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SOME C^*–ALGEBRAS WHOSE EXT IS NOT A GROUP

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Abstract. It has been known since 1977 that, for a unital, separable C^*-algebra A, $Ext_u(A)$ is a group if and only if every unital $*$-monomorphism of A with values in the Calkin algebra $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ has a unital completely positive lifting to $\mathbb{B}(\ell^2)$. While Ext_u is a group for all C^*-algebras with the Local Lifting Property, the information in the opposite direction is rather scarce, with only two examples being known to this day whose Ext_u is *not* a group.

In this note we present several new examples of separable C^*-algebras whose Ext_u is not a group. These examples are a consequence of the existence of a finite dimensional operator system in the Calkin algebra whose identity map has no completely positive lifting to $B(\ell^2)$.

1. Introduction

Lifting a map $\varphi : A \to B/J$ from a C^*-algebra A with values in a quotient C^*-algebra B/J means the existence of a map $\psi : A \to B$ such that $\varphi = q \circ \psi$, where $q : B \to B/J$ is the quotient map, and has long been an important goal in the theory of C^*-algebras. While a variety of maps are of particular interest, it is the completely positive ones that have received the most attention. We say that a unital C^*-algebra A has the Lifting Property (LP) if, for every unital completely positive (u.c.p.) map $\varphi : A \to B/J$, there exists a u.c.p. map $\psi : A \to B$ such that $\varphi = q \circ \psi$. A u.c.p. map φ with this property is said to be u.c.p. liftable to B. A u.c.p. map $\varphi : A \to B/J$ is locally u.c.p. liftable if, for every finite dimensional operator system $E \subset A$, there exists a u.c.p. map $\psi : E \to B$ such that $\varphi|_E = q \circ \psi$. A C^*-algebra has the Local Lifting Property (LLP) if every u.c.p. map from A to B/J is locally u.c.p. liftable to B.

The origins of this circle of ideas can be traced back to the work of Brown, Douglas and Fillmore [4, 5], Arveson [2, 3], and Voiculescu [19] in the 1970's, which was largely motivated by the study of essentially normal operators, a topic that goes back to Weyl in the early 1900's. The two central questions were: (1) Under what conditions is an essentially normal operator $T \in \mathbb{B}(\ell^2)$ (that is, the image of T in $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ is normal) a compact perturbation of a normal operator in $\mathbb{B}(\ell^2)$? (2) Under what conditions are two normal operators in $\mathbb{B}(\ell^2)$ unitarily equivalent modulo $\mathbb{K}(\ell^2)$? Addressing these questions led to the study of $*$-monomorphisms of abelian C^*-algebras into the Calkin algebra and to the necessity of lifting them to completely positive maps with values in $\mathbb{B}(\ell^2)$. This, in turn, led to the notion of Ext and from there two new and

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important directions emerged. Brown, Douglas and Fillmore pursued the topological aspects of the problem, marking the moment when topological methods were introduced in the study of C^*-algebras, while Arveson initiated the theory of completely positive maps for C^*-algebras. Connecting the two trends was Arveson's theorem stating that $Ext_u(A)$ is a group if and only if every unital $*$-monomorphism of A into the Calkin algebra has a u.c.p. lifting. Subsequently, Anderson [1] found the first example of a C^*-algebra for which Ext_u is not a group.

After Choi and Effros [6] settled in the affirmative the u.c.p. lifting problem for separable nuclear C^*-algebras, by proving in effect that these algebras have the LP, Kirchberg [11] exhibited a non-nuclear C^*-algebra with the LP, namely $C^*(\mathbb{F}_\infty)$, the full C^*-algebra of the free group on countably many generators. To this date, nuclear C^*-algebras and $C^*(\mathbb{F}_\infty)$ are the only basic examples known to have the LP. In [10], Kirchberg introduced the LLP, which turned out to be very successful in establishing surprising connections with tensor products and with Connes' embedding problem. We recall that Kirchberg's (still open) conjecture, stating that every C^*-algebra is QWEP, is equivalent to Connes' embedding problem (every finite von Neumann algebra with separable predual is isomorphic to a subalgebra of R^ω), and, in turn, equivalent to the uniqueness of the C^*-norm on $C^*(\mathbb{F}_\infty) \otimes C^*(\mathbb{F}_\infty)$. We refer the reader to [10] and [17] for more information and a wealth of related results.

