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DISJOINTNESS OF THE DIFFERENTIATION OPERATOR TUPLE ON WEIGHTED BANACH SPACES OF ENTIRE FUNCTIONS

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Abstract. When the differentiation operator is continuous on weighted Banach spaces of entire functions, several characterizations for the disjoint mixing, disjoint hypercyclicity and disjoint supercyclicity of finitely many differentiation operators on $H_{v,0}(\mathbb{C})$ are presented. Especially, we deduce an equivalence between the hypercyclicity of a single differentiation operator and the disjoint mixing of tuple $D^{r_1},\cdots,D^{r_N}$ for $1 \leqslant r_1 < r_2 < \cdots < r_N$ on $H_{v,0}(\mathbb{C})$, which strengthens some existing results.

1. Introduction

A weigh v on the complex plane $\mathbb{C}$ is a strictly positive continuous function on $\mathbb{C}$ which is radial, i.e. $v(z) = v(|z|)$, $z \in \mathbb{C}$, such that $v(r)$ is non-increasing on $[0,\infty)$ and satisfies $\lim\limits_{r\to\infty} r^m v(r) = 0$ for each $m \in \mathbb{N}$. For such a weight, the weighted Banach spaces of entire functions are defined by

$$H_v := H_v(\mathbb{C}) = \{f \in H(\mathbb{C}),\ \|f\|_v = \sup_{z\in\mathbb{C}} v(z)|f(z)| < +\infty\},$$

$$H_{v,0} := H_{v,0}(\mathbb{C}) = \{f \in H(\mathbb{C}),\ \lim_{|z|\to\infty} v(z)|f(z)| = 0\},$$

endowed with the norm $\|f\|_v$. It is obvious that $H_{v,0}$ is a closed subspace of H_v which contains all polynomials. We refer the readers to [6, 7, 23, 24, 25] and the references therein to learn more about this type of spaces.

Harutyunyan and Lusky [18] have shown the continuity of the differentiation operator $Df = f'$ on the space H_v under the assumption that the space H_v is isomorphic to the Banach space l_∞. After that, Bonet [4] further completed the results of Harutyunyan and Lusky, and showed that the differentiation operator $D : H_v \to H_v$ is continuous if and only if $D : H_{v,0} \to H_{v,0}$ is continuous. Moreover, he obtained the following three theorems concerning the dynamical behavior of differentiation operators on weighted Banach spaces of entire functions.

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THEOREM 1.1. [4, Theorem 2.3] *Assume that the differentiation operator* $D: H_{v,0} \to H_{v,0}$ *is continuous. The following conditions are equivalent,*

(1) D *satisfies the Hypercyclicity criterion;*

(2) D *is hypercyclic on* $H_{v,0}$;

(3) $\liminf_{n\to\infty} \frac{\|z^n\|_v}{n!} = 0$.

THEOREM 1.2. [4, Theorem 2.4] *Assume that the differentiation operator* $D: H_{v,0} \to H_{v,0}$ *is continuous. The following conditions are equivalent,*

(1) D *is topologically mixing;*

(2) $\lim\limits_{n\to\infty} \frac{\|z^n\|_v}{n!} = 0$.

THEOREM 1.3. [4, Proposition 2.7] *Every continuous differentiation operator* $D: H_{v,0} \to H_{v,0}$ *is supercyclic.*

If a Banach space X admits a hypercyclic operator, then X must be separable. This is why we investigate the differentiation operators on the space $H_{v,0}$. Throughout this paper, we always assume that $D: H_{v,0} \to H_{v,0}$ is continuous. Our results provide several equivalent conditions for the disjoint properties of finitely many differentiation operators acting on $H_{v,0}$. Therefore we generalize the above three theorems to N-tuple of differentiation operators $D^{r_1}, \cdots, D^{r_N}$ acting on $H_{v,0}$.

The paper is organized as follows. We first recall some basic definitions, related propositions and a well-known lemma in section 2. After that we present some interesting conditions for the disjointness in mixing, hypercyclicity and supercyclicity of the N differentiation operators $D^{r_1}, \ldots, D^{r_N}$ in sections 3-5.

2. Notation and preliminaries

For a positive integer n, the $n-$th iterate of $T \in L(X)$ denoted by T^n, is the function obtained by composing T with itself n times. Let $L(X)$ denote the space of linear continuous operators on a separable infinite dimensional Fréchet space X.

A continuous linear operator T on a topological vector space X is called hypercyclic (respectively, supercyclic) provided there is some $f \in X$ such that the orbit $Orb(T, f) = \{T^n f : n = 0, 1, \cdots\}$ (respectively, the projective orbit $\{\lambda T^n f : \lambda \in \mathbb{C}, n = 0, 1, 2, \cdots\}$) is dense in X. Such a vector f is said to hypercyclic (respectively, supercyclic) for T.

It is well known that an operator T on a separable Banach space X is hypercyclic if and only if it is topologically transitive in the sense of dynamical systems, i.e. for every pair of non-empty open subsets U and V of X there is $n \in \mathbb{N}$ that $T^n(U)$ meets V. A stronger condition is the following: the operator T on X is called topologically mixing if for every pair of non-empty open subsets U and V of X there is $N \in \mathbb{N}$ such that $T^n(U)$ meets V for each $n \geqslant N$.

For motivation, examples and background about linear dynamics of single operators, we refer the readers to the books [9] by Bayart and Matheron, [14] by Grosse-Erdmann and Peris, and the article by Godefroy and Shapiro [16] etc.

