\mathscr{M}_\varphi\|_{cb} \leqslant \|[|\varphi_{ij}|]\|_{B(\ell_2)}.$$

This proves the “if” part.

Conversely, assume that $M_\varphi$ is completely co-bounded with $\|M_\varphi\|_{cob} \leqslant 1$. Let $x = [x_{ij}]$ be an $n\times n$ matrix viewed as sitting in $B(\ell_2)$. Let $B = B(\ell_2)$. Note that

$$\left\|\sum_{ij=1}^n e_{ij}\otimes e_{ij}x_{ij}\right\|_{M_n(B)} = \|x\|_B$$

while

$$\left\|\sum_{ij=1}^n e_{ij}\otimes e_{ij}x_{ij}\right\|_{M_n(B^{op})} = \left\|\sum_{ij=1}^n e_{ji}\otimes e_{ij}x_{ij}\right\|_{M_n(B)} = \sup|x_{ij}|.$$

By definition of $\|M_\varphi\|_{cob} \leqslant 1$, we have

$$\|\sum e_{ij}\otimes e_{ij}\varphi_{ij}x_{ij}\|_{M_n(B)} = \|\sum e_{ji}\otimes e_{ij}\varphi_{ij}x_{ij}\|_{M_n(B^{op})} \leqslant \|\sum e_{ji}\otimes e_{ij}x_{ij}\|_{M_n(B)}$$

which yields

$$\|[\varphi_{ij}x_{ij}1_{\{i,j\leqslant n\}}]\|_B \leqslant \sup_{ij}|x_{ij}| \leqslant 1.$$

This implies

$$\|[|\varphi_{ij}|]_{ij\leqslant n}\| \leqslant 1$$

and since $n$ is arbitrary we obtain

$$\|[|\varphi_{ij}|]\|_B \leqslant 1.$$

This proves the "only if" part. The proof also yields (2). □

COROLLARY 5. *A Schur multiplier $M_\varphi$ is completely co-bounded on $B(\ell_2)$ iff it factors through a commutative $C^*$-algebra or iff it factors through a minimal operator space and the corresponding factorization norm coincides with $\|M_\varphi\|_{cob}$.*

*Proof.* For any commutative $C^*$-algebra $C$ or for any $E \subset C$, we have clearly $E = E^{op}$, so a $cb$-factorization $M_\varphi\colon\ B \xrightarrow{u_1} E \xrightarrow{u_2} B$ yields

$$\|M_\varphi\colon\ B \to B^{op}\|_{cb} \leqslant \inf\{\|u_1\|_{cb}\|u_2\|_{cb}\}$$

where the infimum runs over all possible factorizations. Conversely, the preceding shows the converse with a factorization through $\ell_\infty(\mathbb{N}\times\mathbb{N})$. □

REMARK 6. Let $G$ be an infinite discrete group. Consider a function $f\colon\ G \to \mathbb{C}$, and the function $\hat{f}$ defined on $G\times G$ by $\hat{f}(s,t) = f(st^{-1})$. By the well known Kesten-Hulanicki criterion (cf. e.g. [7, Th. 2.4]) $G$ is amenable iff there is a constant $C$ such that for any finitely supported $f$ we have $\sum_{t\in G}|f(t)| \leqslant C\|[|\hat{f}(s,t)|]\|_{B(\ell_2(G))}$ and when $G$ is amenable this holds with $C=1$. Thus, by Theorem 4, the inequality

$$\sum_{t\in G}|f(t)| \leqslant C\|M_{\hat{f}}\|_{cob}$$

characterizes amenable groups. This should be compared with Bożejko's and Wysoczanski's criteria described in [7, p. 54] and [7, p. 38].

We now generalize Theorem 4 to the Schur multipliers that are bounded on the Schatten $p$-class $S_p$. We assume $S_p$ equipped with the "natural" operator space structure introduced in [6] using the complex interpolation method. We will use freely the notation and results from [6].

