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ON THE DOUBLE COMMUTANT EXPECTATION PROPERTY FOR OPERATOR SYSTEMS

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Abstract. In this note we present an alternative viewpoint to the Double Commutant Expectation Property (DCEP) for operator systems, introduced by Kavruk, Paulsen, Todorov and Tomforde. Our approach is based on the universal C^*-algebra of an operator system and is used to obtain new proofs of several results for operator systems analogue to the properties C^*-algebras have.

1. Introduction

The notion of a weak expectation goes back to E. C. Lance [12], where he considered, for a C^*-algebra $A \subset B(H)$, a unital completely positive map (u.c.p. for short) $\varphi : B(H) \to A''$ which acts identically on A, and he called φ a weak expectation for A. A C^*-algebra is said to have the Weak Expectation Property (WEP) if it has a weak expectation in every faithful representation on a Hilbert space. The WEP turned out to play a key role in Kirchberg's seminal work [9, 10] and, together with the Local Lifting Property (LLP), it now stands out as a central topic in the study of C^*-algebras.

In [18] Pisier generalized the WEP to arbitrary operator spaces. If $i_X : X \to X^{**}$ is the canonical inclusion, then X is said to have the λ-WEP if there exist completely bounded maps $T_1 : B(H) \to X^{**}$ and $T_2 : X \to B(H)$ such that $i_X = T_1T_2$ and $||T_1||_{cb}||T_2||_{cb} \leqslant \lambda$.

More recently [7, 8], Kavruk, Paulsen, Tomforde and Todorov established a general theory of tensor products of operator systems and were, perhaps inevitably, lead to the WEP, which they defined in very much the same way as Pisier, by requiring an injective operator system T and u.c.p. maps $\varphi_1 : S \to T$ and $\varphi_2 : T \to S^{**}$ such that $i_S = \varphi_2 \circ \varphi_1$, where $i_S : S \to S^{**}$ is the canonical inclusion. Subsequently, they introduced the Double Commutant Expectation Property (DCEP), which turned out to be a more flexible notion, and which was successful in extending a number of results, mostly due to Kirchberg [9], from C^*-algebras to operator systems. In particular [8], Kirchberg's conjecture found its prominent place in the category of operator systems. An operator system S has the double commutant expectation property (DCEP) provided that for every completely order isomorphic inclusion $S \subset B(H)$, there exists a completely positive map $\varphi : B(H) \to S''$ such that $\varphi(s) = s$ for all $s \in S$. As shown in [8], the DCEP is strictly weaker than the WEP.

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The goal of the present paper is to adopt a C^*-algebraic viewpoint for the results in [8]. We make extensive use of the universal C^*-algebra $C_u^*(S)$ of an operator system S introduced by Kirchberg and Wassermann in [11], and use some of the groundbreaking methods in Kirchberg's seminal paper [9]. Our relying on $C_u^*(S)$ lead us to a new notion of WEP, which we call UWEP, and which requires for every faithful representation $C_u^*(S) \subset B(H)$ the existence of a u.c.p. map $\varphi : B(H) \to C_u^*(S)^{**}$ such that $\varphi(s) = s$ for all $s \in S$. As it turns out, the UWEP and DCEP are equivalent, and thus the methods in this paper augment the operator system techniques in [7, 8] to enlarge the framework in which operator systems are studied and provide a diversified range of options for further investigation. As it is often the case, the analogy between operator systems and C^*-algebras has its limitations, as we shall see in section 5 where we present a somewhat surprising subsystem of $M_n(\mathbb{C})$, namely the Paulsen system $S_{\ell_\infty^n} \subset \ell_\infty^n \otimes M_2(\mathbb{C})$ associated with ℓ_∞^n, which, at least for $n \geqslant 5$, fails to have either the lifting property LP or the DCEP.

While most of the results in this note are not new, the methods of proof are different from [8]. We must, however, point out that a few things here are new even in the C^*-algebra context, like Proposition 5.2 and the proof of $(2) \Rightarrow (3)$ in Proposition 3.5, which is inspired by an elegant argument of Pisier [17].

For more details and information on Kirchberg's results, as well as a wider background, we refer to [9, 10, 13, 2]. For the general theory of operator spaces and systems we relied on [16, 18], while for tensor products of operator systems [7] is the basic reference. For more details on the WEP we refer the reader to Chapter 15 in [18], as well as to [8] and [5].

2. Background and preliminary results

Throughout this paper all C^*-algebras are assumed to be unital. By an isomorphism of operator systems we will always mean a unital, completely isometrical isomorphism. In particular, such a map and its inverse are u.c.p. maps, so they establish a complete order isomorphism. A sum of elementary tensors in a tensor product will be called an elementary operator.

The min and max norms. We begin by recalling the definitions of the minimal and maximal C^*-cross-norms and refer the reader to [16] for more details. Let A_1 and A_2 be unital C^*-algebras. A C^*-cross-norm on the algebraic tensor product $A_1 \otimes A_2$ is a C^*-norm γ satisfying the additional condition $||a \otimes b||_\gamma = ||a|| \cdot ||b||$. If $\pi_1 : A_1 \to B(H_1)$ and $\pi_2 : A_2 \to B(H_2)$ are unital *-homomorphisms, we get a unital, *-preserving homomorphism $\pi_1 \otimes \pi_2 : A_1 \otimes A_2 \to B(H_1 \otimes H_2)$ by letting $\pi_1 \otimes \pi_2(a \otimes b) = \pi_1(a) \otimes \pi_2(b)$. Thus, if for $x \in A_1 \otimes A_2$ we set $||x||_{min} = \sup\{\pi_1 \otimes \pi_2(x) : \ \pi_i : A_i \to B(H_i)$ unital, *-homomorphism, $i = 1,2\}$, then we obtain a C^*- cross-norm on $A_1 \otimes A_2$. The completion of $A_1 \otimes A_2$ in this norm is denoted by $A_1 \otimes_{min} A_2$ and is called the minimal (or the spatial) tensor norm. It is the smallest possible C^*-cross-norm on the algebraic tensor product $A_1 \otimes A_2$.