There are several remarkable advances in understanding dense orbits of (linear) operators. Ansari showed that an operator $T \in L(X)$ and its iterates T^n $(n = 1,2...)$ have the same set of hypercyclic vectors in [1]. Bourdon and Feldman [8] verified that a somewhere dense orbit must be everywhere dense. Each of these two results also holds for d-hypercyclicity (Definition 2.1). Based on these foundations, we study the hypercyclic behaviour of the orbit of $(f,...,f) \in X^N$ under an operator of the form $T_1 \oplus ... \oplus T_N$.

The first example of hypercyclic operators was on the space $H(\mathbb{C})$ of entire functions on the complex plane $\mathbb{C}$, endowed with the topology of uniform convergence on compact subsets of $\mathbb{C}$. Birkhoff [3] showed the translation operator $T_a f(z) = f(z+a)$, $z \in \mathbb{C}$, $f \in H(\mathbb{C})$ is hypercyclic whenever $0 \neq a \in \mathbb{C}$. The hypercyclicity of differentiation operator $Df = f'$ on $H(\mathbb{C})$ was shown in [26] by MacLane. For a broad view of the field of linear dynamics of the differentiation operator, see, e.g. [4, 5] and the references therein.

Now given $N \geqslant 2$ operators $T_1,...,T_N$ on a Fréchet space X, there are a lot of papers concerning the dynamical properties of their direct sum $T_1 \oplus \cdots \oplus T_N$. In 2007, Bès and Peris [11] and, independently, Bernal [2] investigated the property of the orbits

$$\{(f,f,...,f),(T_1 f,T_2 f,\cdots,T_N f),(T_1^2 f,T_2^2 f,\cdots,T_N^2 f),\cdots\}\ (f \in X).$$

They studied the case when one of these orbits is dense in X^N endowed with the product topology for some $z \in X$. If there is some vector satisfying the above condition, the operators $T_1,\cdots,T_N$ are called disjoint hypercyclic which is a weaker notion than the notion of disjointness of Furstenberg (see, e.g. [13]). In this dissertation, we will pay attention to the dynamics of $N \geqslant 2$ differentiation operators $D^{r_1},\cdots,D^{r_N}$ acting on $H_{v,0}$. Some necessary definitions follow.

DEFINITION 2.1. [27, Definition 1.3.1] We say that $N \geqslant 2$ sequences of operators $(T_{1,n})_{n=1}^{\infty},...,(T_{N,n})_{n=1}^{\infty}$ in $L(X)$ are disjoint hypercyclic or d-hypercyclic (respectively, disjoint supercyclic or d-supercyclic) provided the sequence of direct sums $(T_{1,n} \oplus ... \oplus T_{N,n})_{n=1}^{\infty}$ has a hypercyclic (respectively, supercyclic) vector of the form $(f,...,f) \in X^N$. Then f is called a d-hypercyclic (respectively, d-supercyclic) vector for the sequences $(T_{1,n})_{n=1}^{\infty},...,(T_{N,n})_{n=1}^{\infty}$. The operators $T_1,...,T_N$ in $L(X)$ are called d-hypercyclic (respectively, d-supercyclic) if the sequences of iterations $(T_1^n)_{n=1}^{\infty},...,$ $(T_N^n)_{n=1}^{\infty}$ are d-hypercyclic (respectively, d-supercyclic).

It is well-know that two d-hypercyclic operators must be substantially different (see, e.g. [11]). For instance, an operator can not be d-hypercyclic with a scalar multiple of itself. For some recent related results on disjointness in hypercyclicity, see, e.g.[10, 12, 17, 19, 20, 21, 22, 28, 30, 31] and their references therein.

DEFINITION 2.2. [11, Definition 2.1] We say that $N \geqslant 2$ sequences of operators $(T_{1,n})_{n=1}^{\infty},\cdots,$ $(T_{N,n})_{n=1}^{\infty}$ in $L(X)$ are d-topologically transitive (respectively, d-mixing)

provided for every non-empty open subsets $V_0,\cdots,V_N$ of X there exists $m\in\mathbb{N}$ such that
$$V_0\bigcap T_{1,m}^{-1}(V_1)\bigcap\cdots\bigcap T_{N,m}^{-1}(V_N)\neq\emptyset$$
(respectively, so that $V_0\bigcap T_{1,j}^{-1}(V_1)\bigcap\cdots\bigcap T_{N,j}^{-1}(V_N)\neq\emptyset$ for each $j\geqslant m$). Also, we say that $N\geqslant 2$ operators $T_1,\cdots,T_N$ in $L(X)$ are d-topologically transitive (respectively, d-mixing) provided $(T_1^n)_{n=1}^{\infty},\cdots,(T_N^n)_{n=1}^{\infty}$ are d-topologically transitive (respectively, d-mixing) sequences.

The next proposition shows that when X is a Fréchet (therefore, a Baire) space, the concepts of d-hypercyclicity and d-topological transitivity coincide.

