A NOTE ON 2–LOCAL REPRESENTATIONS OF C*–ALGEBRAS

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Abstract. We survey the results on linear local and 2-local homomorphisms and zero products preserving operators between C*-algebras, and we incorporate some new precise observations and results to prove that every bounded linear 2-local homomorphism between C*-algebras is a homomorphism. Consequently, every linear 2-local *-homomorphism between C*-algebras is a *-homomorphism.

1. Introduction

Most of the authors agree in acknowledging the papers of R. V. Kadison [24] and D. R. Larson and A. R. Sourour [30] as the pioneering contributions to the theory of local derivations and local automorphisms on Banach algebras, respectively. We recall that a linear mapping T from a Banach algebra A into a Banach algebra B is said to be a *local homomorphism* if for every a in A there exists a homomorphism $\Phi_a : A \to B$, depending on a, satisfying $T(a) = \Phi_a(a)$. When A and B are C*-algebras, and for each a in A there exists a *-homomorphism $\Phi_a : A \to B$, depending on a, satisfying $T(a) = \Phi_a(a)$, the mapping T is called a *local *-homomorphism*. *Local automorphisms*, *local *-automorphisms*, and *local derivations* are similarly defined.

R. V. Kadison proved in [24] that every bounded local derivation on a von Neumann algebra (i.e. a C*-algebra which is also a dual Banach space) is a derivation. After Kadison's contribution, a multitude of researchers explored the same problem for general C*-algebras (see, for example, [1, 4, 21, 31, 44], and [50]). The definitive answer is due to B. E. Johnson [23], who proved that every local derivation from a C*-algebra A into a Banach A-bimodule is a derivation. Much more recently, local triple derivations on C*-algebras and JB*-triples have been studied in [33, 8, 9] and [14].

The knowledge about local homomorphisms and local *-homomorphisms between C*-algebras is less conclusive. D.D. Larson and A. R. Sourour proved in [30] that for an infinite dimensional Banach space X, every surjective local automorphism T on the Banach algebra $B(X)$, of all bounded linear operators on X, is an automorphism. When X is a separable Hilbert space M. Brešar and P. Šemrl showed that the hypothesis concerning the surjectivity is superfluous (cf. [5]). A related result was established

by C. Batty and L. Molnar in [3], where they proved that for a properly infinite von Neumann algebra $\mathcal{M}$, the group, $\mathrm{Aut}(\mathcal{M})$, of all *-automorphisms on $\mathcal{M}$ is strong topologically reflexive, i.e. if a bounded linear mapping $T : \mathcal{M} \to \mathcal{M}$ satisfies that for every $a \in \mathcal{M}$, $T(a)$ belongs to the strong-closure of the set $\{\Phi(a) : \Phi \in \mathrm{Aut}(\mathcal{M})\}$, then T lies in $\mathrm{Aut}(\mathcal{M})$ [3, Proposition 2]. Furthermore, for each separable Hilbert space H of dimension $n \geqslant 3$, the group $\mathrm{Aut}(B(H))$ is strong topologically reflexive [3, Theorem 3]. In [41, §2], F. Pop provides an example of a local homomorphism from $M_2(\mathbb{C})$ into $M_4(\mathbb{C})$ which fails to be multiplicative (cf. Example 3.13).

In 1997, P. Šemrl [43] introduces 2-local derivations and 2-local automorphisms in the following sense: Let A be a Banach algebra, a mapping $T : A \to A$ is a *2-local automorphism* if for every $a, b \in A$ there is an automorphism $T_{a,b} : A \to A$, depending on a and b, such that $T_{a,b}(a) = T(a)$ and $T_{a,b}(b) = T(b)$ (no linearity, surjectivity or continuity of T is assumed). In the just quoted paper, Šemrl proves that for every infinite-dimensional separable Hilbert space H, every 2-local automorphism $T : B(H) \to B(H)$ is an automorphism. In [28], S. O. Kim and J. S. Kim show that every surjective 2-local *-automorphism on a prime C*-algebra or on a C*-algebra such that the identity element is properly infinite is a *-automorphism (see [12, 15, 34, 35, 36, 37] and [27] for other related results).

A closer look at Šemrl's paper [43] shows that the connections with the Gleason-Kahane-Żelazko theorem (cf. [19, 25]) didn't go unnoticed to him. Borrowing a paragraph from [43, Introduction], we notice that Gleason-Kahane-Żelazko theorem can be reformulated in the following sense: every unital linear local homomorphism from a unital complex Banach algebra A into $\mathbb{C}$ is multiplicative (cf. [2]). S. Kowalski and Z. Slodkowski [29] established a 2-local version of the Gleason-Kahane-Żelazko theorem, showing that every 2-local homomorphism $T : A \to \mathbb{C}$ is linear and multiplicative.

In order to keep coherence with the terminology employed by P. Šemrl, a mapping T between C*-algebras A and B is called a *2-local homomorphism* (respectively, *2-local *-homomorphism*) if for every $a, b \in A$ there exists a bounded homomorphism (respectively, a **-homomorphism*) $\Phi_{a,b} : A \to B$, depending on a and b, such that $\Phi_{a,b}(a) = T(a)$ and $\Phi_{a,b}(b) = T(b)$. *2-local Jordan homomorphisms*, *2-local Jordan *-homomorphisms* and *2-local automorphisms* are defined in a similar fashion. We recall that a linear mapping $\Phi : A \to B$ is said to be a Jordan homomorphism whenever $\Phi(a^2) = \Phi(a)^2$ (equivalently, Φ preserves the Jordan products of the form $a \circ b := \frac{1}{2}(ab + ba)$).