Let now $\pi_1 : A \to B(H)$ and $\pi_2 : B \to B(H)$ be unital *-homomorphisms such that

$\pi_1(a)\pi_2(b) = \pi_2(b)\pi_1(a)$ for all $a \in A_1$ and $b \in A_2$. We may then define a unital *-homomorphism $\pi : A \otimes B \to B(H)$ by $\pi(x) = \sum_{i=1}^n \pi_1(a_i)\pi_2(b_i)$, where $x = \sum_{i=1}^n a_i \otimes b_i$. Conversely, if we have a unital *-homomorphism $\pi : A \otimes B \to B(H)$ and we define $\pi_1(a) = \pi(a \otimes I)$, $\pi_2(b) = \pi(I \otimes b)$, we obtain a pair of unital *-homomorphisms of A and B, respectively, with commuting ranges such that $\pi(a \otimes b) = \pi_1(a)\pi_2(b)$. We define $||x||_{max} = \sup\{||\pi(x)|| : \pi : A \otimes B \to B(H)$ unital *-homomorphisms$\}$. The completion of $A \otimes B$ in this norm is denoted by $A \otimes_{max} B$, is called the maximal tensor norm, and is the largest possible C^*-cross-norm on the algebraic tensor product $A \otimes B$. A C^*-algebra is called nuclear if $A \otimes_{min} B = A \otimes_{max} B$ for evey C^*-algebra B. It is well known that abelian algebras and $M_n(\mathbb{C})$ are nuclear.

It can be proved without difficulty that if $A_i \subset B_i$, $i = 1,2$, then $A_1 \otimes_{min} A_2 \subset B_1 \otimes_{min} B_2$, but this is no longer the case for the max norm. Specifically, if $A \subset B$, then it is possible that the inclusion $A \otimes C \subset B \otimes_{max} C$ does not induce the max norm on $A \otimes C$. If it does, we write $A \otimes_{max} C \subset B \otimes_{max} C$.

In [12] Lance observed that $A \otimes_{max} C \subset A^{**} \otimes_{max} C$ for all C^*-algebras A, C, and introduced the following property: given a unital inclusion $A \subset B$, we say that A is weakly c.p. complemented in B if there exists a u.c.p. map $\psi : B \to A^{**}$ such that $\psi(a) = a$ for all $a \in A$. We say that A is c.p. complemented in B if the above map ψ takes values in A. Lance proved in [12] that if A is weakly c.p. complemented in B, then $A \otimes_{max} C \subset B \otimes_{max} C$ for any C. A C^*-algebra A has the weak expectation property (WEP) if it can be faithfully represented on a Hilbert space H such that in this representation A is weakly c.p. complemented in $B(H)$. It can be seen ([12]) that this definition does not depend on the particular representation of A. The following is an important characterization of the WEP due to Kirchberg [9], [10].

PROPOSITION 2.1. *A C^*-algebra A has the WEP if and only if $A \otimes_{max} C^*(F_\infty) = A \otimes_{min} C^*(F_\infty)$.*

Next we recall the three basic tensor norms for operator systems and refer to [7] for a detailed presentation.

(1) *The minimal tensor product* min. If $S \subset B(H)$ and $T \subset B(K)$ are operator systems acting on the Hilbert spaces H, respectively K, then $S \otimes_{min} T$ is the operator system arising from the natural inclusion of $S \otimes T$ into $B(H \otimes K)$.

(2) *The maximal tensor product* max [7] is the operator system structure on $S \otimes T$ obtained from the Archimedeanization of the matrix order given by positive cones

$$D_n = \left\{A^*(P \otimes Q)A : \ A \in M_{n,km}(\mathbb{C}), P \in M_k(S)_+, Q \in M_m(T)_+\right\}.$$

(3) *The commuting tensor product* "c" was introduced in [16] (where it was referred to as max). If $\theta_i : S_i \to B(H)$ are u.c.p. maps with commuting ranges, we have a well-defined map $\theta_1 \otimes \theta_2 : S_1 \otimes S_2 \to B(H)$. For $(x_{ij}) \in M_n(S_1 \otimes S_2)$, we set

$$||(x_{ij})|| = \sup\{||(\theta_1 \otimes \theta_2)(x_{ij})|| : \theta_k : S_k \to B(H),\ k = 1,2\}$$

where θ_1 and θ_2 are u.c.p. maps with commuting ranges and H is an arbitrary Hilbert space.

The universal C^*-algebra of an operator system. A fundamental role in the study of tensor products of operator systems is played by the universal C^*-algebra $C_u^*(S)$ of an operator system S [11]. It is, up to isomorphism, the unique C^*-algebra A satisfying:

(1) There exists a unital completely isometric map $u : S \to A$ such that $A = C^*(u(S))$, that is, A is generated by $u(S)$.

(2) If $\theta : S \to B$ is a u.c.p. map into a C^*-algebra B, then there is a *-homomorphism $\pi : A \to B$ such that $\theta(s) = \pi(u(s))$ for every $s \in S$.

Throughout this paper, we will always identify S and $u(S)$.

Two important facts were proved in [7]: first, $S \otimes_c T \subset C_u^*(S) \otimes_{max} C_u^*(T)$, which shows in particular that if A is a C^*-algebra, then in the definition of $S \otimes_c A$ the u.c.p. maps θ_2 pertaining to A can be chosen to be *-monomorphisms. The second result is that if A is a C^*-algebra, then $S \otimes_c A = S \otimes_{max} A$. Throughout this paper, consequently, we will always use the notation "*max*" when one of the operator systems is a C^*-algebra.

We continue with a few useful results which will be used subsequently, collected in

LEMMA 2.2. *(i) If A and B are C^*-algebras, then $A \otimes_{max} B \subset C_u^*(A) \otimes_{max} B$.*

(ii) If S is an operator system and A is a C^*-algebra then $A \otimes_{max} S \subset A \otimes_{max} C_u^*(S)$.

(iii) If S and T are operator systems, then $S \otimes_c T \subset S \otimes_{max} C_u^*(T)$.

(iv) Let A and B be C^*-algebras such that A is weakly c.p. complemented in B. Then $S \otimes_{max} A \subset S \otimes_{max} B$ for every operator system S.