PROPOSITION 2.3. [11, Proposition 2.3] *Let $(T_{1,n})_{n=1}^{\infty},\cdots,(T_{N,n})_{n=1}^{\infty}$ be sequences of operators in $L(X)$. Then $(T_{1,n})_{n=1}^{\infty},\cdots,(T_{N,n})_{n=1}^{\infty}$ are d-topologically transitive if and only if the set of d-hypercyclic vectors for $(T_{1,n})_{n=1}^{\infty},\cdots,(T_{N,n})_{n=1}^{\infty}$ is a dense G_δ.*

DEFINITION 2.4. [11, Definition 2.2] We say that $N\geqslant 2$ sequences $(T_{1,n})_{n=1}^{\infty},\cdots,$ $(T_{N,n})_{n=1}^{\infty}$ in $L(X)$ are d-universal (respectively, densely d-universal) if
$$\{(T_{1,n}f,T_{2,n}f,\cdots,T_{N,n}f):\ n\in\mathbb{N}\}$$
is dense in X^N for some vector $f\in X$ (respectively, for each vector f in a given dense subset of X). We call such vector f a d-universal vector for $(T_{1,n})_{n=1}^{\infty},\cdots,(T_{N,n})_{n=1}^{\infty}$. Also, we say that $(T_{1,n})_{n=1}^{\infty},\cdots,(T_{N,n})_{n=1}^{\infty}$ are hereditarily universal (respectively, hereditarily densely universal) provided for each increasing sequence of positive integers (n_k) the sequence $(T_{1,n_k})_{k=1}^{\infty},\cdots,(T_{N,n_k})_{k=1}^{\infty}$ are d-universal (respectively, densely d-universal). If additionally X is a topological vector space and $T_j\in L(X)$, then the d-universality goes under the name d-hypercyclicity.

REMARK 2.5. [10, Remark 1.8] An application of the Baire theorem shows that if X is a Baire and second countable, then a sequence $(T_{1,n})_{n=1}^{\infty},\cdots,$ $(T_{N,n})_{n=1}^{\infty}$ in $L(X)$ are d-transitive if and only if they are densely d-universal. Moreover, the sequence $(T_{1,n})_{n=1}^{\infty},\cdots,$ $(T_{N,n})_{n=1}^{\infty}$ in $L(X)$ are d-mixing if and only if they are d-transitive for every subsequence.

The following d-*Hypercyclicity criterion* is a generalization of the *Hypercyclicity criterion* (see,e.g. [15]) to the setting of disjointness of hypercyclic operators.

DEFINITION 2.6. [11, Definition 2.5] Let (n_k) be an increasing sequence of positive integers. We say that $N\geqslant 2$ operators $T_1,\cdots,T_N$ in $L(X)$ satisfy the d-Hypercyclicity criterion with respect to (n_k) provided there exist dense subsets $X_0,\cdots,$ X_N of X and mappings $S_{l,k}:X_l\to X$ $(k\in\mathbb{N},\ 1\leqslant l\leqslant N)$ satisfying

(i) $T_l^{n_k}\underset{k\to\infty}{\longrightarrow}0$ pointwise on X_0,

(ii) $S_{l,k}\underset{k\to\infty}{\longrightarrow}0$ pointwise on X_l, and

(iii) $(T_l^{n_k}S_{i,k}-\delta_{i,l}Id_{X_l})\underset{k\to\infty}{\longrightarrow}0$ pointwise on $X_l(1\leqslant i\leqslant N)$.

In general, we say that $T_1,\cdots,T_N$ satisfy the d-Hypercyclicity criterion if there exists some increasing sequence of positive integers (n_k) for which the above conditions are satisfied.

REMARK 2.7. The d-Hypercyclicity criterion is a sufficient condition for d-hypercyclicity.

PROPOSITION 2.8. [11, Proposition 2.6] *Let $T_1,..,T_N$ satisfy the d-Hypercyclicity criterion with respect to a sequence (n_k). Then the sequences $\{T_1^{n_k}\}_{k=1}^{\infty},\cdots,\{T_N^{n_k}\}_{k=1}^{\infty}$ are d-mixing. In particular, $T_1,\cdots,T_N$ are d-hypercyclic. Indeed, if $(n_k)=(k)$, then $T_1,\cdots,T_N$ are d-mixing.*

The following d-*Supercyclicity criterion* generalizes the *Supercyclicicty criterion* of Salas [29] to the setting of disjointness.

DEFINITION 2.9. [27, Definition 4.1.1] Let X be a Banach space and (n_k) be a strictly increasing sequence of positive integers and $N\geqslant 2$. We say that $T_1,\cdots,T_N$ in $L(X)$ satisfy the d-Supercyclicity criterion with respect to (n_k) provided there exist dense subsets $X_0,X_1,\ldots,X_N$ of X and mappings

$$S_l : X_l \to X, (1\leqslant l\leqslant N)$$

so that for $1\leqslant i\leqslant N$

(i) $(T_l^{n_k}S_i^{n_k}-\delta_{i,l}I_{X_i})\underset{k\to\infty}{\longrightarrow} 0$ pointwise on X_i,

(ii) $\lim\limits_{k\to\infty}\|T_l^{n_k}x\|\cdot\|\sum\limits_{j=1}^{N}S_j^{n_k}y_j\|=0$ for $x\in X_0$ and $y_i\in X_i$.

REMARK 2.10. The d-Supercyclicity criterion is a sufficient condition for d-supercyclicity.

We finally recall a well known lemma, which will be used in the sequel. The transpose of an operator T is denoted by T^t.

LEMMA 2.11. [4, Lemma 2.2] *Let T be an operator on a separable Banach space X. If T is topologically mixing, then $\lim\limits_{n\to\infty}\|(T^t)^n(g)\|=\infty$ for each $g\in X^*$, $g\neq 0$.*

3. Disjoint mixing differentiation operator

At the beginning of this section, we first present several equivalent characterizations for the disjoint mixing properties of $D^{r_1},\cdots,D^{r_N}$ $(N\geqslant 2)$ on $H_{v,0}$.