Whether $Ext_u(A)$ being a group implies that A has the LLP is an open problem. The best result in this direction was obtained by Kirchberg in [10]: If $Ext_u(cone(A))$ or $Ext_u(susp(A))$ is a group, then A has the LLP. For a long time, Anderson's example [1] was the only C^*-algebra whose Ext_u was known not to be a group. In 2005 ([8]) Haagerup and Thorbjornsen proved the same for $Ext_u(C^*_{red}(\mathbb{F}_2))$. To this date, these two are the only known examples with this property.

The goal of this paper is to present some new examples of C^*-algebras whose Ext_u is not a group. The examples we present in Proposition 3.4 are based on a finite dimensional operator system $\mathbb{E}$ of the Calkin algebra whose identity map has no u.c.p. lifting to $B(\ell^2)$, and whose existence is proved in Corollary 3.3.

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2. Background and preliminary results

In this section we collect some useful preliminary results. We also present, for the benefit of the reader, the proof of the fact that Ext_u of a C^*-algebra with the LLP is a group (Corollary 2.6). All C^*-algebras except $\mathbb{K}(\ell^2)$ are assumed to be unital.

Whether every locally u.c.p. liftable map from a separable C^*-algebra A with values in B/J is actually u.c.p. liftable is an open question. The next result is Lemma 3.10(ii) in [14].

LEMMA 2.1. *Let A,B,J be C^*-algebras with A unital and separable, B unital, and $J \subset B$ an ideal. Suppose that for every $\varepsilon > 0$, for any finite dimensional operator systems $E \subset F \subset A$, and for every c.p. map $\theta : E \to J$, there exists a c.p. map $\tilde{\theta} : F \to J$*

such that $||\tilde{\theta}|_E - \theta|| < \varepsilon$. *Then every locally u.c.p. liftable map* $\varphi : A \to B/J$ *is u.c.p. liftable.*

The next result shows that the condition in Lemma 2.1 is satisfied if J is nuclear.

LEMMA 2.2. *Let* J *be a nuclear* C^**-algebra, and let* $E \subset F \subset B(H)$ *be finite dimensional operator systems. Let* $\theta : E \to J$ *be a c.p. map. Then, for every* $\varepsilon > 0$*, there exists a c.p. map* $\tilde{\theta} : F \to J$ *such that* $||\tilde{\theta}|_E - \theta|| < \varepsilon$.

Proof. Since E is finite dimensional, its closed unit ball is norm-compact. Hence, since J is a nuclear C^*-algebra, we can find a matrix C^*-algebra M and contractive c.p. maps $\sigma : J \to M$ and $\rho : M \to J$ such that $||\theta - \rho \circ \sigma \circ \theta|| < \varepsilon$. Since M is injective, we can extend $\sigma \circ \theta$ to a c.p. map $\Theta : F \to M$. Then $\tilde{\theta} = \rho \circ \Theta$ is the desired map.
From Lemmas 2.1 and 2.2 we obtain

COROLLARY 2.3. *Let* A, B, J *be* C^**-algebras with* A *unital and separable,* B *unital, and* $J \subset B$ *a nuclear ideal. If* $\varphi : A \to B/J$ *is locally u.c.p. liftable, then it is u.c.p. liftable. In particular, every locally u.c.p. liftable u.c.p. map* $\varphi : A \to \mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ *is u.c.p. liftable.*

We say that a C^*-algebra A has the Weak Expectation Property (WEP) if, for every faithful representation $\pi : A \to B(H_\pi)$, there exists a u.c.p. map $\psi : B(H_\pi) \to \pi(A)''$ such that $\psi(\pi(a)) = \pi(a)$ for all $a \in A$ (see [13]). Such a map ψ is called a weak expectation. Equivalently, A has the WEP if and only if for some faithful representation $A \subset B(H)$, there exists a u.c.p. map $\Psi : B(H) \to A^{**}$ such that $\Psi(a) = \rho(a)$ for all $a \in A$, where ρ is the universal representation. It is therefore obvious that $B(H)$ has the WEP. A is said to be QWEP if it is a quotient of a C^*-algebra with the WEP. Recall that A has the LLP if, for every unital completely positive (u.c.p.) map $\varphi : A \to B/J$ and every finite dimensional operator subsystem $E \subset A$, there exists a u.c.p. map $\psi : E \to B$ such that $q \circ \psi = \varphi$ on E, where $q : B \to B/J$ is the quotient map.
The following proposition contains two fundamental results of Kirchberg [10].

THEOREM 2.4. *(i) If* A *has the LLP and* B *has the WEP, then* $A \otimes_{max} B = A \otimes_{min} B$.
(ii) A *has the WEP if and only if* $A \otimes_{max} C^*(\mathbb{F}_\infty) = A \otimes_{min} C^*(\mathbb{F}_\infty)$.

