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ERRATUM TO ESSENTIAL NORM OF WEIGHTED COMPOSITION FOLLOWED AND PROCEEDED BY DIFFERENTIATION OPERATOR FROM BLOCH–TYPE INTO BERS–TYPE SPACES, OPERATORS AND MATRICES, 15 (3) (2021), 853–870

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Abstract. In this note, we provide changes in the proof of Theorem 5 in [1] and present the modified version of this Theorem.

In the proof of the Theorem 5 of our published paper, there are two mistakes as follows:

1. In proof of lower estimate in Page 866, line -4, we defined f_n as follows:

$$f_n(w) = \frac{w^n}{n||w||_{B^\alpha}},$$

and then in Page 867, line 2, we calculate

$$\lim_{n\to\infty} \min_{w\in A_n} |f_n''(w)|(1-|w|^2)^\alpha,$$

that was mistake. Becuse, we had to calculate

$$\lim_{n\to\infty} \min_{w\in A_n} |f_n''(w)|(1-|w|^2)^{\alpha+1},$$

to replace it in Page 867, line 10. In this case we will obtain

$$||DC_\varphi^u D||_e \geqslant 0.$$

So, defining $f_n(w) = \dfrac{w^n}{n||w||_{B^\alpha}}$, we did not get a useful result.
We must define f_n according to [9, Proposition 1, Page 1442], as follows:

$$f_n(w) = \frac{w^n}{||w||_{B^\alpha}}.$$

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Then, we obtain

$$\limsup_{n\to\infty} \min_{w\in A_n} |f_n'(w)|(1-|w|^2)^{\alpha} = 1$$

and

$$\lim_{n\to\infty} \sup_{\varphi(w)\in A_n} |u'(w)|(1-|w|^2)^{\beta}|f_n'(\varphi(w))| = 1.$$

Therefore

$$\begin{aligned} ||DC_{\varphi}^{u}D||_e &\geqslant \lim_{n\to\infty} \sup_{\varphi(w)\in A_n} |u(w)||\varphi'(w)|\frac{(1-|w|^2)^{\beta}}{(1-|\varphi(w)|^2)^{\alpha+1}} \\ &\quad - \lim_{n\to\infty} \sup_{\varphi(w)\in A_n} |u'(w)|\frac{(1-|w|^2)^{\beta}}{(1-|\varphi(w)|^2)^{\alpha}} \\ &= A(u,\varphi,\alpha,\beta) - B(u,\varphi,\alpha,\beta). \end{aligned}$$

2. In proof of upper estimate, in Page 868, line 13, by using the Lemma 3, we have assumed that

$$\sup_{||f||_{B^\alpha}\leqslant 1} \sup_{w\in\mathbb{D}} |u'(w)||((I-L_n)f)'(\varphi(w))|(1-|w|^2)^{\beta} = 0,$$

which is not true, because the Lemma 3, says that

$$\sup_{||f||_{B^\alpha}\leqslant 1} \sup_{|\varphi(w)|\leqslant t} |u'(w)||((I-L_n)f)'(\varphi(w))|(1-|w|^2)^{\beta} = 0,$$

and for $|\varphi(w)| > t$,

$$\sup_{||f||_{B^\alpha}\leqslant 1} \sup_{|\varphi(w)|> t} |u'(w)||((I-L_n)f)'(\varphi(w))|(1-|w|^2)^{\beta} \neq 0.$$

So, we corrected the Theorem 5, as follows:

THEOREM 5. *Let $u \in H(\mathbb{D})$, φ an analytic self-map on $\mathbb{D}$, α and β positive real numbers with $0<\alpha\leqslant 1$ and $DC_{\varphi}^{u}D : B^{\alpha} \to H_{\beta}^{\infty}$ is bounded. Then,*

$$||DC_{\varphi}^{u}D||_e \geqslant \max\left\{\frac{1}{2^{\alpha+3}(\alpha+1)}B(u,\varphi,\alpha,\beta),\ A(u,\varphi,\alpha,\beta)-B(u,\varphi,\alpha,\beta)\right\}$$

and

$$||DC_{\varphi}^{u}D||_e \leqslant A(u,\varphi,\alpha,\beta)+B(u,\varphi,\alpha,\beta),$$

where

$$A(u,\varphi,\alpha,\beta) = \lim_{t\to 1} \sup_{|\varphi(w)|>t} |u(w)||\varphi'(w)|\frac{(1-|w|^2)^{\beta}}{(1-|\varphi(w)|^2)^{\alpha+1}}$$

and

$$B(u,\varphi,\alpha,\beta) = \lim_{t\to 1} \sup_{|\varphi(w)|>t} |u'(w)|\frac{(1-|w|^2)^{\beta}}{(1-|\varphi(w)|^2)^{\alpha}}.$$

REFERENCES

[1] H. VAEZI AND M. NAGHLISAR, *Essential norm of weighted composition followed and proceeded by differentiation operator from Bloch-type into Bers-type spaces*, Oper. Matrices **15**, 3 (2021), 853–870.

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