THEOREM 3.1. *Let $1\leqslant r_1<r_2<\cdots<r_N$, where $r_i\in\mathbb{N}$, $i=1,\cdots,N$. Assume that the differentiation operator $D:H_{v,0}\to H_{v,0}$ is continuous. The following conditions are equivalent,*

(1)

$$\lim_{n\to\infty} \frac{\|z^n\|_v}{n!} = 0; \tag{3.1}$$

(2) $D^{r_1}, \cdots, D^{r_N}$ *are d-mixing;*

(3) $D^{r_1}, \cdots, D^{r_N}$ *satisfy the d-Hypercyclicity criterion with respect to the sequence* (n);

(4) $D^{r_1}, \cdots, D^{r_N}$ *are hereditarily densely d-hypercyclic with respect to the sequence* (n)*;*

Proof. We will show these implications $(1) \Rightarrow (3) \Rightarrow (2) \Rightarrow (1)$ and $(3) \Rightarrow (4) \Rightarrow (2)$.

$(1) \Rightarrow (3)$. Denote $T_1 = D^{r_1}, \cdots, T_N = D^{r_N}$. Since $D : H_{v,0} \to H_{v,0}$ is continuous, there is a positive constant $C \geqslant 1$ such that

$$\|f^{(j)}\|_v \leqslant C^j \|f\|_v, \tag{3.2}$$

for each $f \in H_{v,0}$ and each $j \in \mathbb{N}$.

Next we verify that the operators $D^{r_1}, \cdots, D^{r_N}$ satisfy d-*Hypercyclicity criterion* with respect to the sequence (n).

Choose $X_0 = X_1 = \cdots = X_N =$ span$\{1, z, z^2, \ldots\}$, and take the mappings

$$S_{l,n} : X_l \to X$$

for each $1 \leqslant l \leqslant N$ and each $n \in \mathbb{N}$, defined by

$$S_{l,n} z^s := \frac{z^{s+nr_l}}{(s+1)(s+2)\ldots(s+nr_l)}, \; s = 0, 1, 2 \cdots. \tag{3.3}$$

Since the set of all polynomials is dense in $H_{v,0}$, then $X_0, X_1, \cdots X_N$ are dense subsets of $H_{v,0}$. It is easy to check that $T_l^n \underset{n\to\infty}{\to}$ pointwise on X_0 for $1 \leqslant l \leqslant N$. Next we verify that

(ii) $S_{l,n} \underset{n\to\infty}{\to} 0$ pointwise on $X_l (1 \leqslant l \leqslant N)$.

Fix $s \in \mathbb{N} \cup \{0\}$. From the definition in (3.3) we have that

$$S_{l,n}(z^s) = \frac{z^{s+nr_l}}{(s+1)\ldots(s+nr_l)} = s! \frac{z^{s+nr_l}}{(s+nr_l)!},$$

further by (3.1) it follows that

$$\|S_{l,n}(z^s)\|_v = s! \frac{\|z^{s+nr_l}\|_v}{(s+nr_l)!} \to 0, \; n \to \infty. \tag{3.4}$$

Thus the condition (ii) follows from (3.4).

It remains to show that

(iii) $(T_l^n S_{i,n} - \delta_{i,l} Id_{X_l}) \underset{n\to\infty}{\longrightarrow} 0$ pointwise on $X_l (1 \leqslant i \leqslant N)$,

which is immediate when $i = l$ since $T_l^n S_{l,n} z^s = z^s$, $s = 0,1,\cdots$. Now, for $1 \leqslant i < l \leqslant N$, it follows that $r_i < r_l$. On the one hand, if we choose n large enough such that $n(r_l - r_i) > s$, it follows that

$$\begin{aligned}\|T_l^n S_{i,n} z^s\|_v &= \left\| T_l^n \Big(\frac{z^{s+nr_i}}{(s+1)(s+2)\cdots(s+nr_i)} \Big) \right\|_v \\ &= \left\| (D^{r_l})^n \Big(\frac{z^{s+nr_i}}{(s+1)(s+2)\cdots(s+nr_i)} \Big) \right\|_v = 0.\end{aligned}$$

On the other hand, using (3.1) we obtain that

$$\begin{aligned}\|T_i^n S_{l,n} z^s\|_v &= \left\| (D^{r_i})^n \Big(\frac{z^{s+nr_l}}{(s+1)(s+2)\cdots(s+nr_l)} \Big) \right\|_v \\ &= \left\| \frac{z^{s+n(r_l-r_i)}}{(s+1)(s+2)\cdots(s+n(r_l-r_i))} \right\|_v = s! \frac{\|z^{s+n(r_l-r_i)}\|_v}{(s+n(r_l-r_i))!} \to 0,\ n\to\infty.\end{aligned}$$

From the above two equations, the condition (iii) holds. Hence $D^{r_1},\cdots,D^{r_N}$ satisfy the *d-Hypercyclicity criterion* with respect to the sequence (n).

(3)$\Rightarrow$ (2). This is due to Proposition 2.8.

(2)$\Rightarrow$(1). Suppose that $D^{r_1},\cdots,D^{r_N}$ are d-mixing. From Definition 2.2, we know that every $T_l = D^{r_l}$ is mixing for $1 \leqslant l \leqslant N$. Considering $T_1 = D^{r_1}$ is mixing and using Lemma 2.11, we have that

$$\lim_{n\to\infty} \|\delta_0 \circ (D^{r_1})^n\| = \infty, \tag{3.5}$$

for the linear functional $\delta_0 : H_{v,0} \to \mathbb{C}$, defined by $\delta_0(f) = f(0)$. Using Cauchy inequality

$$\frac{r^n}{n!}|f^{(n)}(0)| \leqslant \max_{|z|=r} |f(z)|,\ n\in\mathbb{N},\ r>0,$$

it turns out that

$$v(r)\frac{r^{nr_1}}{(nr_1)!}|\delta_0 \circ (D^{r_1})^n(f)| = v(r)|f^{(r_1 n)}(0)|\frac{r^{nr_1}}{(nr_1)!} \leqslant v(r)\max_{|z|=r}|f(z)| \leqslant 1,$$

for every $r>0$, $n\in\mathbb{N}$ and $f\in H_{v,0}$ with $\|f\|_v \leqslant 1$.