In 2004, new studies on 2-local linear maps between C*-algebras were developed by D. Hadwin and J. Li [21] and F. Pop [41], though these papers seem to be mutually disconnected at the moment of publication. Hadwin and Li prove that every bounded linear and unital 2-local homomorphism (respectively, 2-local *-homomorphism) from a unital C*-algebra of real rank zero into itself is a homomorphism (respectively, a *-homomorphism) [21, Theorem 3.6]. As a consequence, every linear and surjective 2-local *-automorphism on a unital C*-algebra of real rank zero is a *-automorphism (cf. [21, Theorem 3.7]). The main contribution of F. Pop in [41] establishes that every bounded linear 2-local homomorphism (respectively, 2-local *-homomorphism) from a von Neumann algebra into another C*-algebra is a homomorphism (respectively, a *-homomorphism) [41, Corollary 3.6].

In 2006, J.-H. Liu and N.-C. Wong made their own contribution to the study of not necessarily continuous nor linear 2-local homomorphisms between standard operator algebras on locally convex spaces [32]. We recall that a standard operator algebra $\mathscr{A}$ on a locally convex space X, is a subalgebra of $B(X)$ containing the algebra $\mathscr{F}(X)$ of all continuous finite rank operators on X. Liu and Wong prove, without assuming linearity, surjectivity or continuity, that every 2-local automorphism of $\mathscr{F}(X)$ is an algebra homomorphism. In the case in which X is a Frechet space with a Schauder basis and $\mathscr{A}$ contains all locally compact operators, it can be concluded that every 2-local automorphism on $\mathscr{A}$ is an automorphism. Furthermore, a 2-local automorphism Θ of a standard operator algebra $\mathscr{A}$ on a locally convex space X is an algebra homomorphism provided that the range of Θ contains $\mathscr{F}(X)$, or Θ is continuous in the weak operator topology (cf. [32]). In the just quoted paper, the authors study the question of when a 2-local automorphism of a C*-algebra is an automorphism, showing that every linear 2-local automorphism T of a C*-algebra whose range is a C*-algebra is an algebra homomorphism.

It seems natural to ask whether the above results of Hadwin-Li and Pop remain true for general C*-algebras. This paper, which has an almost expository aim, combined with new research results, we give a positive answer to this question by showing that every bounded linear 2-local homomorphism between C*-algebras is a homomorphism, and consequently, every linear 2-local *-homomorphism between C*-algebras is a *-homomorphism (Theorem 3.9). In particular, according to the terminology in [41], every 2-local (*-)representation of a C*-algebra is a (*-)representation (Corollary 3.10). In Example 3.14 we present an example of a linear local *-automorphism on $M_2(\mathbb{C})$ which is not multiplicative. We survey the connections between this problem and the theory of linear zero products preservers developed by J. Alaminos, M. Bresar, J. Extremera and A. Villena in [1]. The novelties in this paper include an independent proof which is not based on the result in [1] together with the precise observations to provide a definitive answer to the whole line of problems on linear preservers on C*-algebras presented above. Here we make use of techniques developed in the setting of JB*-triples combined with the use of compact-G_δ projections in the bidual of a C*-algebra. We study of the connections between (linear) 2-local homomorphisms and zero product preserving mappings. Although the results presented here could have been obtained by combining some of the results that we shall review later, the equivalence between bounded linear 2-local homomorphisms and bounded homomorphisms between C*-algebras has not been explicitly established before.

2. Techniques of Jordan algebras and JB*-triples

Every C*-algebra A admits a Jordan product defined by $a \circ b = \frac{1}{2}(ab + ba)$. The Jordan product is commutative but not necessarily associative. Let B be another C*-algebra. A linear map $T : A \to B$ is said to be a *Jordan homomorphism* whenever it preserves Jordan products, or equivalently, when $T(a^2) = T(a)^2$, for every a. A *Jordan *-homomorphism* is a Jordan homomorphism which maps self-adjoint elements into self-adjoint elements. For each element a in A, the symbol U_a will denote the linear map $U_a : A \to A$ defined by $U_a(x) := axa$. Since, for every $a, x \in A$ we

have $U_a(x) = 2(a \circ x) \circ a - a^2 \circ x$, every Jordan homomorphism $T : A \rightarrow B$ satisfies $T(U_a(x)) = U_{T(a)}(T(x))$.

Let $T : A \rightarrow B$ be a Jordan homomorphism between C*-algebras. Since A^{**} and B^{**} are von Neumann algebras, $T^{**} : A^{**} \rightarrow B^{**}$ is weak* continuous, and the product of every von Neumann algebra is separately weak* continuous (cf. [42, Theorem 1.7.8]), we deduce, via Goldstine's theorem, that $T^{**} : A^{**} \rightarrow B^{**}$ is a Jordan homomorphism. Since the involution of a von Neumann algebra is weak* continuous (cf. [42, Theorem 1.7.8]), T^{**} is a Jordan *-homomorphism whenever T is a Jordan *-homomorphism.