THEOREM 7. *Let $2 \leqslant p \leqslant \infty$. Let $T\colon\ S_p \to S_p$ denote again the transposition mapping $x \to {}^t x$. Then a bounded Schur multiplier $M_\varphi\colon\ S_p \to S_p$ is completely co-bounded iff it admits a factorization as follows:*

$$S_p \xrightarrow{J_p} \ell_p(\mathbb{N}\times\mathbb{N}) \xrightarrow{\mathscr{M}_\varphi} S_p$$

*where $J_p$ is the natural (completely contractive) inclusion and where $\|M_\varphi\|_{cob} = \|\mathscr{M}_\varphi\|_{cb}$.*

*Proof.* Note that the fact that $J_p\colon\ S_p \to \ell_p(\mathbb{N}\times\mathbb{N})$ is completely contractive is immediate by interpolation between the cases $p=2$ and $p=\infty$.

The proof can then be completed following the same idea as for Theorem 4. We have for any $[x_{ij}]$ in $M_n(S_p)$

$$\left\|\sum e_{ij}\otimes e_{ji}\otimes x_{ij}\right\|_{S_p^n[S_p^n[S_p]]} = \left(\sum_{ij}\|x_{ij}\|_{S_p}^p\right)^{1/p}$$

while

$$\left\|\sum e_{ij}\otimes e_{ij}\otimes x_{ij}\right\|_{S_p^n[S_p^n[S_p]]} = \|[x_{ij}]\|_{S_p^n[S_p]}.$$

Both of these identities can be proved by routine interpolation arguments starting from $p=\infty$ and $p=2$. This gives us

$$\|[\varphi_{ij}x_{ij}]\|_{S_p^n[S_p]} \leqslant \|M_\varphi\|_{cob}\left(\sum\|x_{ij}\|_{S_p}^p\right)^{1/p}$$

which means (cf. [6]) that

$$\|\mathscr{M}_\varphi\|_{cb} \leqslant \|M_\varphi\|_{cob}.$$

To prove the converse, it suffices to notice again that

$$\ell_p(\mathbb{N}\times\mathbb{N})^{op} = \ell_p(\mathbb{N}\times\mathbb{N}). \qquad \square$$

REMARK 8. Consider Schur multipliers from $B(\ell_2)$ (or the subalgebra of compact operators $K$) into the trace class $S_1$. We refer to [9] for a detailed discusion of when such a multiplier is bounded and when it is c.b. From that discussion follows easily that such a multiplier is completely co-bounded iff it is bounded. Indeed, more generally (see [9, 3]), if $A,B$ are $C^*$-algebras a linear map $u\colon\ A\to B^*$ is completely co-bounded iff there are a constant $c$ and states $f_1,f_2$, $g_1,g_2$ on $A,B$ respectively, such that for any $(a,b)\in A\times B$

$$|\langle u(a),b\rangle| \leqslant c\left((f_1(a^*a)g_1(b^*b))^{1/2} + (f_2(aa^*)g_2(bb^*))^{1/2}\right). \tag{3}$$

Consider a bounded Schur multiplier $\varphi=[\varphi_{ij}]$ from $B(\ell_2)$ (or $K$) to $S_1$, where $S_1$ is equipped with its natural o.s.s. as the dual of $K$. By this we mean that $\langle M_\varphi(a),b\rangle = \sum\varphi_{ij}a_{ij}b_{ij}$. Then (see [9]) there are two nonnegative summable sequences $(\lambda_i)$ and $(\mu_i)$ such that for any $i,j$

$$|\varphi_{ij}| \leqslant \lambda_i + \mu_j.$$

Then, using the Cauchy-Schwarz inequality, we obtain (3) with the states $f_1=g_1=(\sum\lambda_j)^{-1}\sum\lambda_j e_{jj}$ and $f_2=g_2=(\sum\mu_j)^{-1}\sum\mu_j e_{jj}$. This shows that $M_\varphi$ is automatically completely co-bounded. See [10] for an extension to Schur multipliers from $S_p$ to $S_{p'}$ with $2<p<\infty$.

REMARK 9. The preceding two theorems illustrate the following simple observation: Assume that a linear map $u\colon\ E\to F$ is both c.b. and completely co-bounded, then $u$ can be completely boundedly factorized through an operator space $G$ for which the identity map $I_G$ is completely co-bounded with $\|I_G\|_{cob}=1$. Indeed, we just consider $G=F\cap F^{op}$ in the sense of [6] (this means $G=F$ equipped with the o.s.s. induced by the diagonal embedding $F\subset F\oplus F^{op}$), then $u$ can be viewed as $u\colon\ E\to G\to F$, with $\|u\colon\ E\to G\|_{cb}=\max\{\|u\|_{cb},\|u\|_{cob}\}$ and $\|G\to F\|_{cb}=1$.