This implies that for every $nr_1 \in \mathbb{N}$ and $r>0$,

$$v(r)\frac{r^{nr_1}}{(nr_1)!}\|\delta_0 \circ (D^{r_1})^n\| \leqslant 1.$$

Hence for every $n\in\mathbb{N}$,

$$\frac{\|z^{nr_1}\|_v}{(nr_1)!}\|\delta_0 \circ (D^{r_1})^n\| \leqslant 1,\ z\in\mathbb{C}.$$

Using (3.5) it is clear that

$$\lim_{n\to\infty} \frac{\|z^{nr_1}\|_v}{(nr_1)!} = 0. \tag{3.6}$$

Since $D : H_{v,0} \to H_{v,0}$ is continuous, for $C > 1$ we have that

$$\frac{(n+1)!}{n!}\|z^n\|_v = (n+1)\|z^n\|_v = \|D(z^{n+1})\|_v \leqslant C\|z^{n+1}\|_v.$$

Thus it follows that

$$\frac{\|z^n\|_v}{n!} \leqslant C\frac{\|z^{n+1}\|_v}{(n+1)!}. \tag{3.7}$$

Since for each $m \in \mathbb{N}$, there is $n \in \mathbb{N}$ such that $nr_1 < m < (n+1)r_1$, then employing (3.7) we have that

$$\frac{\|z^{nr_1}\|_v}{(nr_1)!} \leqslant C^{m-nr_1}\frac{\|z^m\|_v}{m!} \leqslant C^{r_1}\frac{\|z^m\|_v}{m!},$$

and

$$\frac{\|z^m\|_v}{m!} \leqslant C^{(n+1)r_1-m}\frac{\|z^{(n+1)r_1}\|}{((n+1)r_1)!} \leqslant C^{(n+1)r_1-nr_1}\frac{\|z^{(n+1)r_1}\|}{((n+1)r_1)!} = C^{r_1}\frac{\|z^{(n+1)r_1}\|}{((n+1)r_1)!}.$$

From the above two inequalities and (3.6), it follows that

$$\lim_{m\to\infty} \frac{\|z^m\|_v}{m!} = 0.$$

That is the desired condition (3.1).

(3)$\Rightarrow$(4). Suppose that $D^{r_1},\cdots,D^{r_N}$ satisfy the *d-Hypercyclicity criterion* with respect to the sequence (n). If (n_k) is any subsequence of (n), $D^{r_1},\cdots,D^{r_N}$ clearly satisfy the *d-Hypercyclicity criterion* with respect to (n_k) and hence by Proposition 2.8 it follows that $\left((D^{r_1})^{n_k}\right)_{k=1}^{\infty},\cdots,\left((D^{r_N})^{n_k}\right)_{k=1}^{\infty}$ are d-mixing. By Remark 2.5 and Proposition 2.3, we have that $D^{r_1},\cdots,D^{r_N}$ are hereditarily densely d-hypercyclic with respect to (n).

(4)$\Rightarrow$(2). Suppose that $D^{r_1},\cdots,D^{r_N}$ are hereditarily densely d-hypercyclic with respect to the sequence (n). That is, for every subsequence (n_k) of (n) the set of d-universal vectors for $\left((D^{r_1})^{n_k}\right)_{k=1}^{\infty},\cdots,\left((D^{r_N})^{n_k}\right)_{k=1}^{\infty}$ is a dense set. By Proposition 2.3 we have that $\left((D^{r_1})^{n_k}\right)_{k=1}^{\infty},\cdots,\left((D^{r_N})^{n_k}\right)_{k=1}^{\infty}$ are d-transitive. Hence $D^{r_1},\cdots,D^{r_N}$ are d-transitive for every subsequence of (n). Further by Remark 2.5 it follows that $D^{r_1},\cdots,D^{r_N}$ are d-mixing. This completes the proof. $\square$

In particular, combining Theorem 1.1 and Theorem 1.2 with Theorem 3.1, we deduce interesting equivalently dynamical characterizations between hypercyclicity (mixing) of a single differentiation operator and disjoint mixing of tuple $D^{r_1},\cdots,D^{r_N}$ for $1 \leqslant r_1 < r_2 < \cdots < r_N$.

THEOREM 3.2. *Let $1 \leqslant r_1 < r_2 < \cdots < r_N$, where $r_i \in \mathbb{N}$, $i = 1, \cdots, N$. Assume that the differentiation operator $D : H_{v,0} \to H_{v,0}$ is continuous. The following conditions are equivalent,*

(1) D satisfies the hypercyclicity criterion.

(2) D is hypercyclic on H_v^0.

(3)

$$\lim_{n\to\infty} \frac{\|z^n\|_v}{n!} = 0;$$

(4) D is topologically mixing;

(5) $D^{r_1}, \cdots, D^{r_N}$ are d-mixing;

(6) $D^{r_1}, \cdots, D^{r_N}$ satisfy the d-Hypercyclicity criterion with respect to the sequence (n);

(7) $D^{r_1}, \cdots, D^{r_N}$ are hereditarily densely d-hypercyclic with respect to the sequence (n).

Furthermore, based on the results in Theorem 3.1, the following corollary holds.