There is some benefit in considering a C*-algebra as an element in the wider class of JB*-triples. A *JB*-triple* is a complex Banach space E equipped with a triple product $\{\cdot,\cdot,\cdot\} : E \times E \times E \rightarrow E$ which is linear and symmetric in the outer variables, conjugate linear in the middle variable and satisfies the following conditions:

(a) (Jordan identity)

$$\{a,b,\{x,y,z\}\} = \{\{a,b,x\},y,z\} - \{x,\{b,a,y\},z\} + \{x,y,\{a,b,z\}\},$$

for a,b,x,y,z in E;

(b) For each $a \in E$, the mapping $L(a,a) : E \rightarrow E$, $x \mapsto \{a,a,x\}$ is a hermitian (linear) operator with non-negative spectrum;

(c) $\|\{x,x,x\}\| = \|x\|^3$ for all $x \in E$.

Every C*-algebra is a JB*-triple via the triple product given by

$$\{x,y,z\} = \frac{1}{2}(xy^*z + zy^*x).$$

It was shown by Poincaré in the early 1900s, that the Riemann mapping theorem fails when the complex plane is replaced by a complex Banach space of higher dimension. Although, a complete holomorphic classification of bounded simply connected domains in arbitrary complex Banach spaces is unattainable, bounded symmetric domains in finite dimensions were studied and classified by E. Cartan [10]. In the setting of complex Banach spaces of arbitrary dimension, W. Kaup proved, in [26], that a complex Banach space is a JB*-triple if, and only if, its open unit ball is a bounded symmetric domain, and every bounded symmetric domain in a complex Banach space is biholomorphically equivalent to the open unit ball of a JB*-triple; showing that the category of all bounded symmetric domains with base point is equivalent to the category of JB*-triples. We refer to monographs [46] and [11] for the basic theory of JB*-triples and JB*-algebras.

Spectral resolutions of non-normal elements in a C*-algebra is a completely hopeless goal. However, in every JB*-triple E, the JB*-subtriple E_a generated by a single element $a \in E$ is (isometrically) JB*-isomorphic to $C_0(L)$ for some locally compact Hausdorff space $L \subseteq (0, \|a\|]$, such that $L \cup \{0\}$ is compact. It is also known that there

exists a JB*-triple isomorphism $\Psi_a : E_a \to C_0(L)$, satisfying $\Psi(a)(t) = t$ $(t \in L)$ (compare [26, Lemma 1.14]). In particular, there exists a unique element $a^{[\frac{1}{3}]} \in E_a$ such that $\{a^{[\frac{1}{3}]}, a^{[\frac{1}{3}]}, a^{[\frac{1}{3}]}\} = a$. When $E = A$ is a C*-algebra,

$$a^{[\frac{1}{3}]}(a^{[\frac{1}{3}]})^* a^{[\frac{1}{3}]} = \{a^{[\frac{1}{3}]}, a^{[\frac{1}{3}]}, a^{[\frac{1}{3}]}\} = a.$$

In order to simplify notation, for each element a in a JB*-triple E we write $a^{[1]} = a$ and $a^{[2n+1]} := \{a, a^{[2n-1]}, a\}$ $(\forall n \in \mathbb{N})$. It is known that JB*-triples are power associative, that is, $\{a^{[2k-1]}, a^{[2l-1]}, a^{[2m-1]}\} = a^{[2(k+l+m)-3]}$, for every $k, l, m \in \mathbb{N}$ (cf. [11, Lemma 1.2.10]).

3. Local and 2-local representations of C*-algebras

Let A and B be C*-algebras. Clearly, every local *-homomorphism $T : A \to B$ is automatically continuous and contractive. Indeed, since for each $a \in A$, there exists a *-homomorphism $\Phi_a : A \to B$ satisfying $T(a) = \Phi_a(a)$, we have $\|T(a)\| = \|\Phi_a(a)\| \leqslant \|a\|$. Concerning (local) homomorphisms, many basic questions still being open, like automatic continuity of homomorphisms between C*-algebras ([13, Question 5.4.D] or [47, Question 1]).

The next result summarizes some clear facts about local homomorphisms.

LEMMA 3.1. *Let A, B and C denote C*-algebras, $T : A \to B$ a local homomorphism (respectively, a local *-homomorphism) and $\Phi : B \to C$ a homomorphism (respectively, a *-homomorphism), then ΦT is a local homomorphism (respectively, a local *-homomorphism). Every local *-homomorphism between C*-algebras is positive.*

By Gelfand theory the set of extreme points in the positive part of the unit ball in the dual, $\mathscr{B}^*$, of a commutative C*-algebra $\mathscr{B}$ is precisely the set X of non-zero multiplicative functionals on $\mathscr{B}$, and thus the identification $\mathscr{B} = C_0(X)$ was established. Therefore, non-zero homomorphisms from $C_0(L)$ into $\mathbb{C}$ identify with those functionals $\delta_s : C_0(L) \to \mathbb{C}$, $\delta_s(f) = f(s)$, where s runs in L.

The refinement of the Gleason-Kahane-Żelazko theorem established by Żelazko in [49] asserts that for every complex Banach algebra $\mathscr{B}$ (not necessarily unital nor commutative), every linear selection from the spectrum $\varphi : \mathscr{B} \to \mathbb{C}$ (i.e. $\varphi(a) \in \sigma(a)$, for every $a \in \mathscr{B}$) is multiplicative. Another interesting result, implicitly established by J. P. Kahane and W. Żelazko in [25, Theorem 3], will be applied in the next proposition.