Conversely, any mapping of the form $u\colon\ E\xrightarrow{v}G\xrightarrow{w}F$, with c.b. maps $v,w$ and $G$ such that the identity $I=I_G$ on $G$ is completely co-bounded, must be both c.b. and completely co-bounded (since $u\colon\ E\xrightarrow{v}G\xrightarrow{I}G^{op}\xrightarrow{w}F^{op}$ is c.b.).

Let us say that an operator space $G$ is self-transposed if $I_G$ is completely co-bounded. This property passes obviously to subspaces, quotients (and hence subquotients) and dual spaces. It is also stable under ultraproducts. Examples include any commutative $C^*$-algebra (or any minimal operator space), by duality any $L_1$-space (or any maximal operator space) and by interpolation any $L_p$-space ($1\leqslant p\leqslant\infty$). Perhaps there is a nice characterization of self-transposed operator spaces?

Let $G$ be a finite group. Let $f,g\colon\ G\to\mathbb{C}$ be functions on $G$ and let $\lambda(f)\colon\ x\to f*x$ and $\rho(g)\colon\ x\to x*g$ be the associated convolutors on $\ell_2(G)$.

The Fourier transform of $f$ is defined as follows: for any irreducible representation $\pi$ on $G$ (i.e. $\pi\in\widehat{G}$)

$$\hat{f}(\pi)=\int f(t)\pi(t)^*dm(t)$$

where $m$ is the normalized Haar measure on $G$. We have then

PROPOSITION. *With the above notation, we have*

$$\|\lambda(f)\rho(g)\|_{cob}=\sup_{\pi\in\widehat{G}}\|\hat{f}(\pi)\|_2\|\hat{g}(\pi)\|_2$$

*where* $\|\ \|_2$ *denotes the Hilbert–Schmidt norm on* $H_\pi$. *In particular,*

$$\|Id\colon\ C^*(G)\to C^*(G)\|_{cob}=\sup_{\pi\in\widehat{G}}\dim(\pi).$$

*Proof.* Passing to Fourier transforms, we see that $\lambda(f)\rho(g)$ coincides with $\bigoplus_{\pi\in\widehat{G}}L(\hat{f}(\pi))R(\hat{g}(\pi))$ where $L(a)$ (resp. $R(a)$) denotes left (resp. right) multiplication by $a$ on $H_\pi$. Thus the result follows from the next lemma. □

LEMMA. *Let $H$ be a Hilbert space and let $u\colon\ B(H)\to B(H)$ be defined by $u(x)=axb$. Then $u$ is completely co-bounded iff $a,b$ are both Hilbert–Schmidt operators and $\|u\|_{cob}=\|a\|_2\|b\|_2$.*

*Proof.* We may easily reduce this to the finite dimensional case. So we assume $B(H)=M_n$. Then again we can write

$$\left\|\sum e_{ij}\otimes ae_{ij}b\right\|\leqslant\|u\|_{cob}\left\|\sum e_{ji}\otimes e_{ij}\right\|=\|u\|_{cob}.$$

But now, $T = \sum e_{ij} \otimes ae_{ij}b$ is a rank one operator on $\ell_2^n \otimes \ell_2^n = \ell_2^n(H)$ with $H = \ell_2^n$. Indeed, for any $h = (h_i)$, $k = (k_j) \in \ell_2^n(H)$ we have:

$$\begin{aligned}\langle Th, k\rangle &= \sum_{ij} \langle ae_{ij}bh_j, k_i\rangle \\ &= \sum_i \langle ae_i, k_i\rangle \sum_j \langle h_j, b^* e_j\rangle \\ &= \langle (ae_i), k\rangle \langle h, (b^* e_j)\rangle\end{aligned}$$

and hence

$$\|T\| = \left(\sum \|ae_i\|^2\right)^{1/2} \left(\sum \|b^* e_j\|^2\right)^{1/2} = \|a\|_2 \|b\|_2. \qquad \square$$

Let us denote by $C_\lambda^*(G)$ the reduced $C^*$-algebra of a discrete group $G$, i.e. the $C^*$-algebra generated by the left regular representation $\lambda\colon\ G \to B(\ell_2(G))$. By a Herz-Schur multiplier on $C_\lambda^*(G)$, we mean a bounded linear map $T$ on $C_\lambda^*(G)$ for which there is a function $f\colon\ G \to \mathbb{C}$ such that, for any $t \in G$

$$T(\lambda(t)) = f(t)\lambda(t).$$

We then denote $T_f = T$.