Two unital $*$-monomorphisms φ, ψ defined on a unital, separable C^*-algebra A with values in the Calkin algebra $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ are said to be equivalent if there exists a unitary operator $u \in \mathbb{B}(\ell^2)$ such that $\psi(x) = q(u)\varphi(x)q(u^*)$ for all $x \in A$, where q denotes the quotient map. The set of equivalence classes is denoted by $Ext_u(A)$ and is a commutative semigroup with respect to the additive operation $[\varphi] + [\psi] = [\varphi \oplus \psi]$. A unital $*$-monomorphism φ is called trivial if there exists a unital $*$-monomorphism $\psi : A \to \mathbb{B}(\ell^2)$ such that $\varphi = q \circ \psi$. A consequence of Voiculescu's theorem [19] is that all trivial unital $*$-monomorphisms are equivalent, thus forming the neutral element of $Ext_u(A)$. We refer the reader to [7] for a good introduction to the subject and further references. The following theorem of Arveson [3] highlights the close connection between liftings and the existence of inverses in Ext.

PROPOSITION 2.5. *If A is a unital, separable C^*-algebra, then $Ext_u(A)$ is a group if and only if every unital $*$-monomorphism of A with values in the Calkin algebra $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ is u.c.p. liftable to $\mathbb{B}(\ell^2)$.*

From Proposition 2.5 and Corollary 2.3 we obtain

COROLLARY 2.6. *If A is a unital, separable C^*-algebra with the LLP, then $Ext_u(A)$ is a group.*

Conversely, Kirchberg [10] proved that if $Ext_u(cone(A))$ or $Ext_u(susp(A))$ is a group, then A has the LLP.

The next results are motivated by Arveson's Proposition 2.5. The question which arises naturally about a C^*-algebra whose Ext_u is a group is what (unital) maps other than unital $*$-monomorphisms are u.c.p. liftable? Some simple observations are collected in the next lemma.

LEMMA 2.7. *(i) If A is unital and separable, then there exists a unital $*$-monomorphism from A into $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$.*

(ii) If $Ext_u(A)$ is a group, then every unital $*$-homomorphism from A with values in the Calkin algebra $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ is u.c.p. liftable.

Proof. (i) This fact is undoubtedly folklore, but we include a proof for the sake of completeness. Assume that A is faithfully represented on a separable Hilbert H and denote by $q : \mathbb{B}(\ell^2) \to \mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ the quotient map. If A contains no nonzero compact operator in $B(H)$, then, by identifying H and ℓ^2, we have $ker\ (q) = 0$, so q is a $*$-monomorphism. If $A \cap K(H) \neq \{0\}$, then A is isomorphic to $A \otimes I$ acting on $H \otimes \ell^2$, and which contains no nonzero compact operator.

(ii) Let $\varphi : A \to \mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ be a unital $*$-homomorphism and, by part(i), let $\theta : A \to \mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ be a unital $*$-monomorphism. Then

$$\psi(x) = \begin{pmatrix} \varphi(x) & 0 \\ 0 & \theta(x) \end{pmatrix}$$

is a $*$-monomorphism defined on A with values in $\mathbb{B}(\ell^2 \oplus \ell^2)/\mathbb{K}(\ell^2 \oplus \ell^2)$. Since ψ is u.c.p. liftable, there exists a uc.p. map $\alpha : A \to \mathbb{B}(\ell^2 \oplus \ell^2) = \mathbb{M}_2(\mathbb{B}(\ell^2))$ such that $\psi = (q \otimes id) \circ \alpha$, where $q \otimes id$ is the quotient map from $\mathbb{B}(\ell^2 \oplus \ell^2)$ to $\mathbb{B}(\ell^2 \oplus \ell^2)/\mathbb{K}(\ell^2 \oplus \ell^2)$. It is easy to see that the (1,1) corner of α is the desired lifting of φ.

3. The main results

We begin this section by recalling Proposition 2.1(ii) in [18].

LEMMA 3.1. *If A and B are C^*-algebras, then the C^*-algebras $M_2(A) \otimes_{max} B$ and $M_2(A \otimes_{max} B)$ are isomorphic.*

Given an operator space $E \subset B(H)$, the Paulsen operator system $S_E \subset M_2(B(H))$ associated with E is

$$S_E = \left\{ \begin{pmatrix} \lambda I & a \\ b^* & \mu I \end{pmatrix}; \; a,b \in E, \; \lambda, \mu \in \mathbb{C} \right\}.$$

The following is the main technical result of the paper.