PROPOSITION 3.2. *Let L_1 and L_2 be locally compact Hausdorff spaces and let $T : C_0(L_1) \to C_0(L_2)$ be a local homomorphism. Then, for each $s \in L_2$, the mapping $\delta_s T : C_0(L_1) \to \mathbb{C}$ is a *-homomorphism. In particular, T is a *-homomorphism.*

Proof. Let us assume that $\delta_s T \neq 0$. We shall prove that $\delta_s T = \delta_t$ for a unique $t \in L_1$. Since T is a local homomorphism, $\delta_s T$ is a local homomorphism. So, given

$f \in C_0(L_1)$ there exists a homomorphism $\Phi_f : C_0(L_1) \to \mathbb{C}$, and hence an element $t_f \in L_1$, satisfying $\delta_s T(f) = \Phi_f(f) = f(t_f)$. Now, Theorem 3 in [25] proves that $\delta_s T$ is a multiplicative functional. $\square$

COROLLARY 3.3. *Let $T : A \to B$ be a local homomorphism between C^*-algebras, where B is commutative. Then T is a Jordan *-homomorphism and, consequently, continuous.*

Proof. Let a be a self adjoint element in A. Considering the C^*-subalgebra, $\mathscr{C}$, generated by a, the mapping $T|_{\mathscr{C}} : \mathscr{C} \to B$ is a local homomorphism between commutative C^*-algebras. Proposition 3.2 assures that $T|_{\mathscr{C}}$ is a *-homomorphism. Therefore $T(a^2) = T(a)^2$ for every $a \in A_{sa}$, and hence T is a Jordan *-homomorphism. $\square$

When $C(K)$ is replaced with the real C^*-algebra $C(K, \mathbb{R})$, of all real-valued continuous functions on K, the corresponding versions of the above results are not, in general, true. For example, the linear operator $T : C([0,1], \mathbb{R}) \to \mathbb{R}$, $T(a) := \int_0^1 a(t) dt$ is not multiplicative. However, the mean-value theorem implies that T is a local homomorphism.

The existence of bounded linear operators between C^*-algebras which are local homomorphisms and fail to be multiplicative (cf. [41, example in §2]) led F. Pop to focus his attention on 2-local homomorphisms (called 2-local representations by Pop). The just-mentioned counter-example, provided by Pop, is not multiplicative but it is a Jordan homomorphism (see Example 3.13). The latter property is actually satisfied by every linear 2-local homomorphism between C^*-algebras (cf. [32, Lemma 2.1]).

PROPOSITION 3.4. *Every linear 2-local homomorphism between C^*-algebras is a Jordan homomorphism. Every linear 2-local *-homomorphism between C^*-algebras is a Jordan *-homomorphism.*

Proof. When T is a linear 2-local homomorphism, for each $a \in A$, there exists a homomorphism $\Phi_{a,a^2} : A \to B$ such that $T(a) = \Phi_{a,a^2}(a)$ and $T(a^2) = \Phi_{a,a^2}(a^2)$. Then, $T(a)^2 = \Phi_{a,a^2}(a)^2 = \Phi_{a,a^2}(a^2) = T(a^2)$, confirming that T is a Jordan homomorphism. $\square$

In the setting of von Neumann algebras, the hypothesis in Proposition 3.4 can be relaxed. Indeed, in [41, Proposition 1.4], F. Pop establishes that every bounded linear local homomorphism from a commutative von Neumann algebra into $B(H)$ is multiplicative, and hence a representation. This result applies to get:

COROLLARY 3.5. *Let $T : M \to B$ be a bounded linear local homomorphism from a von Neumann algebra into a C^*-algebra. Then T is a Jordan homomorphism. Consequently, every linear local *-homomorphism from a von Neumann algebra into a C^*-algebra is a Jordan *-homomorphism.*

Proof. Let $T : M \to B$ be a bounded linear local homomorphism. Making use of the representation theory and Lemma 3.1, we can assume that $B = B(H)$ for a suitable complex Hilbert space H. Let a be a self-adjoint element in M, and let $\mathscr{C}$ denote the von Neumann subalgebra of M generated by a and 1. Clearly, $T|_{\mathscr{C}} : \mathscr{C} \to B(H)$ is a bounded linear local homomorphism. By [41, Proposition 1.4], $T|_{\mathscr{C}}$ is multiplicative. Therefore, $T(a^2) = T(a)^2$, for every $a \in M_{sa}$. This implies that $T(a \circ b) = T(a) \circ T(b)$ for every $a, b \in M_{sa}$ and hence $T((a + ib)^2) = T(a^2 - b^2 + 2ia \circ b) = T(a)^2 - T(b)^2 + 2iT(a) \circ T(b) = T(a + ib)^2$, for every $a, b \in M_{sa}$, which proves the statement. $\square$

It seems natural to ask whether the above mentioned results of Hadwin-Li and Pop hold when the domain is a general C*-algebra. We shall see that the answer is intrinsically related to zero-products preserving operators between C*-algebras.