COROLLARY 10. *If $G$ is a finite group and $f$ is the function constantly equal to 1, then $\|T_f\|_{cob}$ (i.e. the identity map on $C_\lambda^*(G)$) is equal to the supremum of the dimensions of the irreducible representations of $G$.*

REMARK 11. By Theorem 4, if $f \equiv 1$ as above, then the function defined on $G \times G$ by $\hat{f}(s,t) = f(st^{-1})$ (that is also constantly equal to 1), satisfies (when viewed as a Schur multiplier on $B(\ell_2(G))$ $\|M_{\hat{f}}\|_{cob} = |G|$ and in general this is different from $\|T_f\|_{cob} = \sup\{\dim(\pi) \mid \pi \in \widehat{G}\}$.

Now let $G$ be an infinite discrete group. By Remarks 2 and 3, the identity map on $A = C_\lambda^*(G)$ is completely co-bounded iff the same is true for $A^{**}$, and this implies that the latter is a direct sum of finitely many algebras of the form $M_n \otimes A_n$, with $A_n$ commutative. In particular, of course $A^{**}$ is injective, $A$ is nuclear and hence $G$ is amenable.

It would be interesting to describe the completely co-bounded Herz-Schur multipliers on the reduced $C^*$-algebra of a discrete group $G$ in analogy with what is known for the c.b. ones (see [1]).

**Acknowledgement**

I am grateful to Marek Bożejko and Marcin Marciniak for stimulating communication.

## REFERENCES

[1] M. BOŻEJKO AND G. FENDLER, *Herz-Schur multipliers and completely bounded multipliers of the Fourier algebra of a locally compact group*, Boll. Unione Mat. Ital. (6) **3-A** (1984), 297–302.

[2] E. G. EFFROS AND Z. J. RUAN, *Operator Spaces*, The Clarendon Press, Oxford University Press, New York, 2000, xvi+363 pp.

[3] U. HAAGERUP AND M. MUSAT, *The Effros–Ruan conjecture for bilinear forms on $C^*$-algebras*, Invent. Math. **174** (2008), 139–163.

[4] W. MAJEWSKI AND M. MARCINIAK, *$k$-decomposability of positive maps*, Quantum probability and infinite dimensional analysis, 362–374, QP–PQ: Quantum Probab. White Noise Anal., 18, World Sci. Publ., Hackensack, NJ, 2005.

[5] M. MARCINIAK, *On extremal positive maps acting between type I factors*, arXiv:0812.2311.

[6] G. PISIER, *Non-commutative vector valued $L_p$-spaces and completely $p$-summing maps*, Astérisque **247** (1998), vi+131 pp.

[7] G. PISIER, *Similarity problems and completely bounded maps*, Springer Lecture Notes 1618, Second Expanded Edition. (Incl. the solution to "the Halmos Problem") (2001), 1–198.

[8] G. PISIER, *Introduction to operator space theory*, London Mathematical Society Lecture Note Series, 294, Cambridge University Press, Cambridge, 2003, viii+478 pp.

[9] G. PISIER AND D. SHLYAKHTENKO, *Grothendieck's theorem for operator spaces*, Invent. Math. **150**, 1 (2002), 185–217.

[10] Q. XU, *Operator-space Grothendieck inequalities for noncommutative $L_p$-spaces*, Duke Math. J. **131** (2006), 525–574.

(Received December 28, 2010)

*Gilles Pisier*
*Texas A&M University*
*College Station*
*TX 77843, U. S. A.*
*and*
*Université Paris VI*
*Equipe d'Analyse*
*Case 186, 75252*
*Paris Cedex 05, France*