COROLLARY 3.3. *Let $1 \leqslant r_1 < r_2 < \cdots < r_N$, and let $\lambda_1, \cdots, \lambda_N$ be unimodular scalars, where $r_i \in \mathbb{N}$, $i = 1, \cdots, N$. Assume that the differentiation operator $D : H_{v,0} \to H_{v,0}$ is continuous. Then the following statements are equivalent,*

(1)

$$\lim_{n\to\infty} \frac{\|z^n\|_v}{n!} = 0;$$

(2) $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ are d-mixing;

(3) $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ satisfy the d-Hypercyclicity criterion with respect to the sequence (n);

(4) $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ are hereditarily densely d-hypercyclic with respect to the sequence (n).

Proof. Let $T_1 = \lambda_1 D^{r_1}, \cdots, T_N = \lambda_N D^{r_N}$, and they clearly satisfy the *d-Hypercyclicity criterion* with respect to the sequence (n), if we let $X_0 = \cdots = X_N = \operatorname{span}\{1, z, z^2, \cdots\}$, and consider for each $1 \leqslant l \leqslant N$ and each $n \in \mathbb{N}$ the linear map

$$S_{l,n} z^s := \frac{1}{\lambda_l^n} \frac{z^{s+r_l n}}{(s+1)(s+2)\cdots(s+r_l n)}, \; s = 0, 1, 2 \cdots$$

Then the remaining proof is similar to Theorem 3.1, so we omit the details. $\square$

The corollary below generalizes [4, Corollary 2.5], we include the proof for readers' convenience.

COROLLARY 3.4. *Let v be a weight such that the differentiation operator $D: H_{v,0} \to H_{v,0}$ is continuous. If there are $A > 0, \alpha \geqslant 1, r_0 > 0$ such that $v(r) \leqslant A\exp(-\alpha r)$ for each $r \geqslant r_0$, then $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ are d-mixing on $H_{v,0}$. In particular, $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ are d-mixing for $v(r) = \exp(-\alpha r)$, $\alpha \geqslant 1$, where $|\lambda_i| = 1$, $i = 1, \cdots, N$. Also, $D^{r_1}, \cdots, D^{r_N}$ are d-mixing on $H_{v,0}$ and then $D^{r_1}, \cdots, D^{r_N}$ are d-mixing for $v(r) = \exp(-\alpha r)$, $\alpha \geqslant 1$.*

Proof. Combining Theorem 3.1 and Corollary 3.3, we only need to show (3.1) holds. Since $v(r)$ is non-increasing, and then we have that

$$\begin{aligned} \|z^n\|_v &= \sup_{z\in\mathbb{C}} v(|z|)|z|^n = \sup_{r\geqslant 0} r^n v(r) = \sup_{0\leqslant r<r_0} r^n v(r) + \sup_{r\geqslant r_0} r^n v(r) \\ &\leqslant r_0^n v(0) + A\sup_{r\geqslant r_0} r^n e^{-\alpha r} = r_0^n v(0) + A\frac{n^n e^{-n}}{\alpha^n}. \end{aligned}$$

By Stirling's formula it follows that (3.1) holds. This completes the proof. □

4. Disjoint hypercyclic differentiation operators

In this section, we offer a sufficient condition ensuring the tuple $D^{r_1}, \cdots, D^{r_N}$ satisfies the d-Hypercyclicity criterion, hence they are also disjoint hypercyclic.

THEOREM 4.1. *Let $1 \leqslant r_1 < r_2 < \cdots < r_N$, where $r_i \in \mathbb{N}$, $i = 1, \cdots, N$ and $N \geqslant 2$. Assume that the differentiation operator $D: H_{v,0} \to H_{v,0}$ is continuous. If for every $\varepsilon > 0$ and $k \in \mathbb{N}$ there exists $m \in \mathbb{N}$ satisfying*

$$\|z^{r_l m+k+1}\|_v < (r_l m + k + 1)!\varepsilon \quad (1 \leqslant l \leqslant N), \tag{4.1}$$

and

$$\|z^{(r_l - r_i)m+k+1}\|_v < ((r_l - r_i)m + k + 1)!\varepsilon \quad (1 \leqslant i < l \leqslant N). \tag{4.2}$$

Then $D^{r_1}, \cdots, D^{r_N}$ satisfy the d-Hypercyclicity criterion. In particular, $D^{r_1}, \cdots, D^{r_N}$ are d-hypercyclic.

Proof. Denote $T_1 = D^{r_1}, \cdots, T_N = D^{r_N}$. Since $D: H_{v,0} \to H_{v,0}$ is continuous, then (3.2) still holds for $C \geqslant 1$. From the conditions (4.1) and (4.2), for every $k \in \mathbb{N}$, there exists a strictly increasing sequence $(n_k) \subset \mathbb{N}$ with $n_{k+1} > n_k + k + 1$ such that

$$\frac{\|z^{r_l n_k+k+1}\|_v}{(r_l n_k + k + 1)!} < \frac{1}{kC^{k+1}}, \ (1 \leqslant l \leqslant N), \tag{4.3}$$

and

$$\frac{\|z^{(r_l - r_i)n_k+k+1}\|_v}{((r_l - r_i)n_k + k + 1)!} < \frac{1}{kC^{k+1}}, \ (1 \leqslant i < l \leqslant N). \tag{4.4}$$

We will show that $D^{r_1},\cdots,D^{r_N}$ satisfy the *d-Hypercyclicity criterion* for the strictly increasing sequence (n_k). Similarly, take dense subsets $X_0 = X_1 = \cdots = X_N = \operatorname{span}\{1,z,z^2,\dots\}$ of X. It is obvious that the condition (i) of Definition 2.6 holds. For each $1 \leqslant l \leqslant N$ and each $k \in \mathbb{N}$, define the mappings $S_{l,k} : X_l \to X$ by

$$S_{l,k}z^s := \frac{z^{s+r_l n_k}}{(s+1)(s+2)\cdots(s+r_l n_k)},\ s = 0,1,2\cdots. \tag{4.5}$$

Next we show that

(ii) $S_{l,k} \underset{k\to\infty}{\to} 0$ pointwise on $X_l(1 \leqslant l \leqslant N)$.