Let A and B be C*-algebras. A mapping $f : A \to B$ is said to be *orthogonality preserving* on a subset $U \subseteq A$ when the implication

$$a \perp b \Rightarrow f(a) \perp f(b),$$

holds for every $a, b \in U$. We recall that elements a, b in A are said to be *orthogonal* (denoted by $a \perp b$) whenever $ab^* = b^*a = 0$. When the implication

$$ab = 0 \Rightarrow f(a)f(b) = 0$$

holds for every $a, b \in U$, we shall say that f *preserves zero products* or is *zero products preserving* on U. In the case $A = U$, we shall simply say that f is *orthogonality preserving* or that f *preserves zero products*, respectively. Every homomorphism between C*-algebras preserves zero products and every *-homomorphism is orthogonality preserving.

LEMMA 3.6. (cf. [32, Lemma 2.1]) *Let* $T : A \to B$ *be a map between* C*-*algebras. Suppose* T *is a 2-local* *-*homomorphism (respectively, a 2-local homomorphism), then* T *is orthogonality preserving (respectively, zero products preserving).*

Proof. Given $a, b \in A$ with $a \perp b$, we take a *-homomorphism $\Phi_{a,b} : A \to B$ satisfying $T(a) = \Phi_{a,b}(a)$ and $T(b) = \Phi_{a,b}(b)$. Clearly, $T(a) = \Phi_{a,b}(a) \perp \Phi_{a,b}(b) = T(b)$. The other statement follows similarly. $\square$

Orthogonality preserving bounded linear maps between C*-algebras have been completely described in [6, Theorem 17] (see [7] and [18] for completeness).

Let A be a C*-algebra. An element x in the von Neumann algebra A^{**} is a *multiplier* for A if $xA \subseteq A$ and $Ax \subseteq A$. The symbol $M(A)$ will denote the set of all multiplier of A in A^{**}. It is known that $M(A)$ is a unital C*-subalgebra of A^{**}. Multipliers are uninteresting if the algebra A possesses a unit, because in such a case $M(A) = A$ (see [40, §3.12] for more details).

Let W be a von Neumann algebra. For each normal positive functional $\varphi \in W_*$ the mapping $W \times W \to \mathbb{C}$, $(x, y)_\varphi := \frac{1}{2}\varphi(xy^* + y^*x)$ defines a semi-positive sesquilinear form on W. The corresponding prehilbertian seminorm on W is defined by

$$\|x\|_\varphi := (x, x)_\varphi^{\frac{1}{2}} = \left(\frac{1}{2}\varphi(xx^* + x^*x)\right)^{\frac{1}{2}}.$$

The *strong* topology* of W (denoted by $s^*(W, W_*)$) is the locally convex topology on W defined by all the seminorms $\|.\|_\varphi$, where φ runs in the set of all positive functionals in W_* (cf. [42, Definition 1.8.7]). It is known that the strong* topology of W is compatible with the duality (W, W_*), that is, a functional $\psi : W \to \mathbb{C}$ is $s^*(W, W_*)$ if and only if it is weak* continuous (see [42, Corollary 1.8.10]). It is also known, from the above fact together with the Grothedieck-Pisier-Haagerup inequality (cf. [20]), that a linear map between von Neumann algebras is strong* continuous if and only if it is weak* continuous. We also recall that the product of every von Neumann algebra is jointly strong* continuous on bounded sets (see [42, Proposition 1.8.12]).

The next result is a subtle variant of [48, Lemma 2.2]. The proof applies techniques of JB*-triples in a similar fashion to the arguments given in the proofs of [7, Proposition 3.1], [16, Proposition 1.3], and [48, Lemma 2.2].

PROPOSITION 3.7. *Let $T : A \to B$ be a bounded linear map between C^*-algebras sending zero products in A to zero products in B. Then the restricted map $T^{**}|_{M(A)} : M(A) \to B^{**}$ sends zero products in $M(A)$ to zero products in B^{**}.*

Proof. We fix $a, b \in M(A)$ with $ab = 0$. For each natural, n, the odd triple power $a^{[3]} = aa^*a$, $a^{[2n+1]} = a(a^{[2n-1]})^*a$, satisfies that $a^{[2n-1]}b = 0$. Thus, we deduce that, $\alpha b = 0$, for every α in the JB*-subtriple, $M(A)_a$, of $M(A)$ generated by a. The same argument shows that

$$\alpha\beta = 0 \tag{3.1}$$

for every $\alpha \in M(A)_a$ and $\beta \in M(A)_b$. Consequently, we have $a^{[\frac{1}{3}]}b^{[\frac{1}{3}]} = 0$.

Since $M(A)$ is a C*-subalgebra of A^{**}, by Goldstine's Theorem, we can find bounded nets (x_λ) and (y_μ) in A, converging in the weak* topology of A^{**} to $a^{[\frac{1}{3}]}$ and $b^{[\frac{1}{3}]}$, respectively. The nets $\left(a^{[\frac{1}{3}]}x_\lambda^* a^{[\frac{1}{3}]}\right)$ and $\left(b^{[\frac{1}{3}]}y_\mu^* b^{[\frac{1}{3}]}\right)$ lie in A, and

$$\left(a^{[\frac{1}{3}]}x_\lambda^* a^{[\frac{1}{3}]}\right)\left(b^{[\frac{1}{3}]}y_\mu^* b^{[\frac{1}{3}]}\right) = 0,$$

for every λ and μ.