(v) Let A be a C^*-algebra with a closed, two-sided ideal J. If S is an operator system such that $S \otimes_{max} A = S \otimes_{min} A$, then $S \otimes_{max} J = S \otimes_{min} J$.

(vi) Let A be a C^*-algebra with a closed, two-sided ideal J. If S is an operator system, then $(S \otimes_{max} A)/(S \otimes_{max} J)$ and $S \otimes_{max} A/J$ are isomorphic.

Proof. (i) The inclusion $A \subset C_u^*(A)$ induces a u.c.p. map from $A \otimes_{max} B$ with values in $C_u^*(A) \otimes_{max} B$ acting identically on elementary operators. If $a_1, \dots, a_n \in A$ and $b_1, \dots, b_n \in B$, this shows that $||\sum_{i=1}^n a_i \otimes b_i||_{C_u^*(A) \otimes_{max} B} \leqslant ||\sum_{i=1}^n a_i \otimes b_i||_{A \otimes_{max} B}$. The identity map of A extends, by universality, to a *-homomorphism $\pi : C_u^*(A) \to A$, so we obtain a u.c.p. $\pi \otimes id : C_u^*(A) \otimes_{max} B \to A \otimes_{max} B$ acting identically on elementary operators. It follows that $||\sum_{i=1}^n a_i \otimes b_i||_{A \otimes_{max} B} \leqslant ||\sum_{i=1}^n a_i \otimes b_i||_{C_u^*(A) \otimes_{max} B}$.

(ii) By part (i) we have $A \otimes_{max} C_u^*(S) \subset C_u^*(A) \otimes_{max} C_u^*(S)$. Then the algebraic tensor product $A \otimes S$ inherits from $A \otimes_{max} C_u^*(S)$ the same operator system structure as the one inherited from $C_u^*(A) \otimes_{max} C_u^*(S)$. But the latter is precisely $A \otimes_c S = A \otimes_{max} S$ by Theorem 6.4 in [7], so the conclusion follows.

(iii) By part (ii) we have $S \otimes_{max} C_u^*(T) \subset C_u^*(S) \otimes_{max} C_u^*(T)$. It follows that $S \otimes T$ inherits from $S \otimes_{max} C_u^*(T)$ the same structure as the one from $C_u^*(S) \otimes_{max} C_u^*(T)$, namely $S \otimes_c T$.

(iv) We have $C_u^*(S) \otimes_{max} A \subset C_u^*(S) \otimes_{max} B$ and now apply part (ii) of this lemma.

(v) Is a consequence of (iv), as J is weakly c.p. complemented in A (recall that $A^{**} = J^{**} \oplus (A/J)^{**}$).

(vi) The C^*-algebras $(C_u^*(S) \otimes_{max} A)/(C_u^*(S) \otimes_{max} J)$ and $C_u^*(S) \otimes_{max} A/J$ are isomorphic, so the conclusion will follow once we prove that

$$dist(\sum_{i=1}^{n} s_i \otimes a_i, S \otimes_{max} J) = dist(\sum_{i=1}^{n} s_i \otimes a_i, C_u^*(S) \otimes_{max} J)$$

for all $a_1, ..., a_n \in A$ and $s_1, ..., s_n \in S$, and it is clear that the right-hand side is the smaller of the two. To prove the reverse inequality, assume that

$dist(\sum_{i=1}^{n} s_i \otimes a_i, C_u^*(S) \otimes_{max} J) = 1$ and choose $x_1, ..., x_m \in C_u^*(S)$ and $j_1, ..., j_m \in J$ such that, for fixed but arbitrary $\varepsilon > 0$, we have $||\sum_{i=1}^{n} s_i \otimes a_i + \sum_{k=1}^{m} x_k \otimes j_k||_{C_u^*(S) \otimes_{max} A} < 1 + \varepsilon/2$. If (e_α) is an approximate unit for J we have

$$||(\sum_{i=1}^{n} s_i \otimes a_i + \sum_{k=1}^{m} x_k \otimes j_k)(I \otimes (I - e_\alpha))||_{C_u^*(S) \otimes_{max} A} < 1 + \varepsilon/2$$

Choose α such that $j_k(I - e_\alpha)$ are small enough to ensure that

$$||\sum_{i=1}^{n} s_i \otimes a_i(I - e_\alpha)||_{C_u^*(S) \otimes_{max} A} < 1 + \varepsilon$$

which shows that $dist(\sum_{i=1}^{n} s_i \otimes a_i, S \otimes_{max} J) < 1 + \varepsilon$, which concludes the proof. □

We continue with a result which will be used repeatedly.

LEMMA 2.3. *If S is an operator system and B is a C^*-algebra, then $M_2(S) \otimes_{max} B$ and $M_2(S \otimes_{max} B)$ are isomorphic.*

Proof. The operator systems $(S_1 \otimes_{max} S_2) \otimes_{max} S_3$ and $S_1 \otimes_{max} (S_2 \otimes_{max} S_3)$ are isomorphic by Theorem 5.5 in [7]. In particular, it follows that $M_2(S) \otimes_{max} B = (M_2(\mathbb{C}) \otimes_{min} S) \otimes_{max} B = (M_2(\mathbb{C}) \otimes_{max} S) \otimes_{max} B$ and $M_2(S \otimes_{max} B) = M_2(\mathbb{C}) \otimes_{min} (S \otimes_{max} B) = M_2(\mathbb{C}) \otimes_{max} (S \otimes_{max} B)$ are isomorphic. In other words, each of the four corners of $M_2(S) \otimes_{max} B$ is isomorphic to $S \otimes_{max} B$. □

Lifting Properties. An operator system S, in particular a C^*-algebra, has the Lifting Property (LP) if for every C^*-algebra B with a closed, two-sided ideal J, and every u.c.p. map $\varphi : S \to B/J$ there exists a u.c.p. map $\psi : S \to B$ such that $\varphi = \pi \circ \psi$, where π is the quotient map $\pi : B \to B/J$. S has the Local Lifting Property (LLP) if, in the above definition, for any finite dimensional operator subsystem $E \subset S$, there exists a u.c.p. map $\psi : E \to B$ such that $\varphi = \pi \circ \psi$ on E.