Choose $s \in \mathbb{N}\cup\{0\}$ and take $k \geqslant s$. From (4.5) it is obvious that

$$S_{l,k}(z^s) = \frac{z^{s+r_l n_k}}{(s+1)\cdots(s+r_l n_k)} = \frac{s!}{(s+r_l n_k)!} z^{s+r_l n_k}, \tag{4.6}$$

and

$$D^{k+1-s}(z^{r_l n_k+k+1}) = \frac{(r_l n_k+k+1)!}{(s+r_l n_k)!} z^{s+r_l n_k} = \frac{(r_l n_k+k+1)!}{s!} S_{l,k}(z^s). \tag{4.7}$$

From (4.6), (4.7), (3.2), (4.3) and $C \geqslant 1$ we have that

$$\begin{aligned}\|S_{l,k}(z^s)\|_v &= \frac{s!}{(s+r_l n_k)!}\|z^{s+r_l n_k}\|_v = \frac{s!}{(r_l n_k+k+1)!}\|D^{k+1-s}(z^{r_l n_k+k+1})\|_v \\ &\leqslant s!C^{k+1-s}\frac{\|z^{r_l n_k+k+1}\|_v}{(r_l n_k+k+1)!} \leqslant s!C^{k+1}\frac{\|z^{r_l n_k+k+1}\|_v}{(r_l n_k+k+1)!} \\ &< s!\frac{1}{k} \to 0,\ \ k\to\infty. \end{aligned} \tag{4.8}$$

Then the condition (ii) follows from (4.8).

It remains to verify that

(iii) $(T_l^{n_k}S_{i,k} - \delta_{i,l}Id_{X_l}) \underset{n\to\infty}{\to} 0$ pointwise on $X_l(1 \leqslant i \leqslant N)$,

which is immediate when $i = l$ since $T_l^{n_k}S_{l,k} = Id_{X_l}(1 \leqslant l \leqslant N)$.

Now, for $1 \leqslant i < l \leqslant N$ it follows that $r_i < r_l$. On the one hand, if we choose n_k large enough such that $n_k(r_l - r_i) > s$, it follows that

$$\|T_l^{n_k}S_{i,k}z^s\|_v = \left\|(D^{r_l})^{n_k}\left(\frac{z^{s+r_i n_k}}{(s+1)(s+2)\cdots(s+r_i n_k)}\right)\right\|_v = 0, \tag{4.9}$$

pointwise on $X_l(1 \leqslant i \leqslant N)$. On the other hand, employing the similar lines of (4.8), we have that

$$\begin{aligned}\|T_i^{n_k}S_{l,k}z^s\|_v &= \left\|(D^{r_i})^{n_k}\left(\frac{z^{s+r_l n_k}}{(s+1)(s+2)\cdots(s+r_l n_k)}\right)\right\|_v = s!\frac{\|z^{s+(r_l-r_i)n_k}\|_v}{(s+(r_l-r_i)n_k)!} \\ &= s!\frac{\|D^{k+1-s}(z^{(r_l-r_i)n_k+k+1})\|_v}{((r_l-r_i)n_k+k+1)!} \leqslant s!C^{k+1-s}\frac{\|z^{(r_l-r_i)n_k+k+1}\|_v}{((r_l-r_i)n_k+k+1)!} \\ &\leqslant s!C^{k+1}\frac{\|z^{(r_l-r_i)n_k+k+1}\|_v}{((r_l-r_i)n_k+k+1)!} < s!\frac{1}{k} \to 0,\ \ k\to\infty, \end{aligned} \tag{4.10}$$

pointwise on $X_l(1 \leqslant i \leqslant N)$, where the last line follows from (4.4). Hence the condition (iii) follows from (4.9) and (4.10). Therefore $D^{r_1}, \cdots, D^{r_N}$ satisfy the *d-Hypercyclicity criterion* with respect to (n_k). Particularly, $D^{r_1}, \cdots, D^{r_N}$ are d-hypercyclic. This finishes the proof. $\square$

THEOREM 4.2. *Given $N \geqslant 2$ unimodular scalars $\lambda_1, \cdots, \lambda_N$, $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ also satisfy the d-Hypercyclicity criterion under the assumptions of Theorem 4.1. Particularly, $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ are d-hypercyclic on $H_{v,0}$.*

Proof. $T_1 = \lambda_1 D^{r_1}, \cdots, T_N = \lambda_N D^{r_N}$ satisfy the *d-Hypercyclicity criterion* with respect to the increasing sequence of positive integer (n_k) that we used in Theorem 4.1. We let $X_0 = \cdots = X_N = \operatorname{span}\{1, z, z^2, \cdots\}$, and consider for each $1 \leqslant l \leqslant N$ and each $k \in \mathbb{N}$ the linear mappings

$$S_{l,k} z^s := \frac{1}{\lambda_l^{n_k}} \frac{z^{s+r_l n_k}}{(s+1)(s+2)\cdots(s+r_l n_k)}, \ s = 0, 1, 2 \cdots.$$

Then the proof can be analogously obtained, so we omit the details. $\square$

5. Disjoint supercyclic differentiation operators

Here we include a parallelly sufficient condition ensuring the disjoint supercyclicity of N differentiation operators for the sake of completeness.