By hypothesis, T is zero products preserving, and hence,

$$T\left(a^{[\frac{1}{3}]}x_\lambda^* a^{[\frac{1}{3}]}\right) T\left(b^{[\frac{1}{3}]}y_\mu^* b^{[\frac{1}{3}]}\right) = 0, \tag{3.2}$$

for every λ and μ. Finally, taking weak*-limits in λ and μ, the weak* continuity of T^{**} and the separate weak*-continuity of the product of A^{**}, together with (3.2), give

$$0 = T^{**}\left(a^{[\frac{1}{3}]}(a^{[\frac{1}{3}]})^* a^{[\frac{1}{3}]}\right) T^{**}\left(b^{[\frac{1}{3}]}(b^{[\frac{1}{3}]})^* b^{[\frac{1}{3}]}\right) = T^{**}(a)T^{**}(b),$$

which completes the proof. $\square$

Let A be a C*-algebra, a projection p in A^{**} is called *compact-G_δ* (relative to A) whenever there exists a positive, norm-one element a in A such that p coincides with the weak*-limit (in A^{**}) of the sequence $(a^n)_n$. Following standard notation, we

shall say that p is a *range projection* when there exists a positive, norm-one element $a \in A$ for which p is the weak*-limit of the sequence $(a^{\frac{1}{n}})_n$.

Our next result can be derived from [1, Theorem 4.1] (compare Remark 3.11). To our knowledge, it has never been stated in the form presented here. We also include a new proof which is independent from the arguments in [1].

THEOREM 3.8. *Let A and B be C^*-algebras with A unital. Let $J : A \to B$ be a bounded Jordan homomorphism preserving zero products. Then J is a homomorphism.*

Proof. Since J is a Jordan homomorphism, we deduce that $J(1) = e$ is an idempotent in B and

$$J(a) = J(U_1(a)) = U_{J(1)}(J(a)) = U_e(J(a)) = eJ(a)e,$$

for every $a \in A$. Since $J^{**} : A^{**} \to B^{**}$ is a Jordan homomorphism too, we can actually assure that

$$J^{**}(a) = eJ^{**}(a)e = eJ^{**}(a) = J^{**}(a)e, \tag{3.3}$$

for every $a \in A$.

Since J preserves zero products, given $a, b \in A$ with $ab = 0$, we have

$$J(ba) = J(ab + ba) = J(a)J(b) + J(b)J(a) = J(b)J(a),$$

and consequently

$$J(bza) = J(bz)J(a) = J(b)J(za), \tag{3.4}$$

for every $a, b, z \in A$ with $ab = 0$.

Let us consider a compact-G_δ projection $p \in A^{**}$, that is, there exists a positive, norm-one element a in A such that $p = w^* - \lim_n a^n$. We can identify the C^*-subalgebra of A generated by 1 and a with $C(K)$, where $K \subseteq [0, 1]$, $1 \in K$, and $a(t) = t$ in the corresponding identification. Let us define two sequences (y_n) and (z_n) in $C(K)$ given by

$$y_n(t) := \begin{cases} 1, & \text{if } t \in K \cap [0, 1 - \frac{1}{n}]; \\ -2nt + 2n - 1, & \text{if } t \in K \cap [1 - \frac{1}{n}, 1 - \frac{1}{2n}]; \\ 0, & \text{if } t \in K \cap [1 - \frac{1}{2n}, 1], \end{cases}$$

and

$$z_n(t) := \begin{cases} 0, & \text{if } t \in K \cap [0, 1 - \frac{1}{3n}]; \\ 3nt - 3n + 1, & \text{if } t \in K \cap [1 - \frac{1}{3n}, 1] \end{cases}.$$

It is easy to check that $0 \leqslant y_n, z_n$, (y_n) is increasing, (z_n) is decreasing, $y_n z_m = z_m y_n = 0$ for every $n, m \in \mathbb{N}$, $m \geqslant n$, $w^* - \lim_n y_n = 1 - p$, and $w^* - \lim_n z_n = p$ in A^{**}.

By hypothesis, for every z in A, and every n, m in $\mathbb{N}$ with $m \geqslant n$, we have $J(zz_m)J(y_n) = 0$, and thus

$$0 = w^* - \lim_{m \geqslant n} J(zz_m)J(y_n) = J^{**}(zp)J(y_n), \quad \text{for every } n \in \mathbb{N},$$

which implies that

$$0 = w^* - \lim_n J^{**}(zp)J(y_n) = J^{**}(zp)J(1-p),$$

and hence

$$J^{**}(zp)e = J^{**}(zp)J(1) = J^{**}(zp)J(p).$$

It follows from (3.3) that

$$J^{**}(zp) = J^{**}(zp)e = J^{**}(zp)J^{**}(p), \tag{3.5}$$

for every $z \in A^{**}$.

Applying (3.4) we deduce that

$$J(y_n)J(zz_m) = J(y_n zz_m),$$

and

$$J(z_m)J(zy_n) = J(z_m zy_n),$$

for every $z \in A$, $n, m \in \mathbb{N}$ with $m \geqslant n$. Taking weak*-limits in $m, n \to \infty$, we get

$$J^{**}(1-p)J^{**}(zp) = J^{**}((1-p)zp), \tag{3.6}$$

and

$$J^{**}(p)J^{**}(z(1-p)) = J^{**}(pz(1-p)), \tag{3.7}$$

for every z in A or in A^{**}. Combining (3.6) with (3.3) we get

$$J^{**}(zp) - J^{**}(p)J^{**}(zp) = J^{**}(1)J^{**}(zp) - J^{**}(p)J^{**}(zp) = J^{**}(zp) - J^{**}(pzp),$$

and thus

$$J^{**}(p)J^{**}(zp) = J^{**}(pzp), \tag{3.8}$$

for every z in A or in A^{**}.

