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CORRIGENDUM TO "BISHOP'S PROPERTY (β) FOR PARANORMAL OPERATORS" [OPERATORS AND MATRICES 3 (2009), 517—524], ATSUSHI UCHIYAMA, KOTARO TANAHASHI AND "SVEP AND BISHOP'S PROPERTY (β) FOR $k*$–PARANORMAL OPERATORS" [OPERATORS AND MATRICES 5 (2011), 469—472], NAIM L. BRAHA, KOTARO TANAHASHI

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1. There is a fatal gap in the proof of Theorem 3.5 "If an operator T has the property (II), then T also has Bishop's property (β)" in Bishop's property (β) for paranormal operators, Operators and Matrices **3** (2009) 517–524.

Let $D \subset \mathbb{C}$ be an open subset and $f_n : D \to \mathscr{H}$ be a sequence of analytic functions such that $(T-z)f_n(z)$ converges uniformly to 0 on every compact subset of D. We have to prove that $f_n(z)$ converges uniformly to 0 on every compact subset of D.

Our (mistaken) plan is showing that "for any $0 < \varepsilon < 1$ and any $z_0 \in D$ there exist $r > 0$ and $N \in \mathbb{N}$ such that $\overline{B(z_0;r)} = \overline{\{z \in \mathbb{C} : |z - z_0| < r\}} \subset D$ and $\|f_n\|_{\overline{B(z_0;r)}} \leqslant \varepsilon$ for all $n \geqslant N$".

Let $0 < \varepsilon < 1$ and $z_0 \in D$ be given.

(*Case 1*). Assume there exists $R > 0$ such that $\overline{B(z_0;R)} \subset D$ and $\sup_n \|f_n\|_{\overline{B(z_0;R)}} < \infty$. Then we have such $r > 0$ and $N \in \mathbb{N}$ by using the arguement from line 3 to 18 of page 523.

(*Case 2*). Assume $\sup_n \|f_n\|_{\overline{B(z_0;R)}} = \infty$ for any $R > 0$ with $\overline{B(z_0;R)} \subset D$. We choose some $R_1 > 0$ such that $\overline{B(z_0;R_1)} \subset D$ and consider a sequence $g_n = f_n/(1 + \|f_n\|_{\overline{B(z_0;R_1)}}) : \overline{B(z_0;R_1)} \to \mathscr{H}$. Then $\sup_n \|g_n\|_{\overline{B(z_0;R_1)}} \leqslant 1$ and $(T-z)g_n(z)$ converges uniformly to 0 on every compact subsets of $\overline{B(z_0;R_1)}$. Then there exist $0 < r_1 < R_1$ and $N_1 \in \mathbb{N}$ such that $\|g_n\|_{\overline{B(z_0;r_1)}} \leqslant \varepsilon$ for all $n \geqslant N_1$ by the arguement of case 1. We mistook that "this means $\sup_n \|f_n\|_{\overline{B(z_0;r_1)}} < \infty$ and this contradicts to the assumption of case 2". But this is not right.

We regret the following mistakes in the above referenced paper.

The following example was found by an anonymous reviewer for the Bulletin of the Korean Mathematical Society (BKMS) and communicated to us by Torsten Ehrhardt (an editor for BKMS). We are deeply indebted to both of them.

EXAMPLE. Let $f_n(z) = z^n$ for $z \in \{w \in \mathbb{C} : |w-2| < 1\} = B(2;1)$. Let $z_0 \in B(2;1)$ be given. If $0 < r < 1 - |2 - z_0|$, we have $\|f_n\|_{\overline{B(z_0;r)}} = (r + |z_0|)^n$ and $\sup_n \|f_n\|_{\overline{B(z_0;r)}} = \infty$. Let $R_1 = (1 - |z_0 - 2|)/2$ and $g_n = f_n/(1 + \|f_n\|_{\overline{B(z_0;R_1)}})$. Let $r_1 = R_1/2$. Then $\|g_n\|_{\overline{B(z_0;r_1)}} \to 0$, but $\sup_n \|f_n\|_{\overline{B(z_0;r_1)}} = \infty$. Hence "$\|g_n\|_{\overline{B(z_0;r_1)}} \to 0$" says nothing about $\|f_n\|_{\overline{B(z_0;r_1)}}$.

2. Corollary 3.6 "Every paranormal operator on a complex Hilbert space has Bishop's propert (β)" should be deleted. This is still an open problem.

3. Theorem 1 "$k*$-paranormal operators have Bishop's property (β)" in SVEP and Bishop's property (β) for $k*$-paranormal operators, Operators and Matrices **5** (2011) 469–472, should be changed as follows "$k*$-paranormal operators have the property (II)".

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