THEOREM 5.1. *Let $1 \leqslant r_1 < r_2 < \cdots < r_N$, where $r_i \in \mathbb{N}$, $i = 1, \cdots, N$ and $N \geqslant 2$. Assume that the differentiation operator $D : H_{v,0} \to H_{v,0}$ is continuous. If for every $\varepsilon > 0$ and $k \in \mathbb{N}$ there exists $m \in \mathbb{N}$ satisfying*

$$\|z^{(r_l - r_i)m+k+1}\|_v < ((r_l - r_i)m + k + 1)!\varepsilon \ \ (1 \leqslant i < l \leqslant N). \tag{5.1}$$

Then $D^{r_1}, \cdots, D^{r_N}$ are d-supercyclic.

Proof. Denote $T_1 = D^{r_1}, \cdots, T_N = D^{r_N}$. Since $D : H_{v,0} \to H_{v,0}$ is continuous, then (3.2) still holds for $C \geqslant 1$. From the condition (5.1), for every $k \in \mathbb{N}$, there exists a strictly increasing sequence $(n_k) \subset \mathbb{N}$ with $n_{k+1} > n_k + k + 1$ such that

$$\frac{\|z^{(r_l - r_i)n_k+k+1}\|_v}{((r_l - r_i)n_k + k + 1)!} < \frac{1}{kC^{k+1}}, \ (1 \leqslant i < l \leqslant N). \tag{5.2}$$

Next we verify D^{r_1}, $\cdots$, D^{r_N} satisfy d-*Supercyclicity criterion* with respect to (n_k). Similarly, take dense subsets $X_0 = X_1 = \cdots X_N =$ span$\{1, z, z^2, \ldots\}$ of X, and consider the mappings

$$S_l : X_l \to X (1 \leqslant l \leqslant N),$$

given by

$$S_l z^s := \frac{z^{s+r_l}}{(s+1)(s+2)\cdots(s+r_l)}, \ s = 0, 1, 2 \cdots. \tag{5.3}$$

Thus $T_l S_l = Id$ on $X_l (l \leqslant l \leqslant N)$. Now, for $1 \leqslant i < l \leqslant N$ we have $r_i < r_l$. On the one hand, for a fixed $s \in \mathbb{N}$, by (5.3), if we choose n_k large enough such that $n_k(r_l - r_i) > s$, it follows that

$$\|T_l^{n_k} S_i^{n_k} z^s\|_v = \|(D^{r_l})^{n_k} \left(\tfrac{z^{s+n_k r_i}}{(s+1)(s+2)\cdots(s+n_k r_i)}\right)\|_v = 0, \tag{5.4}$$

pointwise on $X_i (1 \leqslant i \leqslant N)$. On the other hand, by (4.10) and (5.2) we obtain that

$$\begin{aligned}\|T_i^{n_k} S_l^{n_k} z^s\|_v &= \|(D^{r_i})^{n_k} \left(\frac{z^{s+n_k r_l}}{(s+1)(s+2)\cdots(s+n_k r_l)}\right)\|_v \\ &= s! \frac{\|z^{s+n_k(r_l - r_i)}\|_v}{(s+n_k(r_l - r_i))!} \to 0,\ k \to \infty,\end{aligned}$$

pointwise on $X_l (1 \leqslant l \leqslant N)$. Then the condition (i) of Definition 2.9 is true.

Next we show the condition (ii) of Definition 2.9 also holds. Take $y_0, \cdots, y_N \in \operatorname{span}\{1, z, z^2, \cdots\}$. We can easily pick $q \in \mathbb{N}$ large enough so that

$$y_i = \sum_{j=0}^{q} y_{i,j} z^j,\ 0 \leqslant i \leqslant N.$$

Let $n_k \geqslant q$ large enough, by (5.3) it follows that

$$\begin{aligned}&\|T_l^{n_k} y_0\|_v \cdot \|\sum_{s=1}^{N} S_s^{n_k} y_s\|_v \\ = &\|\sum_{j=0}^{q} \frac{j!}{(j - n_k r_l)!} y_{0,j} z^{j - n_k r_l}\|_v \cdot \|\sum_{s=1}^{N} \sum_{j=0}^{q} \frac{j!}{(j + n_k r_s)!} y_{s,j} z^{j + n_k r_s}\|_v = 0.\end{aligned}$$

This completes the proof. $\square$

REMARK 5.2. Note the assumptions in Theorem 3.1 are stronger than those in Theorem 4.1 and Theorem 5.1.

THEOREM 5.3. *If the conditions in Theorem 5.1 hold. Then for $N \geqslant 2$ unimodular scalars $\lambda_1, \cdots, \lambda_N$. The operators $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ satisfy the d-Supercyclicity criterion. Particularly, $\lambda_1 D^{r_1}, \cdots, \lambda_N D^{r_N}$ are d-supercyclic on $H_{v,0}$.*

Proof. $T_1 = \lambda_1 D^{r_1}, \cdots, T_N = \lambda_N D^{r_N}$ satisfy the *d-Supercyclicity criterion* with respect to the increasing sequence of positive integer (n_k) that we used in Theorem 5.1. Still letting $X_0 = \cdots = X_N = \operatorname{span}\{1, z, z^2, \cdots\}$, and considering for each $1 \leqslant l \leqslant N$ and each $k \in \mathbb{N}$, define the linear mappings

$$S_{l,k} z^s := \frac{1}{\lambda_l^{n_k}} \frac{z^{s + r_l n_k}}{(s+1)(s+2)\cdots(s+r_l n_k)},\ s = 0, 1, 2 \cdots.$$

Then the proof is similar to the lines of Theorem 5.1, so we omit the details. $\square$

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