PROPOSITION 3.2. *Suppose that A is a unital QWEP C^*-algebra and let $A = B/J$, where B has the WEP. Denote by $q : B \to A$ the quotient map, and by q_n the quotient map from $M_n(B)$ to $M_n(A)$. If, for every $n \geqslant 1$ and for every finite dimensional operator system $E \subset M_n(A)$, there exists a u.c.p. map $\alpha_n : E \to M_n(B)$ such that $q_n \circ \alpha_n(x) = x$ for every $x \in E$, then A has the WEP.*

Proof. Fix $x_0, \dots, x_{n-1} \in A$ and let $U_0, U_1, U_2, \dots$ be the generators of $C^*(\mathbb{F}_\infty)$, where $U_0 = I$. Recall that we have $||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A \otimes_{min} C^*(\mathbb{F}_\infty)} = ||\varphi||_{cb}$ where $\varphi : \ell_\infty^n \to A$ is defined by $\varphi((\lambda_0, \lambda_1, \dots, \lambda_{n-1})) = \sum_{i=0}^{n-1} \lambda_i x_i$ (page 155 in [16]) and assume without loss of generality that $||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A \otimes_{min} C^*(\mathbb{F}_\infty)} = 1$, making φ a complete contraction. Next, we consider the map $\psi : S_{\ell_\infty^n} \to M_2(A)$ defined by

$$\psi\left(\begin{pmatrix} \lambda I_n & a \\ b^* & \mu I_n \end{pmatrix} \right) = \begin{pmatrix} \lambda I_n & \varphi(a) \\ \varphi(b)^* & \mu I_n \end{pmatrix}$$

It is well known that ψ is u.c.p. (Lemma 8.1 in [15]).

We use the hypothesis to obtain a u.c.p. map $\alpha_2 \circ \psi = \Psi : S_{\ell_\infty^n} \to M_2(B)$ such that $q_2 \circ \Psi = \psi$, where q_2 is the quotient map from $M_2(B)$ onto $M_2(A)$ and note that $M_2(B)$ has the WEP, as well as all $M_n(B)$, $n \geqslant 1$. By taking into account Theorem 4.2(ii) we obtain the composition of the sequence of u.c.p. maps

$$S_{\ell_\infty^n} \otimes_{min} C^*(\mathbb{F}_\infty) \xrightarrow{\Psi \otimes id} M_2(B) \otimes_{min} C^*(\mathbb{F}_\infty) = M_2(B) \otimes_{max} C^*(\mathbb{F}_\infty) \xrightarrow{q_2 \otimes id} M_2(A) \otimes_{max} C^*(\mathbb{F}_\infty)$$

and use Lemma 3.1 to focus on the (1,2) corner of this map, namely $\varphi \otimes id$, which represents a complete contraction from $\ell_\infty^n \otimes_{min} C^*(\mathbb{F}_\infty)$ with values in $A \otimes_{max} C^*(\mathbb{F}_\infty)$.

If we denote by $\{e_0, e_1, \dots, e_{n-1}\}$ the canonical basis of ℓ_∞^n, we get

$$||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A \otimes_{max} C^*(\mathbb{F}_\infty)} = ||\sum_{i=0}^{n-1} \varphi(e_i) \otimes U_i||_{A \otimes_{max} C^*(\mathbb{F}_\infty)} \leqslant ||\sum_{i=0}^{n-1} e_i \otimes U_i||_{\ell_\infty^n \otimes_{min} C^*(\mathbb{F}_\infty)} = 1$$

therefore

$$||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A \otimes_{max} C^*(\mathbb{F}_\infty)} = ||\sum_{i=0}^{n-1} x_i \otimes U_i||_{A \otimes_{min} C^*(\mathbb{F}_\infty)}.$$

The above argument can be repeated identically for $M_n(A)$ instead of A.

Denote $E_1 = \text{span}\{x \otimes U_i : x \in A,\ 0 \leqslant i < \infty\} \subset A \otimes_{min} C^*(\mathbb{F}_\infty)$, and let $E_2 \subset A \otimes_{max} C^*(\mathbb{F}_\infty)$ be the corresponding linear subspace. In the first part of the proof we obtained that θ, the identity map on elementary operators, is a complete isometry between E_1 and E_2. Consider $A \otimes_{max} C^*(\mathbb{F}_\infty)$ faithfully represented on some separable Hilbert space H.