The first major result in this direction is due to Choi and Effros [3], stating that separable nuclear C^*-algebras have the LP. Later [10], Kirchberg proved that $C^*(F_\infty)$ also has the LP. Of course, the LP implies the LLP, and an important result of Kirchberg [9] is that A has the LLP if and only if $A\otimes_{max} B(H) = A\otimes_{min} B(H)$.

Miscellaneous results. We conclude this section with two well known results which will be used subsequently. Statements (i) and (ii) are, respectively, Corollary 2.4 and Lemma 3.15 in [13].

PROPOSITION 2.4. *(i) (Arveson) If $\varphi : A \to B$ is a u.c.p. map, then for any C^*-algebra C, the map $\varphi\otimes id$ extends to a u.c.p. map defined on $A\otimes_{max} C$ with values in $B\otimes_{max} C$.*

(ii) Let $U_1, U_2, \ldots$ be the generators of $C^*(F_\infty)$, set $U_0 = I$, and fix $x_0, x_1, \ldots, x_{n-1} \in B$. Then $||\sum_{i=0}^{n-1} x_i\otimes U_i||_{B\otimes_{min} C^*(F_\infty)} = ||\varphi||_{cb}$, where $\varphi : \ell^n_\infty \to B$ is defined by

$$\varphi((\lambda_0,\lambda_1,\ldots,\lambda_{n-1})) = \sum_{i=0}^{n-1} \lambda_i x_i.$$

3. The double commutant expectation property

In this section we extend the notion of WEP to operator systems. We proceed from the Double Commutant Expectation Property of [8] and introduce a new version of WEP, which is more malleable for our approach.

DEFINITION 3.1. [8] We say that an operator system S has the double commutant expectation property (DCEP) provided that for every completely order isomorphic inclusion $S\subset B(H)$, there exists a completely positive map $\varphi : B(H) \to S''$ such that $\varphi(s) = s$ for all $s\in S$.

From a C^*-algebraic viewpoint, this definition encounters the obstacle caused by the fact that $A\otimes_{max} B$ is not always isometrically contained in $A''\otimes_{max} B$, hence the need to involve the second dual A^{**}, which has the nice property that $A\otimes_{max} B \subset A^{**}\otimes_{max} B$. We therefore need another definition for the WEP, one that will rely on the second dual of the universal C^*-algebra of S. For this reason we will call this ad-hoc property "The Universal Weak Expectation Property" (UWEP), and use it for a short while, until Proposition 3.5 proves its equivalence to the DCEP. We also note that a C^*-algebra has the WEP if and only if it has the DCEP as an operator system.

DEFINITION 3.2. We say that an operator system S has the universal weak expectation property (UWEP) provided that for every faithful representation $C^*_u(S)\subset B(H)$, there exists a u.c.p. map $\varphi : B(H) \to C^*_u(S)^{**}$ such that $\varphi(s) = s$ for all $s\in S$.

By arguing like in Proposition 2.11 in [12] we see that for S to have the UWEP it suffices for the property in Definition 3.2 to hold in just one particular faithful representation of $C^*_u(S)$. This is certainly not the case with Definition 3.1, where the property must hold in *every* inclusion $S \subset B(H)$. This aspect will be discussed in more detail in section 5.

LEMMA 3.3. *If $E \subset B(H)$ is an operator system, S is an operator system with the UWEP, and $\varphi : E \to M_n(S)$ is a u.c.p. map, then there exists a u.c.p. map $\psi : B(H) \to M_n(C_u^*(S)^{**})$ such that $\psi(x) = \varphi(x)$ for all $x \in E$.*

Proof. Suppose that $C_u^*(S)$ is faithfully represented in $B(K)$ and let $\theta : B(K) \to C_u^*(S)^{**}$ be the u.c.p. map such that $\theta(s) = s$ for all $s \in S$. Then $\theta \otimes id_n : M_n(B(K)) \to M_n(C_u^*(S)^{**})$ is a u.c.p. map acting identically on $M_n(S)$. If $\Psi : B(H) \to M_n(B(K))$ is any u.c.p. extension of φ given by Arveson's extension theorem, then $\psi = (\theta \otimes id_n) \circ \Psi$ is the desired map. □

While a u.c.p. map $\varphi : A \to B$ induces a u.c.p. map $\varphi \otimes id : A \otimes_{max} C \to B \otimes_{max} C$ for any C^*-algebra C, a complete contraction does not always extend this way. Huruya [6] constructed a complete contraction from A to B such that the map $\varphi \otimes id$ is not even bounded on $A \otimes_{max} C$. It thus becomes very important to find conditions under which such extensions exist, and the next result illustrates the case when the range has the WEP.

The proof relies on the "off diagonal" technique due to Paulsen ([14], [15]). A fundamental role in this technique is played by the Paulsen system S_E associated to an arbitrary operator space $E \subset B(H)$ and defined as

$$S_E = \left\{ \begin{pmatrix} \lambda I & a \\ b^* & \mu I \end{pmatrix} ;\ a, b \in E \right\}$$

PROPOSITION 3.4. *Let A be a C^*-algebra, S an operator system with the UWEP, and $\varphi : A \to M_n(S)$ a completely contractive map. Then, for any C^*-algebra C, the map $\varphi \otimes id$ extends to a completely contractive map from $A \otimes_{max} C$ with values in $M_n(S) \otimes_{max} C$.*

Proof. Consider the operator system $S_A \subset A \otimes M_2(\mathbb{C})$

$$S_A = \left\{ \begin{pmatrix} \lambda I & a \\ b^* & \mu I \end{pmatrix} ;\ a, b \in A \right\}$$

and define the map $\psi : S_A \to M_n(S) \otimes M_2(\mathbb{C})$ defined by

$$\psi\left(\begin{pmatrix} \lambda I & a \\ b^* & \mu I \end{pmatrix}\right) = \begin{pmatrix} \lambda I & \varphi(a) \\ \varphi(b)^* & \mu I \end{pmatrix}$$

It is well-known that φ is completely contractive if and only if ψ is completely positive (Lemma 8.1 in [16]).