Now, combining (3.7) and (3.8), we deduce that

$$J^{**}(p)J^{**}(z) = J^{**}(pz),$$

for every $z \in A^{**}$. We have therefore proved that

$$J^{**}(pz) = J^{**}(p)J^{**}(z), \tag{3.9}$$

for every $z \in A^{**}$ and every compact-G_δ projection $p \in A^{**}$.

Finally, take an arbitrary self adjoint element $b \in A$ and identify the C^*-subalgebra of A generated by 1 and b with a $C(K)$-space for a suitable $K \subset [-\|b\|, \|b\|]$. The property proved in (3.9) shows that

$$J^{**}(pz) = J^{**}(p)J^{**}(z)$$

for every projection p of the form $p = \chi_{[\alpha,\beta] \cap K}$ with $[\alpha, \beta] \subseteq [-\|b\|, \|b\|]$. Having in mind that projections $q \in C(K)^{**} \subseteq A^{**}$ of the form $q = \chi_{(\alpha,\beta) \cap K}$, with $(\alpha, \beta) \subseteq K$ can

be approximated in the strong* topology of A^{**} by sequences of projections $(p_n) = \left(\chi_{[\alpha - \frac{1}{n}, \beta + \frac{1}{n}] \cap K} \right)$, we deduce that $J^{**}(qz) = J^{**}(q)J^{**}(z)$ for every such projection q and every $z \in A^{**}$. It is well known that b (regarded as an element in $C(K) \subseteq A$) can be approximated in norm by a finite linear combinations of mutually orthogonal projections of the form $\chi_{[\alpha,\beta] \cap K}$ and $\chi_{(\alpha,\beta) \cap K}$ with $[\alpha, \beta] \subseteq [-\|b\|, \|b\|]$ (i.e. steps functions). Therefore, $J(b)J(z) = J(bz)$, for every $z, b \in A$ with $b = b^*$ and, by linearity, J is a homomorphism. $\square$

We can now prove the main result concerning 2-local homomorphisms.

THEOREM 3.9. *Every bounded linear 2-local homomorphism between C*-algebras is a homomorphism. Every linear 2-local *-homomorphism between C*-algebras is a *-homomorphism.*

Proof. Let $T : A \to B$ be a bounded 2-local homomorphism between C*-algebras. Proposition 3.4 implies that T is a Jordan homomorphism.

By the 2-local property, we deduce, via Lemma 3.6, that T preserves zero products. Proposition 3.7 implies that $T^{**}|_{M(A)} : M(A) \to B^{**}$ preserves zero products.

Finally, since $T^{**}|_{M(A)} : M(A) \to B^{**}$ is a Jordan homomorphism which preserves zero products and $M(A)$ is unital, the above Theorem 3.8 gives the desired statement. $\square$

Clearly, Theorems 3.6 and 3.7 in [21] are direct consequences of the above Theorem 3.9.

Accordingly to the notation in [41], given a C*-algebra A and a complex Hilbert space H, a bounded linear map $T : A \to B(H)$ is called a *2-local representation* of A whenever it is a 2-local homomorphism. The next result generalizes [41, Corollary 3.6] to the general setting of C*-algebras.

COROLLARY 3.10. *Let A be a C*-algebra. Every 2-local representation of A is a representation.*

REMARK 3.11. It should be noted here that Theorem 3.8 can be derived from [1, Theorem 4.1]. Indeed, in the just commented result the authors prove that for every unital C*-algebra A, every Banach algebra B, and every bounded linear operator $T : A \to B$ preserving zero products, then

$$T(1)T(xy) = T(x)T(y),$$

for all x, y in A. Therefore, if $J : A \to B$ is a bounded Jordan homomorphism preserving zero products, by the first part of the argument in the proof of Theorem 3.8, $J(1) = e$ is an idempotent in B and $J(a) = eJ(a)e = eJ(a) = J(a)e$, for every $a \in A$, and hence $J(xy) = J(1)J(xy) = J(x)J(y)$, for all x, y in A. That is, Theorem 3.8 holds when B is a Banach algebra. Proposition 3.7 is needed for the non-unital version of Theorem 3.9

PROBLEM 3.12. Is every (not necessarily linear) 2-local ($*$-)homomorphism between C^*-algebras a ($*$-)homomorphism? Equivalently, determine whether the hypothesis concerning linearity in Theorem 3.9 is superfluous.

As we have commented before, we cannot expect that a local homomorphism between C^*-algebras is a homomorphism (see [41, §2]). We shall take a closer look at the counter-example provided by F. Pop.