If we view θ as taking values in $B(H)$, then this map extends to a complete contraction Θ with domain $A \otimes_{min} C^*(\mathbb{F}_\infty)$ and values in $B(H)$. Since Θ is unital, it must be a u.c.p. map (2.11 in [15]). Since Θ takes $x \otimes U_i$ to $x \otimes U_i$, the operators $x \otimes U_i$ belong to the multiplicative domain of Θ. By virtue of Theorem 3.18 in [15], Θ becomes a $*$-homomorphism defined on $A \otimes_{min} C^*(\mathbb{F}_\infty)$. Since Θ acts identically on elementary operators, its range is necessarily equal to $A \otimes_{max} C^*(\mathbb{F}_\infty)$. Consequently, Θ is a $*$-homomorphism with domain $A \otimes_{min} C^*(\mathbb{F}_\infty)$ and range $A \otimes_{max} C^*(\mathbb{F}_\infty)$. The conclusion follows.

COROLLARY 3.3. *The Calkin algebra $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ contains a finite dimensional operator system $\mathbb{E}$ with the property that there exists no u.c.p. map $\alpha : \mathbb{E} \to \mathbb{B}(\ell^2)$ such that $q \circ \alpha(x) = x$ for every $x \in \mathbb{E}$, where $q : \mathbb{B}(\ell^2) \to \mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ is the quotient map.*

Proof. It is obvious that the Calkin algebra is QWEP, since $\mathbb{B}(\ell^2)$ clearly has the WEP. The Calkin algebra does not have the WEP, otherwise Proposition 2.4 (i) and (ii) would imply that the sequence

$$0 \to C^*(\mathbb{F}_\infty) \otimes_{min} \mathbb{K}(\ell^2) \to C^*(\mathbb{F}_\infty) \otimes_{min} \mathbb{B}(\ell^2) \to C^*(\mathbb{F}_\infty) \otimes_{min} \mathbb{B}(\ell^2)/\mathbb{K}(\ell^2) \to 0$$

is exact. By [9] this implies that $C^*(\mathbb{F}_\infty)$ is exact, which is a contradiction by [20]. The conclusion follows from Proposition 3.2, after noting that $M_n(\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2))$ and $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ are isomorphic for all n.

At this stage we recall the universal C^*-algebra of an operator system, introduced by Kirchberg and Wassermann. In [12] they proved that, given an operator system E, there exists a C^*-algebra $C^*_u(E)$, unique up to isomorphism, satisfying:

1. There exists a unital completely isometric map $\iota : E \to C^*_u(E)$.
2. $C^*_u(E)$ is the C^*-algebra generated by $\iota(E)$.
3. If $\theta : E \to B$ is a u.c.p. map with values in a C^*-algebra B, then there exists a $*$-homomorphism $\pi : C^*_u(E) \to B$ such that $\theta = \pi \circ \iota$.

We arrived at the main result of this paper.

PROPOSITION 3.4. *(i) If $C^*(\mathbb{E})$ is the sub-C^*-algebra of $\mathbb{B}(\ell^2)/\mathbb{K}(\ell^2)$ generated by $\mathbb{E}$, then $Ext_u(C^*(\mathbb{E}))$ is not a group.*

(ii) If $C^*_u(\mathbb{E})$ is the universal C^*-algebra of $\mathbb{E}$, then $Ext_u(C^*_u(\mathbb{E}))$ is not a group.

(iii) If A is an arbitrary separable C^*-algebra, then there exists a separable C^*-algebra B containing (an isomorphic copy of) A and such that $Ext_u(B)$ is not a group.

Proof. (i) The identity map of $C^*(\mathbb{E})$ represents a $*$-monomorphism of $C^*(\mathbb{E})$ into the Calkin algebra with no u.c.p. lifting to $\mathbb{B}(\ell^2)$ by virtue of Corollary 3.3.

(ii) By the definition of the universal C^*-algebra, there exists a $*$-homomorphism of $C_u^*(\mathbb{E})$ with values in the Calkin algebra taking $\iota(\mathbb{E})$ to $\mathbb{E}$ pointwise. If $Ext_u(C_u^*(\mathbb{E}))$ was a group, then by Lemma 2.7(ii) this homomorphism would lift to $\mathbb{B}(\ell^2)$, thus producing a u.c.p lifting of $\mathbb{E}$.

(iii) By Lemma 2.7(i), the Calkin algebra contains an isomorphic copy of A, call it A_0. Then the C^*-algebra generated by A_0 and $\mathbb{E}$ is the desired algebra B.

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