We apply Lemma 3.3 and denote by $\Psi : A \otimes M_2(\mathbb{C}) \to M_n(C_u^*(S)^{**}) \otimes M_2(\mathbb{C})$ a u.c.p. map extending ψ. The next step is to consider the u.c.p. map $\Psi \otimes id : (A \otimes$

$M_2(\mathbb{C})) \otimes_{max} C \to (M_n(C_u^*(S)^{**}) \otimes M_2(\mathbb{C})) \otimes_{max} C$ and use Lemma 2.3 to look at the (1,2) corners of this map. The map $\Psi \otimes id$ takes the operator

$$\begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix} \otimes c \in (A \otimes M_2(\mathbb{C})) \otimes_{max} C = M_2(A) \otimes_{max} C = M_2(A \otimes_{max} C) \text{ to the operator}$$

$$\begin{pmatrix} 0 & \varphi(a) \\ 0 & 0 \end{pmatrix} \otimes c \in (M_n(S) \otimes M_2(\mathbb{C})) \otimes_{max} C \subset (M_n(C_u^*(S)) \otimes M_2(\mathbb{C})) \otimes_{max} C$$

$$\subset (M_n(C_u^*(S)^{**}) \otimes M_2(\mathbb{C})) \otimes_{max} C$$

thus to an operator in $M_2(M_n(S)) \otimes_{max} C = M_2(M_n(S) \otimes_{max} C)$.

It follows that the u.c.p. map $\Psi \otimes id$ induces a (necessarily completely contractive) map, namely $\varphi \otimes id$, between the $(1,2)$ corners of $M_2(A \otimes_{max} C)$ and $M_2(M_n(S) \otimes_{max} C)$. By taking into account Lemma 2.3, these corners are $A \otimes_{max} C$ and, respectively, $M_n(S) \otimes_{max} C$, which concludes the proof. □

We have come to the main result of this section, which presents several properties equivalent to the UWEP and which are reminiscent of the properties shared by C^*-algebras with the WEP. The equivalence of (1), (3), (4), and (5) was first proved, from a fairly different perspective, in [8].

PROPOSITION 3.5. *The following are equivalent for an operator system S:*

(1) S has the DCEP.

(2) S has the UWEP.

(3) $S \otimes_{max} C^*(F_\infty) = S \otimes_{min} C^*(F_\infty)$.

(4) If $S \subset T$ is an inclusion of operator systems, then for every C^*-algebra A we have $S \otimes_{max} A \subset T \otimes_{max} A$.

(5) If $S \subset T$ is an inclusion of operator systems, then for every operator system E we have $S \otimes_c E \subset T \otimes_c E$.

Proof. $(1) \Rightarrow (2)$ is immediate if one considers S in the universal representation of $C_u^*(S)$ and apply the definition of DCEP. $(2) \Rightarrow (3)$ Let $U_1, U_2, \ldots$ be the generators of $C^*(F_\infty)$, set $U_0 = I$, and fix $x_0, x_1, \ldots, x_{n-1} \in M_n(S) \subset M_n(C_u^*(S))$. Recall from Proposition 2.4(ii) that

$||\sum_{i=0}^{n-1} x_i \otimes U_i||_{M_n(C_u^*(S)) \otimes_{min} C^*(F_\infty)} = ||\varphi||_{cb}$ where $\varphi : \ell_\infty^n \to M_n(S)$ is defined by $\varphi((\lambda_0, \lambda_1, \ldots, \lambda_{n-1})) = \sum_{i=0}^{n-1} \lambda_i x_i$. Without loss of generality we may assume that

$||\sum_{i=0}^{n-1} x_i \otimes U_i||_{M_n(C_u^*(S)) \otimes_{min} C^*(F_\infty)} = 1$, thus φ is a complete contraction.

By Proposition 3.4, the map $\varphi \otimes id$ extends to a complete contraction from $\ell_\infty^n \otimes_{max} C^*(F_\infty) = \ell_\infty^n \otimes_{min} C^*(F_\infty)$ with values in $M_n(S) \otimes_{max} C^*(F_\infty)$ (note that ℓ_∞^n is abelian, thus nuclear). In particular, if $e_i = (0,...,0,1,0,...,0)$ where 1 appears in the i^{th} coordinate, $0 \leqslant i \leqslant n-1$, we have

$$||\sum_{i=0}^{n-1} x_i \otimes U_i||_{M_n(C_u^*(S))\otimes_{max} C^*(F_\infty)} = ||\sum_{i=0}^{n-1} \varphi(e_i) \otimes U_i||_{M_n(C_u^*(S))\otimes_{max} C^*(F_\infty)}$$
$$\leqslant ||\sum_{i=0}^{n-1} e_i \otimes U_i||_{\ell_\infty^n \otimes_{min} C^*(F_\infty)}$$

It follows that $||\sum_{i=0}^{n-1} x_i \otimes U_i||_{M_n(C_u^*(S))\otimes_{max} C^*(F_\infty)} \leqslant 1$ because $\sum_{i=0}^{n-1} e_i \otimes U_i$ is a unitary operator. In combination with the assumption made in the previous paragraph, namely that $||\sum_{i=0}^{n-1} x_i \otimes U_i||_{M_n(C_u^*(S))\otimes_{min} C^*(F_\infty)} = 1$, we obtain

$$||\sum_{i=0}^{n-1} x_i \otimes U_i||_{M_n(C_u^*(S))\otimes_{max} C^*(F_\infty)} \leqslant ||\sum_{i=0}^{n-1} x_i \otimes U_i||_{M_n(C_u^*(S))\otimes_{min} C^*(F_\infty)}$$

and therefore equality holds.

If we denote by $E_1 \subset S \otimes_{min} C^*(F_\infty)$, respectively $E_2 \subset S \otimes_{max} C^*(F_\infty) \subset B(K)$, the operator spaces generated by $s \otimes U_i$ in each of the two tensor products, we proved the fact that θ, the identity map on elementary operators, is a complete isometry between E_1 and E_2. This map extends to a complete contraction between $C_u^*(S) \otimes_{min} C^*(F_\infty)$ and $B(K)$, still denoted by θ. Since θ is unital, it must be a u.c.p. map. Since θ takes U_i to U_i, the unitary operators U_i belong to the multiplicative domain of θ, therefore θ extends to a u.c.p. map between $S \otimes_{min} C^*(F_\infty)$ and $S \otimes_{max} C^*(F_\infty)$, acting identically on elementary operators, which concludes the proof.