EXAMPLE 3.13. We know, from [41, §2], that the mapping $T : M_2(\mathbb{C}) \to M_4(\mathbb{C})$,

$$T\left(\begin{pmatrix} a & b \\ c & d \end{pmatrix}\right) = \begin{pmatrix} a & 0 & b & 0 \\ 0 & a & 0 & c \\ c & 0 & d & 0 \\ 0 & b & 0 & d \end{pmatrix},$$

is a local homomorphism which is not multiplicative. Is easy to check that the above T is a unital Jordan $*$-homomorphism. We claim that T is not a local $*$-homomorphism. Otherwise, there exits a $*$-homomorphism

$$\pi = \pi_{\left(\begin{smallmatrix} 1 & 1 \\ 2 & 0 \end{smallmatrix}\right)} : M_2(\mathbb{C}) \to M_4(\mathbb{C})$$

satisfying

$$\pi\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} = T\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 \\ 2 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}.$$

Therefore

$$\pi\begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix} = \pi\left(\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}^*\right)$$

$$= \pi\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}\pi\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}^* = \begin{pmatrix} 2 & 0 & 2 & 0 \\ 0 & 5 & 0 & 1 \\ 2 & 0 & 4 & 0 \\ 0 & 1 & 0 & 1 \end{pmatrix},$$

$$\pi\begin{pmatrix} 5 & 1 \\ 1 & 1 \end{pmatrix} = \pi\left(\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}^*\begin{pmatrix} 1 & 1 \\ 2 & 0 \end{pmatrix}\right) = \begin{pmatrix} 5 & 0 & 1 & 0 \\ 0 & 2 & 0 & 2 \\ 1 & 0 & 1 & 0 \\ 0 & 2 & 0 & 4 \end{pmatrix},$$

$$\pi\begin{pmatrix} 0 & 0 \\ -2 & 4 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 3 & 0 & -3 \\ -2 & 0 & 4 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix},$$

$$\pi\begin{pmatrix} -8 & 0 \\ 0 & 2 \end{pmatrix} = \pi\left(\begin{pmatrix} 2 & 2 \\ 2 & 4 \end{pmatrix} - 2\begin{pmatrix} 5 & 1 \\ 1 & 1 \end{pmatrix}\right) = \begin{pmatrix} -8 & 0 & 0 & 0 \\ 0 & 1 & 0 & -3 \\ 0 & 0 & 2 & 0 \\ 0 & -3 & 0 & -7 \end{pmatrix},$$

$$\pi\begin{pmatrix} 0 & 0 \\ 0 & 20 \end{pmatrix} = \pi\left(\begin{pmatrix} 0 & 0 \\ -2 & 4 \end{pmatrix}\begin{pmatrix} 0 & 0 \\ -2 & 4 \end{pmatrix}^*\right) = 20\begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 9/10 & 0 & -3/10 \\ 0 & 0 & 1 & 0 \\ 0 & -3/10 & 0 & 1/10 \end{pmatrix},$$

$$\pi\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1/10 & 0 & 3/10 \\ 0 & 0 & 0 & 0 \\ 0 & 3/10 & 0 & 9/10 \end{pmatrix},$$

$$\pi\begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 3/10 & 0 & 9/10 \\ 1 & 0 & 0 & 0 \\ 0 & -1/10 & 0 & -3/10 \end{pmatrix},$$

and

$$\pi\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 3/10 & 0 & -1/10 \\ 0 & 0 & 0 & 0 \\ 0 & 9/10 & 0 & -3/10 \end{pmatrix},$$

which gives $\pi\left(\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\right) \neq \pi\left(\begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}\right)\pi\left(\begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix}\right)$, contradicting that π is a homomorphism.

Furthermore, the elements $a = \begin{pmatrix} 1 & -1 \\ 1 & -1 \end{pmatrix}$ and $b = \begin{pmatrix} 1 & 2 \\ 1 & 2 \end{pmatrix}$ satisfy $ab = 0$ and $T(a)T(b) \neq 0$, which shows that T does not preserves zero products.

It seems natural to ask whether every local *-homomorphism between C*-algebras is multiplicative. We shall see that the answer to this question is, in general, negative. The next example illustrates this fact and provides an easier argument to Pop's counterexample.

EXAMPLE 3.14. A problem posed by P. R. Halmos in [22, Proposition 159] asks whether every square complex matrix is unitarily equivalent to its transpose. In other words, given $a \in M_n(\mathbb{C})$, when does there exist a unitary matrix $u \in M_n(\mathbb{C})$ satisfying $u^*au = a^t$?

More generally, the problem of deciding whether two given square matrices a and b over the field of complex numbers are unitarily equivalent was positively solved by W. Specht [45] who found a (more or less satisfactory) necessary and sufficient condition for two complex square matrices to be unitarily equivalent. In the setting of 2×2 matrices the conditions are much more simple; F. D. Murnaghan [38] showed that, the traces of a, a^2, and a^*a form a complete set of invariants to determine when two matrices in $M_2(\mathbb{C})$ are unitarily equivalent (i.e. two matrices $a, b \in M_2(\mathbb{C})$ are unitarily equivalent if and only if $\mathrm{tr}(a) = \mathrm{tr}(b)$, $\mathrm{tr}(a^2) = \mathrm{tr}(b^2)$, and $\mathrm{tr}(a^*a) = \mathrm{tr}(b^*b)$). Some years later, C. Pearcy [39] obtained a list of nine conditions to determine when $a, b \in M_3(\mathbb{C})$ are unitarily equivalent (see [17] for a recent publication on these topics).

Murnaghan's characterization implies that every matrix in $M_2(\mathbb{C})$ is unitarily equivalent to its transpose, that is for each $a \in M_2(\mathbb{C})$ there exists a unitary matrix $u \in M_2(\mathbb{C})$ (depending on a) satisfying $u^* au = a^t$. Consequently, the mapping

$$T : M_2(\mathbb{C}) \to M_2(\mathbb{C}), \quad T(a) = a^t,$$

is a linear local *-homomorphism and a *-anti-homomorphism which is not multiplicative.

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