$(3) \Rightarrow (4)$ By Proposition 9 in [11] the inclusion $S \subset T$ induces a canonical inclusion $C_u^*(S) \subset C_u^*(T)$, hence a u.c.p. map from $C_u^*(S) \otimes_{max} A$ with values in $C_u^*(T) \otimes_{max} A$ acting identically on elementary operators. If $s_1,...,s_n \in S$ and $a_1,...,a_n \in A$, this shows that $||\sum_{i=1}^{n} s_i \otimes a_i||_{T\otimes_{max} A} \leqslant ||\sum_{i=1}^{n} s_i \otimes a_i||_{S\otimes_{max} A}$. To prove the reverse inequality, assume without loss of generality that $||\sum_{i=1}^{n} s_i \otimes a_i||_{T\otimes_{max} A} = 1$. Writing $A = C^*(F_\infty)/J$, apply Lemma 2.2(vi) to obtain $t_1,...,t_m \in T$ and $j_1,...,j_m \in J$ such that, for fixed but arbitrary $\varepsilon > 0$, we have $||\sum_{i=1}^{n} s_i \otimes a_i + \sum_{k=1}^{m} t_k \otimes j_k||_{T\otimes_{max} C^*(F_\infty)} < 1 + \varepsilon/2$. If (e_α) is an approximate unit for J we have

$$||(\sum_{i=1}^{n} s_i \otimes a_i + \sum_{k=1}^{m} t_k \otimes j_k)(I \otimes (I - e_\alpha))||_{T\otimes_{max} C^*(F_\infty)} < 1 + \varepsilon/2$$

Choose α in such a way that $j_k(I - e_\alpha)$ are small enough to ensure that

$$||\sum_{i=1}^{n} s_i \otimes a_i(I - e_\alpha)||_{T\otimes_{max} C^*(F_\infty)} < 1 + \varepsilon$$

The algebraic tensor product $S \otimes C^*(F_\infty)$ inherits from $T \otimes_{max} C^*(F_\infty)$ an operator system structure. By hypothesis, this structure must be the max, thus $S \otimes_{max} C^*(F_\infty) \subset T \otimes_{max} C^*(F_\infty)$. In particular, we obtain

$$||\sum_{i=1}^{n} s_i \otimes a_i(I - e_\alpha)||_{S \otimes_{max} C^*(F_\infty)} < 1 + \varepsilon$$

Finally, we apply the completely contractive quotient map from $S \otimes_{max} C^*(F_\infty)$ to $S \otimes_{max} A$ (whose kernel is $S \otimes_{max} J$) and get $||\sum_{i=1}^{n} s_i \otimes a_i||_{S \otimes_{max} A} < 1 + \varepsilon$. Since $\varepsilon > 0$ was arbitrary, the proof is complete.

$(4) \Rightarrow (5)$ Follows from (3) since $S \otimes_{max} C_u^*(E) \subset T \otimes_{max} C_u^*(E)$.

$(5) \Rightarrow (1)$ Consider an arbitrary inclusion $S \subset B(H)$ and denote by A the commutant of S'' in $B(H)$. By hypothesis, the inclusion $S \subset S''$ induces the inclusion $S \otimes_{max} A \subset S'' \otimes_{max} A$. By composing this inclusion with the *-representation $x \otimes a \to xa$ from $S'' \otimes_{max} A$ to $B(H)$, we obtain a u.c.p. $\varphi : S \otimes_{max} A \to B(H)$ such that $\varphi(x \otimes a) = xa$. Again by hypothesis we have $S \otimes_{max} A \subset B(H) \otimes_{max} A$, so there exists a u.c.p. extension of φ from $B(H) \otimes_{max} A$ to $B(H)$, still denoted by φ. Since φ acts identically on A, we see that A is in the multiplicative domain of φ, thus the restriction of φ to $B(H)$ takes values in $A' = S''$ and acts identically on S, which concludes the proof. □

Due to the equivalence of (1) and (2), we will stop using the term UWEP, in favor of DCEP.

It is known [9] that if a C^*-algebra A has the WEP, then $A \otimes_{max} B = A \otimes_{min} B$ whenever B has the LLP, and a similar result holds true for operator systems.

PROPOSITION 3.6. *If S is an operator system with the DCEP and A is a C^*-algebra with the LLP, then $S \otimes_{max} A = S \otimes_{min} A$.*

Proof. Since $A = C^*(F_\infty)/J$ has the LLP, the sequence

$$C_u^*(S) \otimes_{min} J \to C_u^*(S) \otimes_{min} C^*(F_\infty) \to C_u^*(S) \otimes_{min} A$$

is exact [9], so $S \otimes_{min} A$ is isomorphic to $(S \otimes_{min} C^*(F_\infty))/(S \otimes_{min} J)$, by using an approximate unit like in the proof of Lemma 2.2(vi). On the other hand, we already know from Lemma 2.2(vi) that $S \otimes_{max} A$ is isomorphic to $(S \otimes_{max} C^*(F_\infty))/(S \otimes_{max} J)$. Since S has the DCEP, we have $S \otimes_{max} C^*(F_\infty) = S \otimes_{min} C^*(F_\infty)$ by Proposition 3.5 and $S \otimes_{max} J = S \otimes_{min} J$ by Lemma 2.2(v). The conclusion follows. □

4. The relationship with exactness and nuclearity

It is known that a C^*-algebra which is exact and has the WEP must be nuclear (cf. [13]). An analogue result holds true for operator systems, but we must pay attention to the appropriate notion of exactness, as there exist several of them [8]. An operator system S is called 1-exact if the operator systems $(S \otimes_{min} A)/(S \otimes_{min} J)$ and $S \otimes_{min} A/J$ are isomorphic for every C^*-algebra A. With this definition of exactness we have

PROPOSITION 4.1. *If the operator system S is 1-exact and has the DCEP, then $S \otimes_{max} A = S \otimes_{min} A$ for every C^*-algebra A.*

Proof. If $A = C^*(F_\infty)/J$, then $(S \otimes_{min} C^*(F_\infty))/(S \otimes_{min} J)$ and $S \otimes_{min} A$ are isomorphic because of 1-exactness. By Lemma 2.2(vi) we also have a complete order isomorphism between $(S \otimes_{max} C^*(F_\infty))/(S \otimes_{max} J)$ and $S \otimes_{max} A$. Since S has the DCEP, $S \otimes_{max} C^*(F_\infty) = S \otimes_{min} C^*(F_\infty)$ and $S \otimes_{max} J = S \otimes_{min} J$ and from here we get $S \otimes_{max} A = S \otimes_{min} A$. $\square$

The nuclearity property for S in the previous result is weaker than the established definition of nuclearity for operator systems, which requires $S \otimes_{max} T = S \otimes_{min} T$ for all operator systems T. Moreover, it was proved in [7] that there exist non-nuclear operator systems S satisfying $S \otimes_{max} A = S \otimes_{min} A$ for every C^*-algebra A. We next observe that the difference between the two notions of nuclearity hinges upon the difference between the tensor norms $S \otimes_{max} T$ and $S \otimes_c T$.

REMARK 4.2. If $S \otimes_{max} A = S \otimes_{min} A$ for every C^*-algebra A, then $S \otimes_{min} T = S \otimes_c T$ for every operator system T.

Proof. By hypothesis we have $S \otimes_{max} C_u^*(T) = S \otimes_{min} C_u^*(T)$ and we look at the closures of the algebraic tensor product $S \otimes T$ in both sides. From Lemma 2.2(iii) we get $S \otimes_c T \subset S \otimes_{max} C_u^*(T) = S \otimes_{min} C_u^*(T)$ and the conclusion follows. $\square$

5. Examples and applications

In this section we present an example of a matricial operator system outside the class of C^*-algebras which does not have the DCEP.

The next lemma captures the essence of a beautiful argument of Kirchberg. We omit the proof, since it is almost identical to the proof of Lemma 2.5, including Sublemma 2.5.1, in [9] (see also the proof of Corollary 3.12 in [13]).

LEMMA 5.1. *Let E be an operator system with the LP and $S \subset E$ a finite dimensional subsystem. If A is a C^*-algebra and $\varphi : E \to A^{**}$ is a u.c.p. map such that $\varphi(S) \subset A$, then for every $\varepsilon > 0$ there exists a u.c.p. map $\psi : E \to A$ such that $||(\psi - \varphi)|_S|| < \varepsilon$.*

PROPOSITION 5.2. *Let E be an operator system with the LP. If $S \subset E$ is a finite dimensional subsystem with the DCEP, then S has the LP.*

Proof. Let $\varphi : S \to B/J$ be a u.c.p. map. By universality, let $\pi : C_u^*(S) \to B/J$ be the *-homomorphism extending φ. Since S has the DCEP, let $\theta : E \to C_u^*(S)^{**}$ be a u.c.p. map such that $\theta(s) = s$ for all $s \in S$. By Lemma 5.1, let $\psi : E \to C_u^*(S)$ be a u.c.p. map such that $||(\psi - \theta)|_S|| < \varepsilon$. Since E has the LP, there exists a u.c.p. map $\alpha : E \to B$ such that, if $q : B \to B/J$ denotes the quotient map, we have $q \circ \alpha = \pi \circ \psi$. If we denote by β the restriction of α to S, we have $||q \circ \beta - \varphi|| = ||(\pi \circ \psi - \pi \circ \theta)|_S|| < \varepsilon$, which shows that φ is uniformly approximable by liftable maps. By a well-known result of Arveson (Theorem 6 in [1]), this implies that φ is itself liftable. $\square$

Consider now the Paulsen system $S_{\ell^n_\infty} \subset \ell^n_\infty \otimes M_2(\mathbb{C})$ associated with ℓ^n_∞

$$S_{\ell^n_\infty} = \left\{ \begin{pmatrix} \lambda I_n & a \\ b^* & \mu I_n \end{pmatrix}; \, a, b \in \ell^n_\infty \right\}$$

We will prove the rather surprising fact that, at least for $n \geqslant 5$, $S_{\ell^n_\infty}$ does not have the DCEP. This emphasizes the remark we made after Definition 3.2, namely that in Definition 3.1 every completely order isomorphic inclusion $S \subset B(H)$ is relevant, as it easily seen that in the above representation, as a subsystem of the injective von Neumann algebra $\ell^n_\infty \otimes M_2(\mathbb{C})$, S clearly satisfies the property in Definition 3.1.

PROPOSITION 5.3. *For $n \geqslant 5$ the operator system $S_{\ell^n_\infty}$ does not have the DCEP.*

Proof. We apply Proposition 5.2 for $S = S_{\ell^n_\infty}$ and $E = \ell^n_\infty \otimes M_2(\mathbb{C})$, which, as a nuclear C^*-algebra, has the LP. The conclusion will be a consequence of $\square$

PROPOSITION 5.4. *For $n \geqslant 5$ the operator system $S_{\ell^n_\infty}$ does not have the LP.*

Proof. The argument is essentially a short cut through the proofs of Proposition 3.4 and $(2) \Rightarrow (3)$ in Proposition 3.5. Let A be a QWEP C^*-algebra (quotient of a C^*-algebra with the WEP) and set $A = B/J$, where B has the WEP. Fix $x_0, \dots, x_{n-1} \in A$ and let $U_1, U_2, \dots$ be the generators of $C^*(F_\infty)$, where $U_0 = I$.

Recall that we have $||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A \otimes_{min} C^*(F_\infty)} = ||\varphi||_{cb}$ where $\varphi : \ell^n_\infty \to A$ is defined by $\varphi((\lambda_0, \lambda_1, \dots, \lambda_{n-1})) = \sum_{i=0}^{n-1} \lambda_i x_i$ and assume without loss of generality that $||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A \otimes_{min} C^*(F_\infty)} = 1$, making φ a complete contraction. Next, we consider the u.c.p. map $\psi : S_{\ell^n_\infty} \to M_2(A)$ defined by

$$\psi\left(\begin{pmatrix} \lambda I_n & a \\ b^* & \mu I_n \end{pmatrix}\right) = \begin{pmatrix} \lambda I_n & \varphi(a) \\ \varphi(b)^* & \mu I_n \end{pmatrix}$$

and suppose that $S_{\ell^n_\infty}$ has the LP. Then there exists a u.c.p. map $\Psi : S_{\ell^n_\infty} \to M_2(B)$ such that $q \circ \Psi = \psi$, where q is the quotient map from $M_2(B)$ onto $M_2(A)$. We obtain the composition of the sequence of u.c.p. maps

$$S_{\ell^n_\infty} \otimes_{min} C^*(F_\infty) \xrightarrow{\Psi \otimes id} M_2(B) \otimes_{min} C^*(F_\infty) = M_2(B) \otimes_{max} C^*(F_\infty) \xrightarrow{q \otimes id} M_2(A) \otimes_{max} C^*(F_\infty)$$

and use Lemma 2.3 to focus on the (1,2) corner of this map, namely $\varphi \otimes id$, which represents a complete contraction from $\ell^n_\infty \otimes_{min} C^*(F_\infty)$ with values in $A \otimes_{max} C^*(F_\infty)$.

Like in the proof of $(2) \Rightarrow (3)$ in Proposition 3.5, we get

$$||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A\otimes_{max}C^*(F_\infty)} = ||\sum_{i=0}^{n-1} \varphi(e_i)\otimes U_i||_{A\otimes_{max}C^*(F_\infty)}$$
$$\leqslant ||\sum_{i=0}^{n-1} e_i \otimes U_i||_{\ell_\infty^n \otimes_{min} C^*(F_\infty)} = 1$$

therefore

$$||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A\otimes_{max}C^*(F_\infty)} = ||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A\otimes_{min}C^*(F_\infty)}$$

This would imply that every QWEP C^*-algebra must have the WEP, which is simply not true (e.g. $C^*_{red}(F_n)$). However, this general contradiction does not shed any light on the values of n for which $S_{\ell_\infty^n}$ fails to have the LP, so we need to look more closely into a particular case.

Let $g_1,\dots,g_n$ be the generators of F_n and λ the left regular representation $\lambda : F_n \to B(\ell^2(F_n))$. If $U_1, U_2, \dots$ are the generators of $C^*(F_\infty)$, we apply Proposition 1.1(2) in [4] to obtain

$$||\sum_{i=1}^{n} \lambda(g_i)\otimes U_i||_{C^*_{red}(F_n)\otimes_{min}C^*(F_\infty)} \leqslant 2\max\left\{||\sum_{i=1}^{n} U_i^*U_i||^{1/2}, ||\sum_{i=1}^{n} U_iU_i^*||^{1/2}\right\} = 2\sqrt{n}$$

On the other hand, there exists a surjective *-homomorphism π from $C^*(F_\infty)$ onto $C^*_{red}(F_n)^{op}$ carrying U_i to $\overline{\lambda(g_i)}, i = 1,\dots,n$, therefore a *-homomorphism $id\otimes\pi$: $C^*_{red}(F_n)\otimes_{max} C^*(F_\infty) \to C^*_{red}(F_n)\otimes_{max} C^*_{red}(F_n)^{op}$ carrying $\sum_{i=1}^{n} \lambda(g_i)\otimes U_i$ to $\sum_{i=1}^{n} \lambda(g_i)\otimes\overline{\lambda(g_i)}$. But in $B(\ell^2(F_n))$ we have $||\sum_{i=1}^{n} \lambda(g_i)\otimes\overline{\lambda(g_i)}|| = n$, as it is easily seen by applying the operator to the vector in $\ell^2(F_n)$ coming from the identity, so consequently

$$||\sum_{i=1}^{n} \lambda(g_i)\otimes U_i||_{C^*_{red}(F_n)\otimes_{max}C^*(F_\infty)} = n$$

For $n \geqslant 5$, this is the contradiction which completes the proof. □

We conclude by proving that $S_{\ell_\infty^n}$ is not isomorphic to a C^*-algebra.

PROPOSITION 5.5. *For $n \geqslant 2$ the operator system $S_{\ell_\infty^n}$ is not (completely order) isomorphic to a C^*-algebra.*

Proof. To get a contradiction, suppose that A is a C^*-algebra and $\varphi : S_{\ell_\infty^n} \to A$ is a complete order isomorphism. Since A is generated as a linear space by unitary operators, φ extends to a *-homomorphism $\psi : \ell_\infty^n \otimes M_2(\mathbb{C}) \to A$ by Lemma 3.16 in [13]. Since $\dim S_{\ell_\infty^n} = \dim A = 2n+2$ and $\dim \ell_\infty^n \otimes M_2(\mathbb{C}) = 4n$, we see that $4n > 2n+2$ so, consequently, $\ker\psi \neq \{0\}$. But $J = \ker\psi$ is an ideal in $\ell_\infty^n \otimes M_2(\mathbb{C})$ so it must be of the form $J = \ell_\infty^k \otimes M_2(\mathbb{C})$ for some $k \geqslant 1$. In particular, J contains a non-zero operator of the form $s = \begin{pmatrix} 0 & a \\ 0 & 0 \end{pmatrix}$ for some $a \neq 0 \in \ell_\infty^n$. But then s is seen to belong to $S_{\ell_\infty^n}$ as well, so $\psi(s) = 0$ implies $\varphi(s) = 0$, contradiction. $\square$

Another example of interest is the seven-dimensional operator subsystem $\mathscr{S} \subset M_3(\mathbb{C})$ studied in [7, 8] and described by the star diagram

$$\mathscr{S} = \left\{ \begin{pmatrix} * & * & 0 \\ * & * & * \\ 0 & * & * \end{pmatrix} \right\}$$

where the stars are any complex numbers. It was proved in [7] and [8] that $\mathscr{S}$ has the DCEP but not the WEP. It follows from Proposition 5.2 that $\mathscr{S}$ has the LP, showing that in the category of operator systems the LP does not imply the WEP. Whether or not the LP (or the LLP) implies the DCEP is equivalent to Kirchberg's conjecture, by Theorem 9.1 in [8].

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